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Emergence of Electromagnetically Induced Absorption in a Perturbation Solution of Optical Bloch Equations[1]

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Abstract—We study phenomenon of electromagnetically induced absorption (EIA) in the Hanle configuration by applying a perturbative method to solve linear system of optical Bloch equations (OBEs) for the case of closed $F_g = 1 \longrightarrow F_e = 2$ transition. The method is applied assuming stationary case and a weak laser fields ($\Omega \ll \Gamma$, i.e., Rabi frequency small compared to spontaneous emission rate). This way, we calculate (both numerically and analytically) higher order corrections to density matrix. Odd corrections give contributions to optical coherences, while even corrections contribute to populations and Zeeman coherences. The method gives insight into mechanism of transfer of coherences and transfer of populations between Zeeman sublevels. We have found that the ground-state coherences (2nd correction to coherences) are crucial for the 4th correction to the change of populations which brings EIA type behavior of the Hanle spectrum. Using exact analytical expressions we further discuss further the role of decoherence of the ground-state sublevels when forming EIA.

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INTRODUCTION

Coherently prepared atomic media are subject of extensive studies over the last decade. A coherent phenomenon which has been thoroughly investigated is electromagnetically induced transparency (EIT). Electromagnetically induced absorption (EIA) [1], phenomena also associated with atomic coherences, is on the other hand, much less studied. Investigations of EIA are important because of the role of low-frequency Zeeman coherences in laser spectroscopy and quantum optics. Atomic scheme required for the observation of EIA is a closed dipole transition for which applies:

$$F_g = F \longrightarrow F_e = F+1, \quad F>0, \tag{1}$$

i.e., the total angular momentum of the excited level F_e must be larger than angular momentum of ground level F_g, $F_e = F_g + 1$. Ground level must be degenerate in order to allow the long-lived Zeeman coherence. Both two-photon resonances in a bichromatic light field (pump-probe spectroscopy) and magneto-optical resonances in the Hanle configuration have been explored. Theoretically, EIA is studied by solving optical Bloch equations numerically [2–4], and several realizations of analytical solving have been done. First explanation for the physical origins of EIA phenomenon was given by Taichenachev [5] using simple analytically tractable model of a four-level N system. According to [5], the EIA resonance is caused by spontaneous transfer of the light-induced low-frequency coherence from the excited level to the ground one.

In this paper, we use the method of perturbations to solve OBEs. This is new and elegant way which allows us to study peculiarities about this phenomenon. Simplest closed multilevel $F_g = 1 \longrightarrow F_e = 2$ transition in the Hanle configuration is analyzed.

METHOD OF PERTURBATIONS FOR THE SYSTEM OF LINEAR EQUATIONS APPLIED TO OBES

The system of linear equations can be solved by applying set of approximation schemes. We consider the system of linear equations in matrix form:

$$Ax = y, \tag{2}$$

where A is the system's matrix, x is the solution and y is right side of the nonhomogeneous system. The main assumption of the method is that the matrix A can be separated in two parts, A_{PERT} which contains relatively "small" elements and A_0 with all other elements of the matrix. The idea is to consider matrix A_{PERT} a perturbation and to solve system of linear equations in a perturbative manner. The goal is to find solution x in terms of corrections:

$$x = x_0 + x_R = x_0 + x_1 + x_2 + x_3 + \dots, \tag{3}$$

where x_0 is the exact solution of unperturbed part. Using solution for x_0 we than calculate the residual part x_R. Inserting Eq. (3) into Eq. (2) gives:

$$(A_0 + A_{\text{PERT}})(x_0 + x_R) = y. \tag{4}$$

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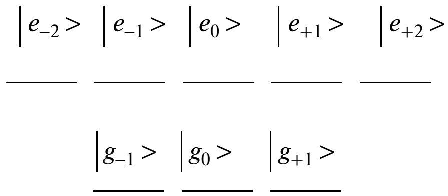

Fig. 1. Atomic transition $F_g = 1 \longrightarrow F_e = 2$ with notation of magnetic sublevels.

We start with a simplified form of the original problem that can be easily solved, i.e. the solution of the unperturbed problem $A_0x_0 = y$ is simply $x_0 = A_0^{-1}y$. To obtain ith—order correction we successively solve Eq. (4) for higher order corrections $x_R = x_1 + \ldots + x_i$. That way we get equation that contains addends of the equations of the previous orders which cancel each other. Approximation is in considering term $A_{\mathrm{PERT}}x_i$ negligible compared to the rest of terms in the equation. This process gives solutions for successive corrections as

$$x_{n+1} = -A_0^{-1}A_{\mathrm{PERT}}x_n. \tag{5}$$

Finally, we get the solution of residual part x_R in terms of power series:

$$\begin{aligned} x_R &= x_1 + x_2 + x_3 + \ldots \\ &= [(-A_0^{-1}A_{\mathrm{PERT}})^1 + (-A_0^{-1}A_{\mathrm{PERT}})^2 \\ &\quad + (-A_0^{-1}A_{\mathrm{PERT}})^3 + \ldots]x_0. \end{aligned} \tag{6}$$

