THE BRST QUANTIZATION OF THE O(2) STRING ☆

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The BRST charge for the $N = 2$, O(2) superstring theory is given. The O(2) algebra is realized a la Kac–Moody, which would seem to cause problems in the BRST formulation, however, the BRST charge is nilpotent due to the supersymmetry of the model. The nilpotency of the BRST charge determines the critical dimension of the model to be $D = 2$, and the intercept of the Regge trajectory becomes zero, making the model tachyon-free. We use the BRST charge to construct a free field theory of the model.

1. Introduction. A covariant approach to the quantization of string theories based on BRST invariance has been proposed recently [1,2]. An alternative and equivalent approach is to use exterior differential calculus over the Virasoro algebra [3]. Using the BRST charge it is possible to build a covariant free string lagrangian [1,2]. The same ideas work in the case of $N = 1$ superstrings [4]. It is plausible to assume that the full interacting covariant string field theory can be formulated and constructed based on a nonlinear extension of the BRST charge [5].

The technique is to build a nilpotent BRST charge, i.e. to first quantize the string theory using the BRST method. Next, use the BRST charge to construct a BRST invariant kinetic operator for the field theory. The gauge invariant string field theory action is then given as the BRST invariant measure on the space of string functionals [1].

In this paper we work out a BRST charge for the $N = 2$ supersymmetric, O(2) charged string theory [6]. The O(2) symmetry is realized as an abelian Kac–Moody symmetry. Based on the result of ref. [7] one would expect that the BRST formulation of the model suffers from a problem that stems from the Kac–Moody part, making it appear as if it were not possible to construct a nilpotent BRST charge. If this is the case then the enforcement of the nilpotency of the BRST charge would make the theory fall into the unphysical sector (for example the states no longer form the unitary representations of the Kac–Moody symmetry and consequently, the energy is no longer bounded from below [8]), or to become inconsistent. Fortunately, this is not true due to the supersymmetry of the model. By a different kind of analysis [6,9] it was shown that the critical dimension of the model is $D = 2$. This enables us to see the subtle way of how the supersymmetry takes care of the anomaly problem for the O(2) group.

Once the nilpotent BRST charge is constructed in the Neveu–Schwarz (NS) and Ramond (R) sectors of the theory it becomes possible to construct a covariant string field lagrangian. Next one could try to find a nonlinear generalization of the BRST charge in order to formulate the interacting theory. The analysis presented here could be relevant to understanding the structure of compactified strings, and to resolving the problems of the BRST formulation of Kac–Moody algebras.

Our paper is organized as follows. In section 2 we briefly review the two-dimensional $N = 2$ supergravity theory that defines the O(2) string and give the BRST charge. We show that it is nilpotent provided that the critical dimension of the model is two and the intercept parameter is zero. In section 3 we show that the ghost structure introduced to make the BRST charge

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nilpotent also makes the partition function equal to one in each sector of the model. This means that there is only one physical state in each sector. In the NS sector it is a ground state scalar and in the R sector it is a ground state fermion. Therefore we must conclude that the free O(2) string theory is trivial. We also give the BRST invariant kinetic operators and construct a gauge-fixed free string action and list its invariances.

2. The BRST charge of the O(2) string. Let us first give a brief review of the first quantized formulation of the O(2) string. The action of a two-dimensional, $N = 2$, O(2) symmetric supergravity (graviton, two Majorana gravitinos, vector particle) interacting with D complex (0,1/2) matter multiplets (complex scalar, two Majorana spinors, complex auxiliary field) that defines the model is given by [10]

$$S = \int d^2\zeta \sqrt{g}\,[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^* + \tfrac{1}{2}i\bar{\Psi}\gamma^\mu \overleftrightarrow{D}_\mu\Psi$$

$$+ A_\mu\bar{\Psi}\gamma^\mu\Psi + (\partial_\mu\Phi^* + \bar{\Psi}\chi_\mu)\,\bar{\chi}_\nu\gamma^\mu\gamma^\nu\Psi$$

$$+ (\partial_\mu\Phi + \bar{\chi}_\mu\Psi)\,\bar{\Psi}\gamma^\nu\gamma^\mu\chi_\nu - \tfrac{1}{2}FF^*]\,, \qquad (2.1)$$

where $D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu\gamma_5$, $\chi_\mu = (1/\sqrt{2})(\chi^1 + i\chi^2)$, $\Phi = \Phi^1 + i\Phi^2$, $\Psi = (1/\sqrt{2})(\Psi^1 + i\Psi^2)$, and other conventions are as in ref. [9].

