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THE EXPLICIT FORM OF THE OPERATOR CONNECTING THE CANESCHI-SCHWIMMER-VENEZIANO AND WITTEN VERTICES *

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We construct, to all levels, the operator connecting the Caneschi-Schwimmer-Veneziano vertex with the Witten vertex. This gives the explicit conformal transformation linking these two interactions in both their coordinate and ghost sectors.

1. Introduction

Recently the operator construction of Witten's interacting string field theory [1] was given [2], and the symmetry properties were established [2, 3]. The theory was defined in terms of δ-function overlaps which were represented in Hilbert space by constructing the Neumann functions on the nontrivial scattering domains. This was achieved through conformal mappings [2-4].

The fact that the problem was related through conformal mappings to the dual model implies that there exists an explicit operator transformation to the dual model vertex of Caneschi, Schwimmer and Veneziano (CSV) [5]. The existence of this transformation assures that all physical couplings of the vertex operators are identical, and it furthermore offers a nontrivial computational tool.

In this paper we give a general procedure for evaluating the coefficients in this operator mapping. Originally the transformations were considered in the light-cone case [6]. We apply our computation scheme to Witten's interaction. We consider explicitly the ghost effects and derive recursion formulas for the calculation of the coefficients to all levels.

This paper is organized as follows. In sect. 2 the operator linking the coordinate sectors of the two theories is found. Useful commutation relations in the coordinate sector are derived. In sect. 3 the work of sect. 2 is extended to the ghost sector as well. The full operator representing the connection between the CSV and Witten interactions is found. Further useful commutators are derived. In appendix A we

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give tables of relevant coefficients used in this work. Finally, in appendix B, we write our conformal operator in an alternate form.

2. Construction in the bosonic sector

The vertex of Witten's covariant, gauge invariant theory [1] is given by [2]

$$\langle V_x^{\mathrm{Witten}}| = \langle 0,0,0| \exp\left(\tfrac{1}{2}\alpha_n^r N_{nm}^{rs} \alpha_m^s\right),$$

where n, m label the modes and take on the values $0,1,2,\dots$, while throughout this paper, if not otherwise specified, r, s stand for string labels and take the values $1,2,3$. The Neumann coefficients N that determine this interaction are given in ref. [2].

The other vertex we consider here was derived by Caneschi, Schwimmer and Veneziano [5] as the operator governing interactions in the dual model

$$\langle V_x^{\mathrm{CSV}}| = \langle 0,0,0| \exp\left(\tfrac{1}{2}\alpha_n^r M_{nm}^{rs} \alpha_m^s\right).$$

The CSV coefficients M have a specially simple form

$$M_{nm}^{12} = (-)^m \frac{1}{n} \binom{n}{m},$$

$$M^{12} = M^{23} = M^{31},$$

while all the other M's vanish. From these expressions we derive the equations

$$\langle V_x^{\mathrm{Witten}}\left(\alpha_{-n}^r - nN_{nm}^{rs}\alpha_m^s\right) = 0, \tag{1}$$

$$\langle V_x^{\mathrm{CSV}}|\left(\alpha_{-n}^r - nM_{nm}^{rs}\alpha_m^s\right) = 0, \tag{2}$$

$$n = 1,2,3,\dots, \qquad m = 0,1,2,\dots .$$

Eqs. (1) and (2) are equivalent to the defining relations for the vertices (in the sense that they determine the coordinate piece of the corresponding vertex), however, for our purposes they will be easier to work with.

Physical on-shell states in the bosonic string are given by the well-known conditions

$$(L_n^r - \delta_{n,0})|\mathrm{phys}\rangle = 0, \qquad n = 0,1,2,\dots .$$

Thus, the statement that both vertices give the same coupling for all physical on-shell states implies the existence of an operator

$$\boldsymbol{O} = \exp\left[A_0^r (L_0^r - 1) + A_1^r L_1^r + A_2^r L_2^r + \cdots \right], \tag{3}$$

such that

$$\langle V_x^{\mathrm{CSV}} | \boldsymbol{O} = \langle V_x^{\mathrm{Witten}} | . \tag{4}$$

Eqs. (2) then give us the relations

$$0 = \langle V_x^{\mathrm{Witten}} \boldsymbol{O}^{-1} (\alpha_{-n}^r - n M_{nm}^{rs} \alpha_m^s) \boldsymbol{O} . \tag{5}$$

Due to the completeness of (1) this is some linear combination of relations (1) for different values of the indices n and r. To work out (5), and calculate this linear combination, we will need to figure out the following commutators

$$\left[\alpha_n, \exp\left(\sum_{m=0}^{\infty} A_m L_m \right) \right]. \tag{6}$$