Here, we emphasize the needs for the invertible matrix A_0 and the nonhomogeneous system of linear equations. For sufficiently small A_{PERT}, approximations for every nth correction ($A_{\mathrm{PERT}}x_n = 0$) are justified, and although not exact, solutions produced by this method converge to the exact values when summed to high enough orders. Using previous formulas, we solve stationary optical Bloch equations:

$$\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}] + \frac{i}{\hbar}[\hat{H}_1, \hat{\rho}] + \widehat{SE} + \gamma\hat{\rho} = \gamma\hat{\rho}_0. \tag{7}$$

Here, H_0 and H_1 are Hamiltonian parts describing interaction with the magnetic field B (Zeeman splitting) and the laser light field (characterized by the Rabi frequency Ω), $\widehat{SE}$ is abbreviation for the spontaneous emission, and γ describes relaxation that is not due to spontaneous emission. In order to apply perturbation method to OBEs, we consider the laser light field a perturbation and all coefficients containing Rabi frequency are part of A_{PERT}. For the y we take $\gamma\hat{\rho}_0$ and the rest are parts of the matrix A_0. For the calculations of the Hanle EIA we used the following for parameter's values: $\Gamma = 2\pi \times 6$ MHz, $\Omega = 0.02\,\Gamma$, $\gamma = 0.001\Gamma$, $l_{Fg} = 1$, $l_{Fe} = 1$. The last two parameters stand for Lande factors. We take linearly polarized light and solve OBEs for $F_g = 1 \longrightarrow F_c = 2$ (see Fig. 1) as a simplest transition for which conditions from Eq. (1) are fulfilled. It is obvious from the list of parameters that the condition $\Omega \ll \Gamma$ is satisfied.

RESULTS AND DISCUSSION

In Fig. 2 we present successive even corrections (second, fourth and sixth) for the real part of $\rho_{g_{-1}, g_{+1}}$ (left) and excited-state population (right). In the bottom row we compare sums of successive corrections with the exact solutions (obtained from exact numerical solution of OBEs). Corrections show few characteristic features. First, we have that the next ascending, either even or odd non-zero correction is nearly by order of magnitude smaller than the previous one. Also, typical profile seems to change the "sign" from one even/odd correction to next one. All this yields that the sum of corrections converges to the solution which agrees perfectly with the exact solution (see bottom row of the Fig. 2). Secondly, the order of appearance of non-zero, even or odd corrections for density matrix elements is schematically (by the column of symbols on the right of the figure) shown in Fig. 3. Our results show that each odd correction is a new contribution to optical coherences, while even corrections bring new contribution to populations and Zeeman coherences via additional level couplings.

The solution of the unperturbed part x_0 is simply the redistribution of the ground state populations by 1/3 for each ground-state sublevel. First correction x_1 is non-zero for optical coherences for which the selection rule $\Delta m = \pm 1$ between magnetic sublevels stands. The 2nd correction corrects all populations and also Zeeman coherences of the sublevels for which $\Delta m_{g,e} = \pm 2$ holds (meaning that matrix elements $\rho_{e-2, e+2}$ and $\rho_{e+2, e-2}$ are not yet perturbed by this correction). The 3rd correction x_3, corrects all optical coherences and 4th all populations and all Zeeman coherences. Please note that here and in the previous text, by all we mean coherences that are radiatively coupled and have physical sense.

Next, some corrections for a few density-matrix elements show complex Lorentzian-like behavior for the dependence of magnetic field B_{scan}. First, by the order of appearance, is for the 2nd correction of ground-state coherences $\rho_{g_{-1}, g_{+1}}$ and $\rho_{g_{+1}, g_{-1}}$. These are also the only elements in the 2nd correction that show this kind of functional dependence. Since each correction depends on the solution of previous ones, the third correction of some elements will also show this kind of behavior. The 4th correction brings complex Lorentzian-like behavior to the populations (see right column of Fig. 2). We find that the narrow peak of the 2nd correction of the ground-state coherences is responsible for the appearance of EIA in the 4th cor-