The action (1) is invariant under coordinate, Lorentz, $N = 2$ supersymmetry, Weyl, vector gauge, chiral gauge and super-conformal transformations [9,10]. As usual, we choose the gauge: $e^a_\mu = \delta^a_\mu$, $\chi_\mu = 0$, $A_\mu = 0$, in order to linearize the action (2.1). This gauge choice is possible due to the symmetries cited. The linearized version of the action thus becomes

$$S = \int d^2\zeta\,(\tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi^* + \tfrac{1}{2}i\bar{\Psi}\gamma^\mu \overleftrightarrow{\partial}_\mu\Psi - \tfrac{1}{2}FF^*)\,. \qquad (2.2)$$

Together with (2.2) one needs a set of constraints and appropriate boundary conditions. They follow from (2.1). The constraints, written in terms of real fields, are given by

$$\Theta_{\mu\nu} = \sum_{i=1}^{2}\,[\partial_\mu\Phi^i\partial^\mu\Phi^i - \tfrac{1}{2}g_{\mu\nu}\partial_\rho\Phi^i\partial^\rho\Phi^i$$

$$+ \tfrac{1}{4}i\bar{\Psi}^i(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)\Psi^i] = 0\,, \qquad (2.3a)$$

$$S^i_\mu = \sum_{j=1}^{2}\,(\delta^{ij}\partial_\nu\Phi^1 - \epsilon^{ij}\partial_\nu\Phi^2)\,\partial^\nu\gamma_\mu\Psi^j = 0\,, \qquad (2.3b)$$

$$T = \tfrac{1}{2}i\sum_{i,j=1}^{2}\,\epsilon^{ij}\Psi^i\Psi^j = 0\,. \qquad (2.3c)$$

The constraints (2.3a), (2.3b) describe conformal invariance and $N = 2$ supersymmetry respectively and (2.3c) is a consequence of the O(2) symmetry of the model.

The constraints form a closed algebra, the symmetry algebra of the model. This means they are first class. Quantization can be implemented by constructing a Fock space realization of this algebra. In the O(2) model this can be accomplished by using two sets of bosonic and two sets of fermionic ladder operators:

$$[\alpha^{i,a}_n, \alpha^{j,b}_m] = n\delta^{ij}\delta_{n+m,0}\eta^{ab}\,,$$

$$\{b^{i,a}_r, b^{j,b}_s\} = \delta^{ij}\delta_{r+s,0}\eta^{ab}\,,$$

where $i, j = 1,2$; $a, b = 0,1, ..., D-1$; $n, m \in \mathbf{Z}$ in the NS sector, and $r, s \in \mathbf{Z} + \frac{1}{2}$ in the R sector. The vacuum is defined by

$$\alpha^{i,a}_n|0\rangle = b^{i,a}_r|0\rangle = 0 \quad (n, r > 0)\,.$$

As a consequence of ordering ambiguities the algebra of constraints acquires central charges, and becomes

$$[L_n, L_m] = (n - m)L_{n+m} + \tfrac{1}{4}D(n^3 - n)\,\delta_{n+m,0}\,,$$

$$\{G^i_r, G^j_s\} = 2\delta^{ij}L_{r+s} + 2i\epsilon^{ij}(r - s)\,T_{r+s}$$

$$+ \tfrac{1}{4}D(4r^2 - 1)\,\delta_{r+s,0}\,,$$

$$[L_n, G^i_r] = (\tfrac{1}{2}n - r)\,G^i_{n+r}\,, \quad [T_n, T_m] = \tfrac{1}{4}Dn\delta_{n+m,0}\,,$$

$$[L_n, T_m] = -mT_{n+m}\,, \quad [T_n, G^i_r] = \tfrac{1}{2}i\epsilon^{ij}G^j_{n+r}\,. \quad (2.4)$$

The L's generate conformal symmetry, G's supersymmetry and the T's generate the O(2) symmetry. Note that the algebra (2.4) is the $N = 2$ super-Virasoro algebra, with an abelian Kac–Moody subalgebra. Physical states are defined by

$$(L_n - \beta\delta_{n,0})|\text{phys}\rangle = T_n|\text{phys}\rangle = G^i_r|\text{phys}\rangle = 0$$

$$(n, r \geqslant 0)\,. \qquad (2.5)$$

Given the algebra (2.4) one can use a standard prescription [11] for forming BRST invariant hamiltonians of constrained systems.

