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η string formation in QCD chiral phase transition*

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Abstract In the paper we discuss the role of the axial $U(1)_{\rm A}$ symmetry in the chiral phase transition using the $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}$ linear sigma model with two massless quark flavors. It is expected that above a certain temperature the axial $U(1)_{\rm A}$ symmetry will be restored. A string-like static solution, the η string can be formed and detected in the ultrarelativistic heavy-ion collision process.

Key words η string, sigma model, chiral phase transition

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1 Introduction

Exploring the phase structure of quantum chromodynamics (QCD) is one of the primary goals of ultrarelativistic heavy-ion collisions. It was argued that at a sufficiently high temperature there should be a transition from ordinary hadronic matter to a chirally symmetric plasma of quark and gluons [1, 2]. For $N_{\rm f}$ massless quark flavors, the QCD Lagrangian has a chiral $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}=SU(N_{\rm f})_{\rm R}\times SU(N_{\rm f})_{\rm L}\times U(1)_{\rm V}\times U(1)_{\rm A}$ symmetry. Here $V=R+L$, while $A=R-L$. The $U(1)_{\rm V}$ symmetry corresponds to baryon number conservation, it is always respected and thus plays no role in the symmetry breaking patterns considered in the following. In vacuum, a non-vanishing expectation value of the quark-antiquark condensate, $\langle\overline{q}_{\rm R}q_{\rm L}\rangle\neq 0$, spontaneously breaks the above symmetry to the diagonal $SU(N_{\rm f})_{\rm V}$ group of vector transformation. This gives to $N_{\rm f}^2$ Goldstone bosons which dominate the low-energy dynamics of the theory. The axial $U(1)_{\rm A}$ symmetry is broken to $Z(N_{\rm f})_{\rm A}$ by a non-vanishing topological susceptibility [3]. Consequently, one of the $N_{\rm f}^2$ Goldstone bosons becomes massive, leaving $N_{\rm f}^2-1$ Goldstone bosons. The $SU(N_{\rm f})_{\rm R}\times SU(N_{\rm f})_{\rm L}\times U(1)_{\rm A}$ group is also explicitly broken by the effects of nonzero quark masses. It was shown by Pisarski and Wilczek that for three or more massless flavors, the phase transition for the restoration of the $SU(N_{\rm f})_{\rm R}\times SU(N_{\rm f})_{\rm L}$ is the first order, while for two massless flavors the phase transition is the second order [1].

The $U(1)_{\rm A}$ symmetry may be effectively restored, if only partially, since the instanton effects will rapidly disappear as the temperature increases. If the chiral condensate $\langle\overline{q}_{\rm R}q_{\rm L}\rangle\neq 0$, also the $U(1)_{\rm A}$ axial symmetry is broken, therefore there are two possibilities: either the $U(1)_{\rm A}$ symmetry is restored at a temperature much greater than the $SU(N_{\rm f})_{\rm R}\times SU(N_{\rm f})_{\rm L}$ symmetry or the two symmetries are restored at (approximately) the same temperature. The lattice gauge theory computations have demonstrated a rapid dropping of the topological susceptibility around the chiral phase transition, seemingly suggesting the simultaneous restoration [4, 5], this is also supported by the random matrix models [6]. On the other hand, the fate of the $U(1)_{\rm A}$ anomaly in nature is not completely clear since instanton liquid model calculations indicate that the topological susceptibility is essentially unchanged at $T_{\rm c}$ [7], also the Lattice results obtained from the $SU(3)$ pure gauge theory show that the topological susceptibility is approximately constant up to the critical temperature $T_{\rm c}$, it has a sharp decrease above the transition, but remains different from zero up to $\sim 1.2T_{\rm c}$ [8]. Additionally, other lattice computations which measure the chiral susceptibility find that the $U(1)_{\rm A}$ symmetry restoration is at or below the 15% level [9, 10]. For simplicity, in this paper we make an assumption

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that the two symmetries are restored at approximately the same temperature and the $U(1)_{\rm A}$ can be effectively restored.