The Virasoro generators are given in terms of the α's by $L_n = \frac{1}{2} \Sigma_k : \alpha_{n-k} \cdot \alpha_k :$, which yields the commutation relations

$$[L_n, \alpha_m] = -m \alpha_{n+m} . \tag{7}$$

Since $\exp(-\Sigma_{m=0}^{\infty} A_m L_m) \alpha_n \exp(\Sigma_{m=0}^{\infty} A_m L_m)$ is some function of the commutators $[L_n, \alpha_m]$, commutators of these commutators etc., eq. (7) then gives us

$$\left[\alpha_n, \exp\left(\lambda \sum_{m=0}^{\infty} A_m L_m \right) \right] = \exp\left(\lambda \sum_{m=0}^{\infty} A_m L_m \right) \left(\sum_k B_k(\lambda) \alpha_k \right),$$

where we have introduced a parameter λ. By differentiating this parameter we will get a set of differential equations for the coefficients $B(\lambda)$. So, after differentiating, we find

$$\frac{\mathrm{d} B_k}{\mathrm{d} \lambda} = \sum_{m=0}^{\infty} A_m (k-m) [B_{k-m} + \delta_{n,k-m}],$$

$$B_k(0) = 0,$$

which immediately tells us that $B_{k<n} = 0$. For the rest of the B's we get a compact equation

$$\frac{\mathrm{d} \tilde{B}_{n+s}}{\mathrm{d} \lambda} = \sum_{k=0}^{s} A_{s-k} (n+k) \tilde{B}_{n+k},$$

$$\tilde{B}_{n+s}(0) = \delta_{s,0}, \qquad s = 0, 1, 2, \dots .$$

We have, for convenience, now introduced a new set of coefficients by

$$\tilde{B}_{n+s} = B_{n+s} + \delta_{s,0}\,.$$

By looking at the first few of these equations it is not hard to see that the general solution will be of the form

$$\tilde{B}_{n+s}(\lambda) = \sum_{k=0}^{s} C_{sk}^{[n]} e^{(n+k)A_0\lambda}\,.$$

By substituting this solution into eq. (10) we determine the coefficients C to be

$$C_{sk}^{[n]} = -\frac{\sum_{l=k}^{s-1} C_{lk}^{[n]}(n+l)\underline{A}_{s-l}}{s-k} \qquad (s>k),$$

$$C_{ss}^{[n]} = \delta_{s,0} - \sum_{l=0}^{s-1} C_{sl}^{[n]}, \tag{8}$$

where we have introduced the reduced coefficients $\underline{A}_m = A_m/A_0$. This gives us all the B's. After setting $\lambda = 1$ we find that

$$\boldsymbol{O}^{-1}\alpha_n^r\boldsymbol{O} = e^{nA_0}\left[\alpha_n^r + \sum_{s=1}^{\infty}\sum_{k=0}^{s} C_{sk}^{[n]} e^{kA_0}\alpha_{n+s}^r\right]. \tag{9}$$

Finally, eq. (5) becomes

$$0 = \langle V_x^{\text{Witten}}| \left[\left(e^{-nA_0}\alpha_{-n}^r + \sum_{a=1}^{\infty}\sum_{b=0}^{a} C_{ab}^{[-n]} e^{bA_0}\alpha_{-n+a}^r \right) \right.$$
$$\left. - nM_{nm}^{rs} e^{mA_0}\left(\alpha_m^s + \sum_{c=1}^{\infty}\sum_{d=0}^{c} C_{cd}^{[m]} e^{dA_0}\alpha_{m+c}^s \right) \right]. \tag{10}$$

As we see this is a linear combination of the first n equations in (1). Let us also note here that since both the M's and N's have cycling symmetry in the string indices it follows that $A_m^1 = A_m^2 = A_m^3$. This is why we have dropped the string label on the A's.

The simplest equation we retreive from (10) is for the $n=1$ mode. As we shall see later we will be able to determine our operator $\boldsymbol{O}$ completely from this equation alone. The higher mode equations that follow from (10) will then become consistency conditions. The existence of the conformal mapping between the dual model and the covariant, gauge invariant theory automatically will guarantee that these consistency conditions will be met. We will also set $r=1$, with no loss of generality due to the cyclic symmetry in string indices. For each m and s we find

$$-N^{1s}_{1m}\alpha^s_m = \sum_{a=1}^{\infty}\sum_{b=0}^{a} C^{[-1]}_{ab}\mathrm{e}^{bA_0}\alpha^1_{a-1}$$

$$-M^{1s}_{1m}\mathrm{e}^{(m+1)A_0}\left(\alpha^s_m + \sum_{c=1}^{\infty}\sum_{d=0}^{c} C^{[m]}_{cd}\mathrm{e}^{dA_0}\alpha^s_{m+c}\right). \tag{11}$$