Let H_0, ϕ_a ($a = 1, ..., k$) be the original hamiltonian and the first class constraints of some system satisfying the Poisson bracket relations

$$\{\phi_a, \phi_b\}_{\mathrm{P.B.}} = u_{ab}^c \phi_c , \quad \{H_0, \phi_a\}_{\mathrm{P.B.}} = v_a^b \phi_b .$$

Here we allow ϕ_a to be bosonic or fermionic, which means that the Poisson brackets are defined with the appropriate Lorentz grading [‡1] (0 for bosons, 1 for fermions). (For our purposes it is enough to assume that the structure constants u_{ab}^c and v_a^b are ordinary c-numbers.) Then there exist classes of quantum hamiltonians H invariant under fermionic (BRST) transformation satisfying nilpotency:

$$H = H_0 + \bar{C}_b v_a^b C^a + [Q_{\mathrm{BRST}}, \Omega] ,$$

$$Q_{\mathrm{BRST}} = \phi_a C^a + \tfrac{1}{2}(-)^{P_a} u_{ab}^c \bar{C}_c C^b C^a ,$$

$$[H, Q_{\mathrm{BRST}}] = 0 ,$$

where C^a and $\bar{C}_a$ are ghost and antighost fields, respectively (corresponding to ϕ_a) satisfying

$$[C^a(x), \bar{C}_b(y)\}|_{x^0=y^0} = \delta_b^a \delta(x-y) .$$

Physical states are given by a single condition:

$$Q_{\mathrm{BRST}}|\mathrm{phys}\rangle = 0 , \tag{2.5'}$$

together with the requirement that they do not contain ghosts, i.e. they are annihilated by the ghost and antighost annihilation operators. This reproduces constraints (2.5). Note that the ghost fields are of the opposite Grassmann type to their corresponding constraint operators.

Applying this formulation to the Virasoro algebra [12] and $N = 1$ super-Virasoro algebra [4] it has been shown that imposing nilpotency on the BRST charge leads to the cancellation of the conformal anomaly, i.e. determines the critical dimension and the Regge intercept of the string. For the algebra (2.4) we have

$$Q_{\mathrm{BRST}} = Q_1 + Q_2 , \tag{2.6}$$

with

$$Q_1 = \sum_n (L_n - \beta\delta_{n,0})\, \eta_{-n} + \sum_n T_n \lambda_{-n} + \sum_r \sum_{i=1}^{2} G_r^i \xi_{-r}^i ,$$

$$Q_2 = \frac{1}{2}\sum_{n,m} (n-m) :\bar{\eta}_{n+m}\, \eta_{-m}\, \eta_{-n}: - \sum_{n,m} m :\bar{\lambda}_{n+m}\, \lambda_{-m}\, \eta_{-n}: + \sum_{n,r} (\tfrac{1}{2}n - r) :\bar{\xi}_{n+r}^i \xi_{-r}^i \eta_{-n}:$$

$$- \frac{1}{4}\sum_{n,r} \epsilon^{ij} :\bar{\xi}_{n+r}^i \xi_{-r}^j \lambda_{-n}: - \sum_{r,s} :\bar{\eta}_{r+s} \xi_{-r}^i \xi_{-s}^i: + \mathrm{i}\sum_{r,s} (r-s)\, \epsilon^{ij} :\bar{\lambda}_{r+s} \xi_{-r}^i \xi_{-s}^j: ,$$

where β is the undetermined parameter present due to normal ordering ambiguities in L_0. We use symbols $\eta, \lambda\ (\bar{\eta}, \bar{\lambda})$ for fermionic ghosts (antighosts) and symbols $\xi_r^i\ (\bar{\xi}_r^i)$ for bosonic ghosts (antighosts). They satisfy

$$\{\eta_n, \bar{\eta}_m\} = \{\lambda_n, \bar{\lambda}_m\} = \delta_{n+m,0} , \quad [\xi_r^i, \bar{\xi}_s^j] = \delta^{ij}\delta_{r+s,0} ,$$

and all other combinations are zero. The colons : : denote normal ordering with respect to the ghost vacuum defined by

$$(\mathrm{ghost})_m|0\rangle = (\mathrm{antighost})_m|0\rangle = 0 , \quad \text{for } m > 0 .$$

We use the convention where η, $\bar{\eta}$, λ, $\bar{\lambda}$ and ξ_r^i are hermitian and $\bar{\xi}_r^i$ are antihermitian.