Several years ago, the issue of finding signals for the restoration of chiral symmetry in ultrarelativistic heavy-ion collisions has received considerable attention. For example, the signals for the restoration of the $SU(2)$ chiral symmetry associated with the σ meson have been proposed in Refs. [11–13]. In particular, signals for detecting the effective restoration of the $U(1)_{\rm A}$ chiral symmetry in ultrarelativistic heavy-ion collisions have been proposed by using the full $SU(3)$ linear sigma model at finite temperature [14].

On the other hand, in QCD, in the chiral limit, spontaneous symmetry breaking $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}\to U(N_{\rm f})_{\rm V}$ allows for existence of topological string defects, and formation of topological and non-topological string defects during the chiral transition in QCD has been invoked in Refs. [15, 16]. These defects and their effects can be taken as signals for detecting the corresponding chiral phase in ultrarelativistic heavy-ion collisions as well as in the early universe. In the following, we are going to study the effects from effective restoration of the $U(1)_{\rm A}$ symmetry by using the $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}$ linear sigma model with chiral symmetry for two flavors.

2 General formalism

The Lagrangian of the $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}$ linear sigma model is given by [11]

$$\begin{aligned}(\Phi) = {}& {\rm Tr}(\partial_\mu\Phi^+\partial^\mu\Phi - m^2\Phi^+\Phi) - \lambda_1[{\rm Tr}(\Phi^+\Phi)]^2 - \\ & \lambda_2{\rm Tr}(\Phi^+\Phi)^2 + c[\det(\Phi)+\det(\Phi^+)] + \\ & {\rm Tr}[H(\Phi+\Phi^+)]. \end{aligned} \tag{1}$$

Φ is a complex $N_{\rm f}\times N_{\rm f}$ matrix parameterizing the scalar and pseudoscalar mesons,

$$\Phi = T_{\rm a}\phi_{\rm a} = T_{\rm a}(\sigma_{\rm a}+{\rm i}\pi_{\rm a}), \tag{2}$$

where $\sigma_{\rm a}$ are the scalar ($J^{\rm p}=0^+$) fields and $\pi_{\rm a}$ are the pseudoscalar ($J^{\rm p}=0^-$) fields. The $N_{\rm f}\times N_{\rm f}$ matrix H breaks the symmetry explicitly and is chosen as

$$H = T_{\rm a}h_{\rm a}, \tag{3}$$

where $h_{\rm a}$ are the external fields, $a=0,1,\cdots,N_{\rm f}^2-1$ and $T_{\rm a}, a\neq 0$ are a basis of generators for the $SU(N_{\rm f})$ Lie algebra. $T_0=1$ is the generator for the $U(1)_{\rm A}$ Lie algebra.

In the above model, the determinant terms correspond to the $U(1)_{\rm A}$ anomaly, as shown by 't Hooft [3], they arise from instantons. These terms are invariant under $SU(N_{\rm f})_{\rm R}\times SU(N_{\rm f})_{\rm L}\cong SU(N_{\rm f})_{\rm V}\times SU(N_{\rm f})_{\rm A}$, but break the $U(1)_{\rm A}$ symmetry of the Lagrangian explicitly. The last term in Eq. (1) which is due to nonzero quark masses breaks the axial and possibly the $SU(N_{\rm f})_{\rm V}$ vector symmetry explicitly.