For $m=0$, using momentum conservation, we find

$$\left(M^{11}_{10}-M^{13}_{10}\right)\mathrm{e}^{A_0}-\left(N^{11}_{10}-N^{13}_{10}\right)=\sum_{b=0}^{1} C^{[-1]}_{1b}\mathrm{e}^{bA_0},$$

$$\left(M^{12}_{10}-M^{13}_{10}\right)\mathrm{e}^{A_0}-\left(N^{12}_{10}-N^{13}_{10}\right)=0.$$

The Neumann coefficients needed are

$$N^{11}_{10}=0,$$

$$N^{12}_{10}=-N^{13}_{10}=\tfrac{2}{3^2}\sqrt{3},$$

$$M^{11}_{10}=M^{13}_{10}=0,$$

$$M^{12}_{10}=1.$$

The first few C's (as functions of the $\underline{A}$'s) have been calculated from (8), and can be found in table 1 of appendix A. The C's needed here are $C^{[-1]}_{10}=-C^{[-1]}_{11}=\underline{A}_1$. We get

$$A_0=-\ln\left(\tfrac{3}{2^2}\sqrt{3}\right),$$

$$\underline{A}_1=-\tfrac{2^3}{11}-\tfrac{2\cdot 3}{11}\sqrt{3}. \tag{12}$$

The CSV coefficients M are remarkably simple, and in order to calculate the rest of the A's we will exploit the fact that all the M's diagonal in string indices vanish, i.e. $M^{rr}=0$. This simplifies (11) for $s=1$ to

$$N^{11}_{1m}=-\sum_{b=0}^{m+1} C^{[-1]}_{m+1\,b}\mathrm{e}^{bA_0},\qquad m=1,2,3,\dots. \tag{13}$$

Using the second equation in (8) this becomes

$$C^{[-1]}_{m+1\,0}=\frac{-N^{11}_{1m}+\sum_{b=1}^{m}C^{[-1]}_{m+1\,b}\left(\mathrm{e}^{(m+1)A_0}-\mathrm{e}^{bA_0}\right)}{1-\mathrm{e}^{(m+1)A_0}}.$$

From (8) it also follows that

$$(m+1)C^{[-1]}_{m+1\,0} = \underline{A}_{m+1} - \sum_{l=2}^{m} C^{[-1]}_{l0}(l-1)\underline{A}_{m-l+1},$$

giving us a recursion relation for the $\underline{A}$'s

$$\underline{A}_{m+1} = \sum_{l=2}^{m} C^{[-1]}_{l0}(l-1)\underline{A}_{m-l+1} + \frac{m+1}{1-\mathrm{e}^{(m+1)A_0}} \times \left[\sum_{b=1}^{m} C^{[-1]}_{m+1\,b}\left(\mathrm{e}^{(m+1)A_0} - \mathrm{e}^{bA_0}\right) - N^{11}_{1m} \right], \qquad m = 1,2,\dots . \tag{14}$$

We note here that the r.h.s. of (14) is a given function (via eqs. (8)) of A_0, $\underline{A}_1, \dots, \underline{A}_m$, and can with (12) give us all the $\underline{A}$'s.

As an illustration of the use of (14) we calculate the next two coefficients of our conformal transformation operator. From tables 1 and 6 of appendix A we get the necessary information to construct (from (14)) the recursion relations

$$\underline{A}_2 = f(A_0, \underline{A}_1),$$

$$\underline{A}_3 = g(A_0, \underline{A}_1, \underline{A}_2).$$

Taking the values of A_0 and $\underline{A}_1$ from eq. (12) we find

$$\underline{A}_2 = \tfrac{2\cdot 5}{11},$$

$$\underline{A}_3 = -\tfrac{2^3\cdot 5\cdot 59}{11\cdot 13\cdot 109} - \tfrac{2\cdot 3\cdot 5}{13\cdot 109}\sqrt{3}. \tag{15}$$

Similarly, the rest of the $\underline{A}$'s follow by straightforward use of the recursion relations (14) and (8).

To conclude this section we recapitulate the calculation scheme. The C's are determined to mth level through recursion relation (8) as functions of A_0, $\underline{A}_1$, $\underline{A}_2, \dots, \underline{A}_m$. These are put in the r.h.s. of eq. (14), which then determines $\underline{A}_{m+1}$. Repeating this procedure we generate the next $\underline{A}$ etc. In this way we can calculate the A's to any specified level.

3. Ghost sector equations

In ref. [2] the Fock space representation of the full Witten vertex was derived, and the symmetries of the vertex were analyzed. The ghost sector was treated in its bosonized form. For calculational reasons it was of interest to recast the ghost sector in its original fermionic form. This has been achieved in ref. [3], using ghost overlap equations, and BRST invariance has been proven. The equivalence of the fermionic and bosonized forms of the ghost vertex has been shown [7].