Alternatively we can write the BRST charge (2.6) in the NS sector as

$$Q = d + \delta + L\eta_0 + T\lambda_0 + A\bar{\eta}_0 + B\bar{\lambda}_0 , \tag{2.7a}$$

and in the R sector as

$$Q = d + \delta + L\eta_0 + T\lambda_0 + A\bar{\eta}_0 + B\bar{\lambda}_0 + G^i \xi_0^i + C^i \bar{\xi}_0^i + P , \tag{2.7b}$$

‡1 The Poisson bracket is defined by

$$\{A, B\}_{\mathrm{P.B.}} = \sum_i \left(\frac{\partial A}{\partial q_i} \frac{\partial B}{\partial p^i} - (-)^{P_A \cdot P_B} \frac{\partial A}{\partial p^i} \frac{\partial B}{\partial q_i} \right)$$

where P_A is called the Lorentz grading.

where

$$L = L_0 + \sum_{n>0} n(\eta_{-n}\bar{\eta}_n + \bar{\eta}_{-n}\eta_n + \lambda_{-n}\bar{\lambda}_n + \bar{\lambda}_{-n}\lambda_n)$$

$$+ \sum_{r>0} r(\bar{\xi}^i_{-r}\xi^i_r - \xi^i_{-r}\bar{\xi}^i_r),$$

$$T = T_0 + \frac{i}{4}\sum_{r>0} \epsilon^{ij}(\xi^i_{-r}\bar{\xi}^j_r - \bar{\xi}^i_{-r}\xi^j_r),$$

$$G^i = G^i_0 + \frac{1}{2}\sum_{r>0} [r(\eta_{-r}\bar{\xi}^i_r - \bar{\xi}^i_{-r}\eta_r) - 4(\xi^i_{-r}\bar{\eta}_r + \bar{\eta}_{-r}\xi^i_r)]$$

$$+ \frac{i}{4}\sum_{r>0} \epsilon^{ij}[(\lambda_{-r}\bar{\xi}^j_r\lambda_r) - 8r(\xi^j_{-r}\bar{\lambda}_r + \bar{\lambda}_{-r}\xi^j_r)],$$

$$A = -2\left(\sum_{n>0} n\eta_{-n}\eta_n + \sum_{r>0} \xi^i_{-r}\xi^i_r\right),$$

$$B = -2\left(\sum_{n>0} n(\eta_{-n}\lambda_n + \lambda_{-n}\eta_n) - 2i\sum_{r>0} r\epsilon^{ij}\xi^i_{-r}\xi^j_r\right),$$

$$C^i = \frac{3}{2}\sum_{r>0} r(\eta_{-r}\xi^i_r - \xi^i_{-r}\eta_r)$$

$$- \frac{i}{4}\sum_{r>0} \epsilon^{ij}(\xi^j_{-r}\lambda_r + \lambda_{-r}\xi^j_r),$$

$$P = -\tfrac{3}{4}i\epsilon^{ij}\bar{\xi}^i_0\xi^j_0\lambda_0 - 3\bar{\eta}_0\xi^i_0\xi^i_0 .$$

The expressions for d and δ are very complicated and we will not display them. d is defined as the operator that creates a net ghost number and δ is the operator that destroys a net antighost number. Nilpotency of the BRST charge Q implies a number of relationships between various terms in (2.7). Since they are easy to obtain we will not write them down. The forms (2.7) of the BRST charge are useful when constructing the field theory.

A very long, but straightforward calculation of the square of the BRST charge gives

$$Q^2 = \frac{1}{4}\sum_{n>0} [(D-2)n^3 - (D-2-8\beta)n]\,\eta_{-n}\eta_n$$

$$+ \frac{1}{4}\sum_{r>0} [(D-2)4r^2 - (D-2-8\beta)]\,\xi^i_{-r}\xi^i_r$$

$$+ \frac{1}{4}\sum_{n>0} (D-2)n\lambda_{-n}\lambda_n .$$

Therefore the BRST charge is nilpotent ($Q^2 = 0$) if the following two equations hold:

$$D-2=0, \quad D-2-8\beta=0 .$$

These conditions give the solution $D = 2$, $\beta = 0$. Therefore the model is free of superconformal anomaly if it lives in two-dimensional spacetime and if the Regge intercept is equal to zero.