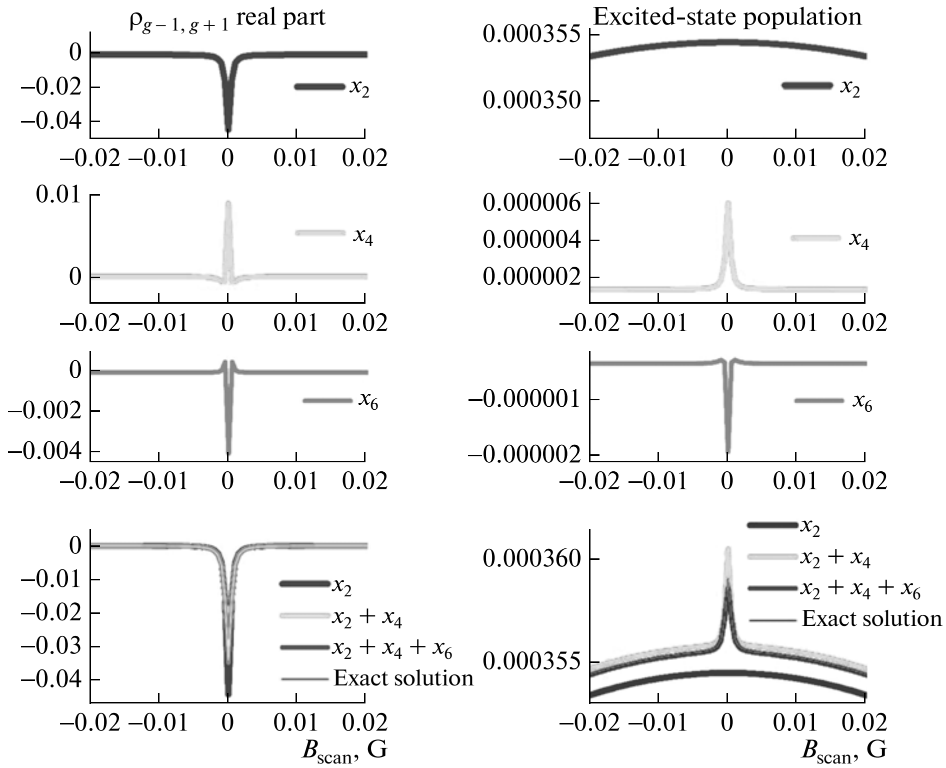

Fig. 2. Successive corrections (top 3 rows) and sum of corrections compared with exact solutions (bottom row) for real part of $\rho_{g_{-1},g_{+1}}$ (left) and excited-state population (right).

rection, and all the latter higher corrections of the populations. Some properties of the EIA (sum of excited-state populations) are therefore governed by the behavior of the 2nd correction of the ground state coherences. We further identify the appearance of the narrow peak of the 2nd correction of the ground-state coherences with the onset of EIA.

As each correction depends on the solution of the previous ones (see Eq. (5)), this 2nd correction of the ground-state coherence depends on previous transfer of coherences and populations. In Fig. 4 we show schematically how $(\rho_{g_{-1},g_{+1}})_{x_2}$ (bottom row) is formed through successive mechanisms of transfer of populations and coherences. Three ground-state populations (top row) are transferred to 6 optical coherences (middle row), and by the next correction, these 6 coherences transfer to ground-state coherence. Altogether, ground-state populations are transferred to 12 optical coherences, but we present here only 6 ones that constitute 2nd correction of the ground-state coherence $(\rho_{g_{-1},g_{+1}})_{x_2}$. By a similar analysis it can be shown how overall behavior of populations in the fourth (and latter) correction depends on the behavior of $(\rho_{g_{-1},g_{+1}})_{x_2}$.

Moreover, since it comes to multiplying matrix with column, each density-matrix element is simply a linear combination of other previously corrected elements. Since our calculations are done with Wolfram Mathematica 5.0, we were able to provide the analytical expressions for 2nd correction of ground-state coherence $(\rho_{g_{-1},g_{+1}})_{x_2}$. The details of this expression will be published elsewhere, but we found that the term that is responsible for the EIA, as mentioned previ-

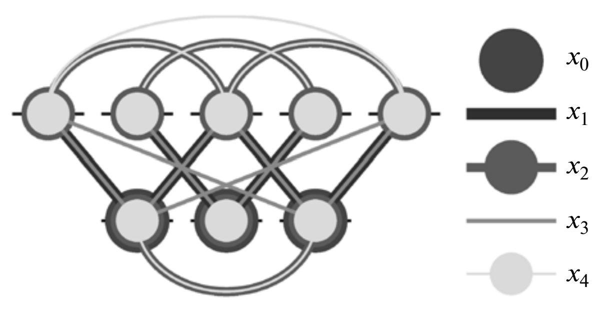

Fig. 3. Schematic diagram which shows which density-matrix elements of successive corrections are non-zero.