The CSV vertex, while originally a vertex of a theory with only a bosonic sector, has been given an appropriate ghost piece [8] by imposing BRST invariance on the full vertex. Taking this approach, and assuming BRST invariance of Witten's theory, the form of its fermionic vertex was recently "guessed at" in ref. [9]. The Fock space representation of the fermionic vertices is given by

$$\langle V_c^{\text{Witten}} = \langle +,+,+|\exp\left(c_n^r n \mathcal{N}_{nm}^{rs} \bar{c}_m^s\right),$$

$$\langle V_c^{\text{CSV}}| = \langle +,+,+|\exp\left(c_n^r n \mathcal{M}_{nm}^{rs} \bar{c}_m^s\right),$$

$$n = 1,2,3,\ldots, \qquad m = 0,1,2,\ldots,$$

where the + ghost vacuum is the one defined by

$$c_{n\geqslant 0}|+\rangle = 0,$$

$$\bar{c}_{n>0}|+\rangle = 0.$$

The coefficients $\mathcal{N}$ for the Witten ghost vertex can be found in ref. [3], while the CSV ghost vertex coefficients $\mathcal{M}$ were derived in ref. [8]. Given this, it is a simple matter to write down the equations

$$\langle V_c^{\text{Witten}}\left(c_{-n}^r - m\mathcal{N}_{mn}^{sr} c_m^s\right) = 0, \tag{16}$$

$$\langle V_c^{\text{CSV}}\left(c_{-n}^r - m\mathcal{M}_{mn}^{sr} c_m^s\right) = 0, \tag{17}$$

$$n = 0,1,2,\ldots, \qquad m = 1,2,3,\ldots .$$

Again, these equations are equivalent to the defining relations for the two vertices. In this they are analogous to eqs. (1) and (2) in the bosonic sector. The ghost vertices also give the $\bar{c}$ equations

$$\langle V_c^{\text{Witten}}\left(\bar{c}_{-n}^r + n\mathcal{N}_{nm}^{rs} \bar{c}_m^s\right) = 0, \tag{18}$$

$$\langle V_c^{\text{CSV}}\left(\bar{c}_{-n}^r + n\mathcal{M}_{nm}^{rs} \bar{c}_m^s\right) = 0, \tag{19}$$

$$n = 1,2,3,\ldots, \qquad m = 0,1,2,\ldots .$$

Finally, we mention a third set of equations satisfied by the full vertices

$$\langle V^{\text{Witten}}|\left(\hat{L}_{-n}^r + n\mathcal{N}_{nm}^{rs} \hat{L}_m^s\right) = 0, \tag{20}$$

$$\langle V^{\text{CSV}}|\left(\hat{L}_{-n}^r + n\mathcal{M}_{nm}^{rs} \hat{L}_m^s\right) = 0, \tag{21}$$

$$n = 1,2,3,\ldots, \qquad m = 0,1,2,\ldots .$$

Eqs. (20) and (21) can also be derived from certain overlap equations, and this has been shown in ref. [3]. We note that the assumption of BRST invariance determines these equations only up to a possible nontrivial center contribution which has been shown by explicit calculation [10] to be absent.

The $\hat{L}$'s in eqs. (20) and (21) represent the full Virasoro generators, i.e. $\hat{L} = L + L^{\mathrm{c}}$.

$$L^{\mathrm{c}}_m = \sum_n (n-m) : \bar{c}_{m+n} c_{-n} : - \alpha\delta_{m,0} . \tag{22}$$

In the critical dimension of the model, which is $D = 26$ for the bosonic string, the algebra of the $\hat{L}$'s closes (the centers of the coordinate and ghost contributions cancel)*.

$$\left[\hat{L}_n, \hat{L}_m\right] = (n-m)\hat{L}_{n+m} . \tag{23}$$

Using (22) we can also easily show

$$\left[\hat{L}_n, c_m\right] = -(2n+m)c_{n+m} , \tag{24}$$

$$\left[\hat{L}_n, \bar{c}_m\right] = (n-m)\bar{c}_{n+m} . \tag{25}$$

Eqs. (23)–(25) are central for our calculation of the commutators of $\hat{\boldsymbol{O}}$ with $\hat{L}$, c and $\bar{c}$. The full operator $\hat{\boldsymbol{O}}$ follows from $\boldsymbol{O}$ by replacing the L's with the full Virasoro generators $\hat{L}$'s. At the end of this section we will show that $\hat{\boldsymbol{O}}$ is the operator of the conformal transformation linking the full Witten and CSV vertices, i.e. that the following equation holds

$$\langle V^{\mathrm{CSV}} | \hat{\boldsymbol{O}} = \langle V^{\mathrm{Witten}} | .$$