It is interesting to note that it is the $\bar{\xi}\xi\lambda$ term in the BRST charge (2.6) which cancels the Kac–Moody anomaly for $D = 2$. This term is present because the algebra (2.4) is supersymmetric. If this was not the case the Kac–Moody anomaly [7] would persist and a modification [13] in the construction of the BRST charge would be necessary. This kind of cancellation of the Kac–Moody anomaly is exactly as expected from the work of Fradkin and Tsetylin [9].

3. The field theory for the O(2) string. We begin the section by calculating the partition function of the model. We will use the covariant formalism introduced in the previous section. The knowledge of the partition function allows us to count the number of the propagating (physical) fields at each mass level of the string theory. At this point we ignore the degeneracy due to ghost zero modes. The calculation of the partition function is the same in both sectors of the model.

Let the number of propagating fields at the mass level N be $T_D(N)$. The number $T_D(N)$ is generated by the function

$$f_D(x) = \sum_{N=0}^{\infty} x^{2N} = \mathrm{Str}(x^{2N}), \tag{3.1a}$$

where

$$\hat{N} = \sum_{n>0} (\alpha^{i\dagger}_{n,\mu}\alpha^{i\mu}_n + n\eta^\dagger_n\bar{\eta}_n + n\bar{\eta}^\dagger_n\eta_n + n\lambda^\dagger_n\bar{\lambda}_n + n\bar{\lambda}^\dagger_n\lambda_n)$$

$$+ \sum_{r>0} r(b^{i\dagger}_{r,\mu}b^{i,\mu}_r + \bar{\xi}^{i\dagger}_r\xi^i_r - \xi^{i\dagger}_r\bar{\xi}^i_r).$$

An easy calculation gives

$$f_D(x) = \left[\prod_{n=1}^{\infty}\left(\frac{1+x^{2n}}{1-x^{2n}}\right)^{2(D-2)}\right]. \quad (3.1b)$$

Therefore, in the critical dimension ($D = 2$) we have $f_{D=2}(x) = 1$. This implies that there is only one propagating state in each sector of the theory because from (3.1a) and (3.1b) $T_2(N > 0) = 0$. In the NS sector of the theory the propagating state is a ground state scalar and in the R sector it is a ground state fermion. This result is in a full agreement with conclusion of Ademollo et al. [6].

In the remainder of this section we describe the construction of the covariant gauge fixed field theory for the model. We closely follow refs. [1,14]. The field theory thus constructed will have an enormous invariance which allows only one propagating state in each sector as expected. We have to treat each sector of the model separately due to the different ghost zero mode structure. We will give the details only for the NS sector because the construction goes the same way in the R sector once the ghost zero mode vacuum is specified.

In the NS sector the ghost zero mode sector is fourfold degenerate (there are four zero mode operators: $\eta_0, \lambda_0, \bar{\eta}_0, \bar{\lambda}_0$). In order that the condition (2.5′) on the physical state reproduces all the constraints (2.5) we must require that the string field is annihilated by the antighost zero mode operators $\bar{\eta}_0$ and $\bar{\lambda}_0$. Defining the ghost zero mode vacuum states by: $\bar{\eta}_0(\bar{\lambda}_0)|0_{\eta(\lambda)}\rangle = 0$ and $\langle 0_{\eta(\lambda)}|\eta_0(\lambda_0)|0_{\eta(\lambda)}\rangle = 1$ we can write the string field vacuum state as

$$|0_{\rm STRING}\rangle = |0\rangle \otimes |0_\eta\rangle \otimes |0_\lambda\rangle. \quad (3.2)$$

The BRST invariant kinetic operator is given by

$$D = \tfrac{1}{2}\{\lambda_0[\eta_0, \bar{\eta}_0], Q\}$$

$$= \lambda_0\eta_0 L - \lambda_0\bar{\eta}_0 A + \tfrac{1}{2}B(2\eta_0\bar{\eta}_0 - 1). \quad (3.3)$$