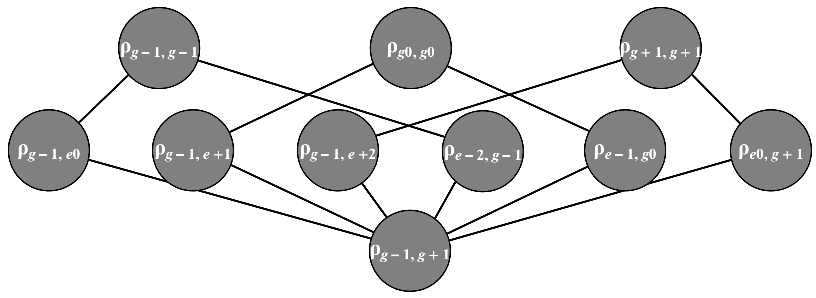

Fig. 4. Visualizing transfer of populations and transfer of coherences through successive solutions $x_0 \longrightarrow x_1 \longrightarrow x_2$ (top $\longrightarrow$ middle $\longrightarrow$ bottom row).

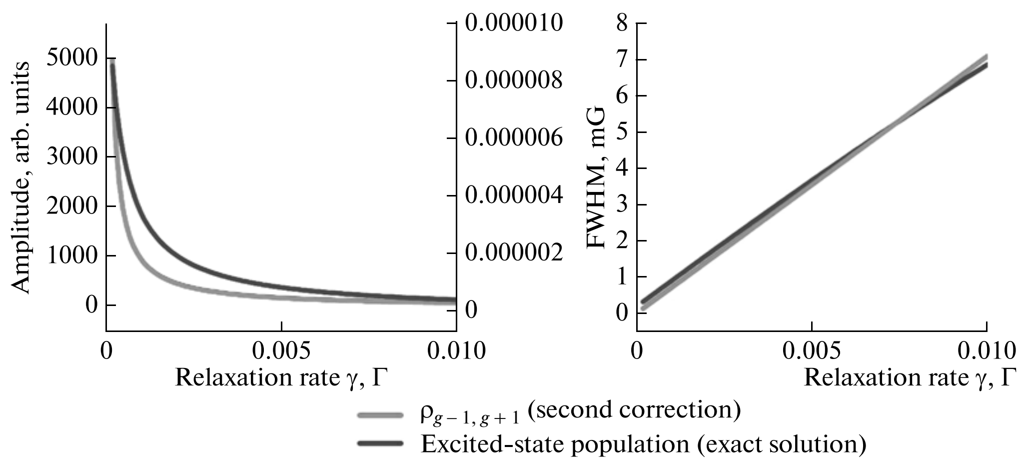

Fig. 5. Dependence of amplitude (left) and full-widths-half-maximum (FWHM) (right) on the relaxation rate γ. Presented results are numerically calculated from real part of $(\rho_{g_{-1}, g_{+1}})_{x_2}$ (Eq. (8)) and from excited-state population (exact solution).

ously is the only narrow Lorentzian in the 2nd correction,

$$(\rho_{g_{-1}, g_{+1}})_{x_2} \propto \frac{1}{(\gamma - 2iBl_{F_g}\mu_B/h)}. \quad (8)$$

Realistically, due to non-perfect isolation from the environment, atoms in a gas are subject to the loss of quantum coherence. We discuss here the role of decoherence of ground-state sublevels due to relaxation in the formation of EIA. The parameter γ in Eq. (7) (and also in Eq. (8)) stands for relaxation of all density matrix elements with the same rate γ due to various processes like "escape" of an atom from the laser light beam, collisions with other atoms etc. We look here how ground-state coherence (only 2nd corrections x_2, but it is the largest contribution) and the overall EIA depend on γ. The amplitude of the real part of the narrow complex Lorentzian in (8) is given by $1/\gamma$, while full-width-half-maximum is $\gamma h/(l_{Fg}\mu_B)$. The comparison between dependence of amplitude and of FWHM on γ, obtained using Eq. (8) and the exact solution for the excited-state population are given in Fig. 5. Within the approximation limitation of the method ($\Omega \ll \Gamma$), widths show full agreement, while amplitudes show qualitative good agreement and that the higher-order corrections for $\rho_{g_{-1}, g_{+1}}$ might also contribute. Our results show that increase of relaxation leads to loss of ground-state coherences and that this wanes overall EIA effect.

In summary, we have presented results of calculating Hanle EIA using perturbative method for solving OBEs for the stationary case. The results presented for near degenerate two-level system, and assuming $\Omega \ll \Gamma$, show that EIA, obtained by the perturbation method converges to the exact solution of the OBEs. The method establishes the role of the ground state coherences (2nd correction in the contribution to ground state coherences) in the latter formation of EIA. Similar dependence of the EIA and of this 2nd correction (as established from analytical expression for this 2nd correction) on decoherence rate can be regarded as the evidence that EIA is developed from the ground state coherences.

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