Let us note that the commutators (7) and (23)–(25) are of the same form, which allows us to use similar derivations as in the previous section. We find

$$\hat{\boldsymbol{O}}^{-1} c^r_n \hat{\boldsymbol{O}} = \mathrm{e}^{nA_0}\left[c^r_n + \sum_{s=1}^{\infty}\sum_{k=0}^{s} D^{[n]}_{sk}\mathrm{e}^{kA_0} c^r_{n+s}\right], \tag{26}$$

$$\hat{\boldsymbol{O}}^{-1} \bar{c}^r_n \hat{\boldsymbol{O}} = \mathrm{e}^{nA_0}\left[\bar{c}^r_n + \sum_{s=1}^{\infty}\sum_{k=0}^{s} \overline{D}^{[n]}_{sk}\mathrm{e}^{kA_0} \bar{c}^r_{n+s}\right], \tag{27}$$

$$\hat{\boldsymbol{O}}^{-1} \hat{L}^r_n \hat{\boldsymbol{O}} = \mathrm{e}^{nA_0}\left[\hat{L}^r_n + \sum_{s=1}^{\infty}\sum_{k=0}^{s} \overline{D}^{[n]}_{sk}\mathrm{e}^{kA_0} \hat{L}^r_{n+s}\right], \tag{28}$$

* Q_{BRST} is the central object in string theory. For the bosonic string we have [11]

$$Q_{\mathrm{BRST}} = \sum_n (L_n - \alpha\delta_{n,0})c_{-n} + \tfrac{1}{2}\sum_{nm}(n-m) : \bar{c}_{n+m} c_{-m} c_{-n} : .$$

The full Virasoro generators can be determined from Q_{BRST} by $\hat{L}_k = \{\bar{c}_k, Q_{\mathrm{BRST}}\}$. The nilpotency of the charge ($Q^2_{\mathrm{BRST}} = 0$) determines the critical dimension and Regge trajectory intercept of the theory ($D = 26$, $\alpha = 1$), and makes the Virasoro algebra of the $\hat{L}$'s anomaly free.

where the coefficients D and $\overline{D}$ play the same role as the C's did in the previous section. D and $\overline{D}$ are given by

$$D_{sk}^{[n]} = -\frac{\sum_{l=k}^{s-1} D_{lk}^{[n]}(n-l+2s)A_{s-1}}{s-k} \qquad (s>k),$$

$$D_{ss}^{[n]} = \delta_{s,0} - \sum_{l=0}^{s-1} D_{sl}^{[n]}, \tag{29}$$

and

$$\overline{D}_{sk}^{[n]} = -\frac{\sum_{l=k}^{s-1} \overline{D}_{lk}^{[n]}(n+2l-s)\underline{A}_{s-l}}{s-k} \qquad (s>k),$$

$$\overline{D}_{ss}^{[n]} = \delta_{s,0} - \sum_{l=0}^{s-1} \overline{D}_{sl}^{[n]}. \tag{30}$$

We note that the same coefficients ($\overline{D}$)'s are used in calculating the similarity transformation with the conformal operator of the $\bar{c}$'s and $\hat{L}$'s. This is due to the fact that the structure constants in the commutators (23) and (24) are the same. This parallel between $\bar{c}$ and $\hat{L}$ extends, as we see, also to the vertex equations (18), (19) compared to (20), (21).

Assuming that $\hat{O}$ is the full conformal operator, we will use formulas (26)–(28) to derive consistency conditions that will validate our assumption.

The $\hat{L}$ equations in (21) give us

$$0 = \langle V^{\text{Witten}}| \hat{O}^{-1}\left(\hat{L}_{-n}^{r} + n\mathscr{M}_{nm}^{rs}\hat{L}_{m}^{s}\right)\hat{O}, \qquad n=1,2,3,\ldots, \qquad m=0,1,2,\ldots. \tag{31}$$

As before we conclude that this is a linear combination of the first n equations in eq. (20), the simplest of which is for $n=1$.

$$\mathscr{N}_{1m}^{1s} = \sum_{b=0}^{m+1} \overline{D}_{m+1\,b}^{[-1]} e^{bA_0}\delta^{1,s} + \mathscr{M}_{1m}^{1s} e^{(m+1)A_0} + \sum_{k=0}^{m-1}\sum_{d=0}^{m-k} \mathscr{M}_{1k}^{1s}\overline{D}_{m-k\,d}^{[k]} e^{(k+d+1)A_0}. \tag{32}$$

For $m=$ formula (32) gives the consistency conditions

$$\mathscr{N}_{10}^{11} - e^{A_0}\mathscr{M}_{10}^{11} = \overline{D}_{10}^{[-1]}(1-e^{A_0}),$$

$$\mathscr{N}_{10}^{12} - e^{A_0}\mathscr{M}_{10}^{12} = 0,$$

$$\mathscr{N}_{10}^{13} - e^{A_0}\mathscr{M}_{10}^{13} = 0.$$

Reading off $\overline{D}_{10}^{[-1]}$ from table 4 of appendix A, and using (12) we find that $\overline{D}_{10}^{[-1]}(1-\mathrm{e}^{A_0}) = -\frac{2^2}{3^2}\sqrt{3}$. Using the following ghost vertex coefficients

$$\mathcal{N}_{10}^{11} = 0\,,$$

$$\mathcal{N}_{10}^{12} = -\mathcal{N}_{10}^{13} = \tfrac{2^2}{3^2}\sqrt{3}\,,$$

$$\mathcal{M}_{10}^{11} = 1\,,$$

$$\mathcal{M}_{10}^{12} = -\mathcal{M}_{10}^{13} = 1\,,$$

we find that these consistency conditions are indeed met.