When $h_{\rm a}=0, c=0$, for $m^2<0$ the global $SU(N_{\rm f})_{\rm V}\times U(N_{\rm f})_{\rm A}$ symmetry is broken to $SU(N_{\rm f})_{\rm V}$, and $\langle\Phi\rangle$ develops a non-vanishing vacuum expectation value, $\langle\Phi\rangle = T_0\overline{\sigma}_0$. Spontaneously breaking $U(N_{\rm f})_{\rm A}$ beads to $N_{\rm f}^2$ Goldstone bosons which form a pseudoscalar, $N_{\rm f}^2$ dimensional multiplet. However when $h_{\rm a}=0, c\neq 0$, the $U(1)_{\rm A}$ is further broken to $Z(N_{\rm f})$ by the axial anomaly, and $SU(N_{\rm f})_{\rm V}\times SU(N_{\rm f})_{\rm A}$ is still the symmetry of the Lagrangian. A non-vanishing $\langle\Phi\rangle$ spontaneously breaks the symmetry to $SU(N_{\rm f})_{\rm V}$, with the appearance of $N_{\rm f}^2-1$ Goldstone bosons which form a pseudoscalar, $N_{\rm f}^2-1$ dimensional multiplet. The $N_{\rm f}^2$ pseudoscalar meson is no longer massless, because the $U(1)_{\rm A}$ symmetry is already explicitly broken, e.g for $N_{\rm f}=2$, the η meson is massive compared with other pseudoscalar mesons. The symmetry is in addition explicitly broken by non-zero quark masses making all the Goldstone bosons massive.

In this paper, since we only concentrate on the effects of the effective restoration of the $U(1)_{\rm A}$ symmetry, we can ignore the possible effects of the restoration of $SU(2)_{\rm R}\times SU(2)_{\rm L}$, this implies that we can forget π and a_0 fields, keeping only the σ and η mesons which are used to specify the $U(1)_{\rm A}$ phase. With this restriction on Φ, the effective Lagrangian we adopt here is

$$\begin{aligned}(\Phi) = {}& {\rm Tr}(\partial_\mu\Phi^+\partial^\mu\Phi - m^2\Phi^+\Phi) - \lambda_1[{\rm Tr}(\Phi^+\Phi)]^2 - \\ & \lambda_2{\rm Tr}(\Phi^+\Phi)^2 + c[\det(\Phi)+\det(\Phi^+)], \end{aligned} \tag{4}$$

where $\Phi = \dfrac{1}{2}(\sigma+{\rm i}\eta)1$.

One expects that above a certain critical temperature $T_{U(1)}$, also the axial $U(1)_{\rm A}$ symmetry will be effective restored. We will try to see if this transition has to do something with the usual chiral transition. As mentioned above, because the chiral condensate $\langle\overline{q}_{\rm R}q_{\rm L}\rangle\neq 0$ also breaks the $U(1)_{\rm A}$ axial symmetry, the scenario with $T_{U(1)}<T_{\rm c}$ is immediately ruled out. Therefore, we are left essentially with the two following scenarios.

SCENARIO 1: $T_{U(1)}\gg T_{\rm c}$, that is, the complete $U(N_{\rm f})_{\rm R}\times U(N_{\rm f})_{\rm L}$ chiral symmetry is restored only well inside the quark gluon plasma domain. In the case of $N_{\rm f}=2$, at $T=T_{\rm c}$ the restoration of $SU(2)_{\rm R}\times SU(2)_{\rm L}\sim O(4)$. Therefore, it is possible to construct two flavor linear sigma models by using only four lightest mesons $\pi^{\pm}$, π^0 and σ. We take

$$\Phi=\frac{1}{2}\sigma+\frac{\mathrm{i}}{2}\vec{\pi}\cdot\vec{\tau}, \tag{5}$$

and the Lagrangian takes the form

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}- \\ & \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-f_\pi^2)^2+H\sigma\ .\end{aligned} \tag{6}$$

During chiral symmetry breaking, the eta string [15] is expected to be produced and the eta string configurations are specified by the $SU(2)_{\mathrm{A}}$ phase.

SCENARIO 2: $T_{U(1)}\sim T_{\mathrm{c}}$. If $T_{U(1)}=T_{\mathrm{c}}$, then in the case of $N_{\mathrm{f}}=2$ light flavors, the restored symmetry across the transition is $U(2)_{\mathrm{R}}\times U(2)_{\mathrm{L}}$, the chiral phase transition can be described by Eq. (4). During chiral symmetry breaking, the field σ takes on a nonvanishing expectation value, which breaks $U(2)_{\mathrm{R}}\times U(2)_{\mathrm{L}}$ down to $U(2)_{\mathrm{V}}$. This results in a massive σ and four massless Goldstone bosons. In addition, we will demonstrate below that there are both a static string-like solution, the η string and a static kink-like solution, the domain walls are expected to be produced during this phase transition.