Expanding the string field functional in ghost zero modes:

$$|\Phi\rangle = [|\phi\rangle + \eta_0|\psi\rangle + \lambda_0|\chi\rangle + \eta_0\lambda_0|\omega\rangle]$$

$$\otimes 0_\eta\rangle \otimes 0_\lambda\rangle, \quad (3.4)$$

the BRST invariant lagrangian is given by $\mathcal{L} = -\frac{1}{2}\langle\Phi|D|\Phi\rangle$. When written in components it reads

$$\mathcal{L} = -\tfrac{1}{2}\langle\phi|L|\phi\rangle - \tfrac{1}{2}\langle\psi|A|\psi\rangle - \tfrac{1}{4}\langle\chi|B|\psi\rangle - \tfrac{1}{4}\langle\psi|B|\chi\rangle$$

$$- \tfrac{1}{4}\langle\omega|B|\phi\rangle + \tfrac{1}{4}\langle\phi|B|\omega\rangle. \quad (3.5)$$

From the expression (3.5) we see that the fields χ and ω are the Lagrange multiplier fields enforcing gauge conditions $B|\phi\rangle = B|\psi\rangle = 0$. The lagrangian (3.5) is invariant under the transformation

$$\delta_E|\Phi\rangle = Q|E\rangle. \quad (3.6)$$

Expanding $|E\rangle = [|e\rangle + \eta_0|f\rangle + \lambda_0|g\rangle + \eta_0\lambda_0|h\rangle] \otimes |0_\eta\rangle \otimes |0_\lambda\rangle$ the transformation (3.6) reads

$$\delta_E|\phi\rangle = (d+\delta)|e\rangle + A|f\rangle + B|g\rangle,$$

$$\delta_E|\psi\rangle = -(d+\delta)|f\rangle - B|h\rangle + L|e\rangle,$$

$$\delta_E|\chi\rangle = -(d+\delta)|g\rangle + A|h\rangle + T|e\rangle,$$

$$\delta_E|\omega\rangle = (d+\delta)|h\rangle + L|h\rangle - T|f\rangle.$$

Using the gauge invariance (3.6) it is only a straightforward exercise to confirm that the only propagating state in the NS sector is the ground state massless scalar.

In the R sector there are additional ghost zero modes $(\xi^i_0, \bar{\xi}^i_0)$. Since the supersymmetry ghosts (antighosts) satisfy the commutator rules, their vacuum is infinitely degenerate. Again we require that the string field functionals are annihilated by the antighost zero mode operators. That lifts half of the infinite degeneracy. The remaining degeneracy can be lifted as in ref. [4] by choosing the commuting ghost zero mode vacuum at the value $\xi^i_0 = 0$. This implies that the string field functional will have only a finite number of components when expanded in ghost zero modes (cf. ref. [4]). Like in the NS sector we can define the BRST invariant kinetic operator and write down the covariant, gauge-fixed lagrangian. The only propagating state is the massless ground state fermion.

It is possible that the spectrum of states of the model is supersymmetric but in order to confirm that one has to study the appropriate vertex operators.

4. Conclusion. In this paper we have quantized the O(2) string in a BRST invariant way. The BRST charge (2.6), (2.7) is nilpotent only if the dimension of the space–time in which the string lives is $D = 2$ and the Regge intercept parameter $\beta = 0$. This follows from the requirement that the corresponding two-dimensional superconformal theory is anomaly free. Our result is in agreement with the previous ones [6,9,15] obtained in a different way. Naively, based on the calculation in ref. [7] one might expect that the Kac–Moody subalgebra would spoil the nilpotency of the BRST charge and that the BRST invariant quantization may not be possible in the standard way. This is not so because the two-dimensional conformal theory is supersymmetric (cf. ref. [16]). Furthermore we have shown using the covariant method that there are only two propagating states in the model. They are the massless ground states in each sector. Since the ground state in the NS sector is bosonic and the ground state in the R sector is fermionic it is possible that the spectrum has the space–time supersymmetry. To examine the space–time supersymmetry in this trivial model one has to construct the appropriate vertex operator. We have learned that this construction is under way [17].

It is easy to understand the triviality of the model because it lives in two-dimensional space–time. Imagine we go to the light-cone gauge. This removes two longitudinal components. In two dimensions this is all we have. Therefore there is no room left for the transverse modes to oscillate. Consequently the only modes that can survive are the ground states, which are the scalar and the spinor respectively.

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