For $m = 1,2,3,\ldots$ the CSV ghost vertex coefficients $\mathcal{M}_{1m}^{11}$ vanish, so that eq. (32) reduces to the much simpler form

$$\mathcal{N}_{1m}^{11} = \sum_{b=0}^{m+1} \overline{D}_{m+1\,b}^{[-1]}\mathrm{e}^{bA_0} + \mathrm{e}^{A_0}\sum_{d=0}^{m} \overline{D}_{md}^{[0]}\mathrm{e}^{dA_0}\,. \tag{33}$$

Let us note in passing the similarity of (33) and eq. (13) derived in the coordinate sector. The essential difference between these two is that in the ghost CSV vertex there exists a nonzero coefficient ($\mathcal{M}_{10}^{11}$) diagonal in string indices, while in the coordinate part of the CSV vertex all such coefficients vanish.

Using tables 4 and 7 in appendix A we have checked the consistency condition in (33) for $m = 1$ and 2. We find a quite nontrivial cancellation process at work. For $m > 2$, eq. (33) has to be checked numerically.

The $\bar{c}$ equations are the same as the $\hat{L}$ equations, so they do not give us any new information. The c equations, however, do give rise to new consistency conditions. Eq. (17) gives

$$0 = \langle V^{\text{Witten}}|\hat{O}^{-1}\left(c_{-n}^{r} - m\mathcal{M}_{mn}^{sr}c_{m}^{s}\right)\hat{O}\,,\qquad n = 0,1,2,\ldots,\qquad m = 1,2,3,\ldots\,. \tag{34}$$

Again, this is a particular linear combination of the first n equations (16). The simplest equation we get now is for $n = 0$, giving us

$$-\mathcal{N}_{m0}^{s1}m = \sum_{b=0}^{m} D_{mb}^{[0]}\mathrm{e}^{bA_0}\delta^{s,1} - \mathcal{M}_{m0}^{s1}m\,\mathrm{e}^{mA_0} - \sum_{k=1}^{m-1}\sum_{d=0}^{m-k}\mathcal{M}_{k0}^{s1}kD_{m-k\,d}^{[k]}\mathrm{e}^{(k+d)A_0}\,. \tag{35}$$

These consistency conditions were examined for $m = 1,2$ and found to work. The equation for $n = 1$, $m = 1$ was also checked. As in the $\hat{L}(\bar{c})$ case the cancellation procedure is far from trivial. The higher level equations again can be checked only numerically.

As a final check of the internal consistency of our procedure in the ghost sector we looked at the commutator of $\hat{O}$ with the total BRST charge

$$Q_{\mathrm{BRST}} = \hat{L}^r_n \cdot c^r_{-n}.$$

Using $\hat{L}_k = \{\bar{c}_k, Q_{\mathrm{BRST}}\}$ we find

$$[\hat{L}_k, Q_{\mathrm{BRST}}] = [\{\bar{c}_k, Q_{\mathrm{BRST}}\}, Q_{\mathrm{BRST}}] = 0,$$

where we have used the nilpotence of the BRST charge to derive this. It follows that independent of the A's we have

$$[\hat{L}_k, \hat{O}] = 0.$$

On the other hand, working with formulas (26)–(28), after a long but straightforward calculation we find that

$$\hat{O}^{-1} Q_{\mathrm{BRST}} \hat{O} = Q_{\mathrm{BRST}} + \sum_n \sum_{c=1}^{\infty} \mathscr{F}_c^{[n]} \hat{L}^r_n c^r_{-n+c},$$

where the coefficients $\mathscr{F}_c^{[n]}$ are given by

$$\mathscr{F}_c^{[n]} = \sum_{d=0}^{c} \left(D_{cd}^{[-n]} + \overline{D}_{cd}^{[n-c]} + \sum_{a=1}^{c} \sum_{l=\max\{0,d+a-c\}}^{\min\{a,d\}} \overline{D}_{al}^{[n-a]} D_{c-a\,d-l}^{[-n+a]} \right) e^{dA_0}. \tag{36}$$

Combining these two results we get that independent of the A's the following relation must hold

$$\mathscr{F}_c^{[n]} = 0. \tag{37}$$

Eq. (37) tells us that D and $\overline{D}$ are in some sense "reciprocal" to each other. This result should not, perhaps, sound so strange, considering that the D's are associated with ghost, and $\overline{D}$'s with antighost fields. We have checked (37), using coefficients given in tables 2–5 of appendix A, for the first 3 levels.