The η string is a static configuration of the Lagrangian of Eq. (4) with $c=0$. In our discussion of the η string and domain walls it proves convenient to define the new fields

$$\phi=\frac{\sigma+\mathrm{i}\eta}{\sqrt{2}}\ . \tag{7}$$

The linear sigma model in Eq. (4) now can be rewritten as

$$\mathcal{L}=(\partial_\mu\phi)^*(\partial^\mu\phi)-\lambda\left(\phi^*\phi-\frac{v^2}{2}\right)^2, \tag{8}$$

where $v^2=\dfrac{-m^2}{\lambda}$ and $\lambda=\lambda_1+\dfrac{\lambda_2}{2}$. For static configurations, the energy functional corresponding to the above Lagrangian is

$$E=\int\mathrm{d}^3x\left[\nabla\phi^*\nabla\phi+\lambda\left(\phi^*\phi-\frac{v^2}{2}\right)^2\right]. \tag{9}$$

The time independent equation of motion is

$$\nabla^2\phi=2\lambda\left(\phi^*\phi-\frac{v^2}{2}\right)\phi. \tag{10}$$

The η string solution extremising the energy functional of Eq. (9) is given by [15, 17]

$$\phi=\frac{v}{\sqrt{2}}\rho(r)\exp(\mathrm{i}\theta), \tag{11}$$

where $\rho(r)=[1-\exp(-\mu r)]$, the coordinates rand are polar coordinates in the x-y plane, the η string is assumed to lie along the z axis and $\mu^2=\lambda\dfrac{89}{144}v^2$. The energy per unit length of the string is

$$E=[0.75+\lg(\mu R)]\pi v^2. \tag{12}$$

For global symmetry in general the energy density of the string solution is logrithmically divergent, R is introduced as a cutoff which in the following numerical calculation we will take to be $O(\mathrm{fm})$.

The evolution of temperature in the small region of critical temperature at the central rapidity in the center of mass frame can be estimated by

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t}\approx-\frac{T(t)}{\tau_{\mathrm{Q}}}\ , \tag{13}$$

where $\mathrm{d}T(t)/\mathrm{d}t\approx 2-6$ MeV/fm at $T_{\mathrm{c}}=140$ MeV [18–25]. This suggests $\tau_{\mathrm{Q}}\approx 20\sim 70$ fm . Kibble-Zurek mechanism states that in a second order phase transition [23, 24], when the QGP fireball cools through the critical temperature, the chiral symmetry will be spontaneously broken and domains of similar orientations will be formed. At the boundaries, where different causality disconnected regions meet, as the order parameter does not necessarily match, this leads to the formation of topological defects [26].

In non-equilibrium phase transition, when the temperature goes below T_{c}, the order parameter fluctuates simultaneously and independently in many parts of the system, and many independent small regions of new low-temperature phase starts to form. Subsequently during further cooling, these independent regions grow together to form the new broken symmetry phase. The fluctuating configuration of the order parameter is frozen out at $\varepsilon_z=(1-T_z/T_{\mathrm{c}})>0$, and the topological defects are formed.