In this section we have found that the conformal operator connecting the full vertices of Witten and of Caneschi, Schwimmer and Veneziano can be constructed from the operator linking the coordinate pieces of these vertices by simply replacing the coordinate Virasoro generators by the corresponding full generators. The byproducts of this work are the commutators of α, c, $\bar{c}$ and $\hat{L}$ with this conformal operator $\hat{O}$ as given by relations (8,9) and (26)–(30). These commutators will be of central importance in all uses of the explicit conformal transformation operator just constructed.

4. Conclusion

The operator giving the conformal transformation between the full (coor and ghost) interaction vertex of Witten's interacting string field theory and tl dual model vertex has been constructed. The commutators of the fields wit conformal operator have been given. Together they represent the basic tool will be needed in order to exploit the connection between the two theories explicit connection is of interest for two basic reasons. First, the vertex of Can Schwimmer and Veneziano is much easier to work with due to the remaı simplicity of the coefficients M and $\mathscr{M}$ of its coordinate and ghost sectors. Se the dual model is a template for all string theories, and the existence of the e: connection to Witten's theory will be of use since it will enable us to "tran many of the important old results of the dual models into the language of cova gauge invariant string field theory. The main goal of this program is to lir Feynman rules of Witten's theory to the "Feynman-like" rules of dual model

The procedure used in this paper can also be applied to constructing the op of the conformal transformation linking the light-cone vertex to the dual mod the operator linking Witten's vertex to the light-cone vertex. The latter operato prove to be very interesting in that it may help us to learn how to fix gauges gauge invariant theory.

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Note added

During the completion of this work we have learned of another attempt [constructing the conformal operator. Using a different procedure and workin in the bosonic sectors of the corresponding theories, the conformal operatc calculated up to second level. The procedure used failed to give a general con tion to all levels.

Appendix A

In this appendix, in tables 1–5, we list the C, D, and $\bar{D}$ coefficients (as calc from the recursion relations (8), (29) and (30)) that were used in the paper. In 6 we list (for the first four levels) the Neumann coefficients of Witten's coor and ghost vertices that are used in eqs. (14) and (33). In table 7 the first ten $\underline{A}$ given.

TABLE 1
The low-order bosonic coefficients C_{sk}

$C_{sk}^{[-1]}$	$k=0$	$k=1$	$k=2$	$k=3$
$s=0$	1			
$s=1$	$\underline{A}_1$	$-\underline{A}_1$		
$s=2$	$\frac{1}{2}\underline{A}_2$	0	$-\frac{1}{2}\underline{A}_2$	
$s=3$	$\frac{1}{3}\underline{A}_3-\frac{1}{6}\underline{A}_1\underline{A}_2$	0	$\frac{1}{2}\underline{A}_1\underline{A}_2$	$-\frac{1}{3}\underline{A}_3-\frac{1}{3}\underline{A}_1\underline{A}_2$

TABLE 2
Ghost coefficients for $n=-1$

$D_{sk}^{[-1]}$	$k=0$	$k=1$	$k=2$	$k=3$
$s=0$	1			
$s=1$	$-\underline{A}_1$	$\underline{A}_1$		
$s=2$	$-\frac{3}{2}\underline{A}_2+\underline{A}_1^2$	$-2\underline{A}_1^2$	$\frac{3}{2}\underline{A}_2+\underline{A}_1^2$	
$s=3$	$-\frac{5}{3}\underline{A}_3+\frac{17}{6}\underline{A}_1\underline{A}_2-\underline{A}_1^3$	$-2\underline{A}_1\underline{A}_2+3\underline{A}_1^3$	$-\frac{9}{2}\underline{A}_1\underline{A}_2-3\underline{A}_1^3$	$\frac{5}{3}\underline{A}_3+\frac{11}{3}\underline{A}_1\underline{A}_2+\underline{A}_1^3$

TABLE 3
Ghost coefficients for $n=0$

$D_{sk}^{[0]}$	$k=0$	$k=1$	$k=2$
$s=0$	1		
$s=1$	$-2\underline{A}_1$	$2\underline{A}_1$	
$s=2$	$-2\underline{A}_2+3\underline{A}_1^2$	$-6\underline{A}_1^2$	$2\underline{A}_2+3\underline{A}_1^2$

TABLE 4
Antighost coefficients for $n=-1$

$\overline{D}_{sk}^{[-1]}$	$k=0$	$k=1$	$k=2$	$k=3$
$s=0$	1			
$s=1$	$2\underline{A}_1$	$-2\underline{A}_1$		
$s=2$	$\frac{3}{2}\underline{A}_2+\underline{A}_1^2$	$-2\underline{A}_1^2$	$-\frac{3}{2}\underline{A}_2+\underline{A}_1^2$	
$s=3$	$\frac{4}{3}\underline{A}_3+\frac{4}{3}\underline{A}_1\underline{A}_2$	$-2\underline{A}_1\underline{A}_2$	0	$-\frac{4}{3}\underline{A}_3+\frac{2}{3}\underline{A}_1\underline{A}_2$

TABLE 5
Antighost coefficients for $n=0$

$\overline{D}^{[0]}_{sk}$	$k=0$	$k=1$	$k=2$
$s=0$	1		
$s=1$	$\underline{A}_1$	$-\underline{A}_1$	
$s=2$	$\underline{A}_2$	0	$-\underline{A}_2$

TABLE 6
The Neumann coefficients for the bosonic and ghost vertex

m	N^{11}_{1m}	$\mathscr{N}^{11}_{1m}$
1	$-\frac{5}{3^3}$	$\frac{11}{3^3}$
2	0	0
3	$\frac{2^5}{3^6}$	$-\frac{2^4\cdot 5}{3^6}$
4	0	0

TABLE 7
Numerical values for the first few coefficients in the conformal operator transformation

m	$\underline{A}_m$
1	−1.67203
2	+0.90909
3	+0.18808
4	+0.14369
5	−0.14743
6	+0.25667
7	−0.12597
8	+0.18019
9	−0.10968
10	+0.19065

Appendix B

In this appendix we will use a generalized form [12] of the well-known Baker-Hausdorff formula in order to write the conformal operator derived in sects. 2 and 3 as a product of exponentials of the Virasoro generators $\hat{L}_0, \hat{L}_1, \hat{L}_2, \dots$. Although the form given in eq. (3) is much more useful for applications, this alternate form may still be of some interest. The formula we will use here states that

$$e^A e^B = \exp(A + \mathscr{L}\cdot [B + (\coth \mathscr{L})\cdot B] + \cdots). \tag{38}$$

This expression contains all the terms linear in the operator B. The Lie derivative $\mathscr{L}$ is defined as

$$\mathscr{L}\cdot X = \left[\tfrac{1}{2}A,\, X\right].$$

The hyperbolic cotangent is understood through its power series expansion.

If we use this formula for operators A and B such that

$$[A, B] = a_0 B, \tag{39}$$

then we get the result

$$e^A e^B = e^{A + \lambda(a_0) B}, \tag{40}$$

where $\lambda(a_0)$ is given by the expression

$$\lambda(a_0) = \frac{a_0}{1 - e^{-a_0}}. \tag{41}$$

If we now look at a set of three operators A, B, C satisfying the commutation relations

$$\begin{aligned} [A, B] &= a_0 B, \\ [A, C] &= 2a_0 C, \\ [B, C] &= \cdots \end{aligned} \tag{42}$$

(where the dots indicate that the commutator is a linear combination of operators other than A, B, C), then by using eqs. (40)–(42) it is easy to show that

$$e^A e^B e^C = \exp\left(A + \lambda(a_0) B + \lambda(2a_0) C + \cdots\right). \tag{43}$$

The operators we use are

$$\begin{aligned} A &= -a_0 \sum_r \left(\hat{L}_0^r - 1\right), \\ B &= -a_1 \sum_r \hat{L}_1^r, \\ C &= -a_2 \sum_r \hat{L}_2^r. \end{aligned} \tag{44}$$

It is very easy to check that the commutation relations (42) are indeed satisfied.

We have

$$\langle V^{\mathrm{CSV}}| = \langle V^{\mathrm{Witten}}|\exp\Big(-A_0\sum_r\big(\hat{L}_0^r - 1\big) - A_1\sum_r \hat{L}_1^r - A_2\sum_r \hat{L}_2^r - \cdots\Big),$$

where the coefficients A_m are given in sect. 2. Using formulas (41)–(44) we find that up to level $n=2$ we have

$$\langle V^{\mathrm{CSV}}| = \langle V^{\mathrm{Witten}}|\exp\Big(-a_0\sum_r\big(\hat{L}_0^r - 1\big)\Big)\exp\Big(-a_1\sum_r \hat{L}_1^r\Big)\exp\Big(-a_2\sum_r \hat{L}_2^r \dots\Big).$$

The new coefficients a_0, a_1 and a_2 are related to our old coefficients in the following way

$$a_0 = A_0\,,$$

$$a_1 = \underline{A}_1\big(1 - \mathrm{e}^{-A_0}\big),$$

$$a_2 = \tfrac{1}{2}\underline{A}_2\big(1 - \mathrm{e}^{-2A_0}\big).$$

Using eqs. (12) and (15) we find

$$a_0 = -\ln\Big(\tfrac{3}{2^2}\sqrt{3}\Big),$$

$$a_1 = \tfrac{1}{2}\,,$$

$$a_2 = -\tfrac{5}{16}\,.$$

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