If the chiral symmetry breaking occurs in QGP phase transition, the topological defects (the η string loops) will produce. The characteristic correlation length ξ_z corresponding to ε_z is

$$\xi_z=\xi_0(\tau_{\mathrm{Q}}/\tau_0)^{1/4}\approx(\tau_{\mathrm{Q}}/m_\sigma^3)^{1/4}, \tag{14}$$

where $\xi_0\approx\tau_0\approx 1/m_\sigma$. For $m_\sigma=400$–600 MeV, $\xi_z\approx 0.918$–1.26 fm ($m_\sigma=600$ MeV), $\xi_z\approx 1.24$–1.70 fm ($m_\sigma=400$ MeV). At Zurek temperature,

$$T_z=T_{\mathrm{c}}(1-\sqrt{\tau_0/\tau_{\mathrm{Q}}}), \tag{15}$$

the η string is formed. $T_z\approx 118$–128 MeV ($m_\sigma=400$ MeV), $T_z\approx 122$–130 MeV ($m_\sigma=600$ MeV). T_z is just in the region of QGP freeze out temperature, $T_f\approx 110$–130 MeV. So we get the result $T_{\mathrm{f}}\approx T_z$. For simplicity, in the following discussion we use the parameters: $m_\sigma=400$ MeV, $v^2=m^2/\lambda=90$ MeV, $\lambda=\lambda_1+\lambda_2/2=9.877$. In heavy ion collisions, the string loops are formed in various independent domains, each domain has the size of $\sim\xi_z^2$, the π string energy of unit length is $E \approx 180$ MeV/fm. The distribution of the string loops can be obtained from the non-equilibrium evolution of cosmic string loops [27],

$$n(l) = K \frac{\exp(-\beta l)}{\xi_z^{3/2} l^{5/2}}, \tag{16}$$

where $K \approx 1$. $\beta \approx \Gamma \approx m_\sigma$ can be approximately taken as the width of the sigma decay width. In heavy-ion collisions only the string loops will be formed. The shortest loops have the length of $l_0 = 2\pi/\mu \approx 5.6$ fm. The number of loops is

$$N \approx V_{\rm f} \int_{l_0}^{2\pi R_{\rm f}} n(l) {\rm d}l \ , \tag{17}$$

where the parameter $R_{\rm f}$ is approximately the radius of the hadronic phase. $R_{\rm f} \approx 10$ fm at RHIC, and $R_{\rm f} \approx 18$ fm at LHC [20]. The volume of hadronic phase is about $V_{\rm f} \approx 4\times10^3$ fm^3 and 2.4×10^4 fm^3 at RHIC and LHC respectively. The total number of loops is $N_{\rm RHIC} \approx 20$–30, $N_{\rm LHC} \approx 70$–110. Below $T_{\rm f}$, all eta strings will decay into the sigma particles and eta mesons. To estimate the number of the particle produced we notice that for the ansatz Eqs. (7) and (12), the sigma field in Eq. (7) contributes about 50% of the total energy of the string. Due to energy conservation half of the string energy should convert into that carried by the sigma particles. The remaining 50% of the string energy will go to the eta mesons. One expects the mesons produced from the decay of the eta strings with length l have a typical momentum $p \sim 1/l \approx 35$ MeV. The total number of sigma mesons is about 50–90 in RHIC Pb-Pb and 190–300 in LHC Pb-Pb collisions. The total number of eta mesons from the decay of eta string is just the same as sigma mesons.

As mentioned above it is expected that the eventually resultant pion and eta spectra will have a non-thermal enhancement at a low momentum region because all produced eta mesons from eta strings are distributed at low momentum. The eta string and sigma particles are dominant in the low momentum region, and the momentum distribution of the pions and eta mesons produced at the decay of the eta string can be taken as a distinctive signal of the formation of the eta string in heavy ion collisions.

3 Conclusions

We have discussed the possible effects of the restoration of the axial $U(1)_{\rm A}$ symmetry during the chiral phase transition by using the $U(N_{\rm f})_{\rm R} \times U(N_{\rm f})_{\rm L}$ linear sigma model with two massless quark flavors. It is pointed out that if the axial $U(1)_{\rm A}$ symmetry is to be restored above a certain temperature, the η string is expected to be formed during the chiral phase transition. These eta strings will decay into the η mesons, and it can be viewed as a signal of restoration of the axial $U(1)_{\rm A}$ symmetry in ultrarelativistic heavy-ion collisions.

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