BRST INVARIANCE OF THE MEASURE IN STRING FIELD THEORY ☆

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The BRST transformations, given by gauge-fixing Witten's string field theory in the Siegel gauge, are applied to the string measure It is shown that the simple measure (just the product of differentials of all the fields) is BRST invariant, thus maintaining the invariance of the gauge-fixed action at the quantum level

1 Introduction Recently, Witten's gauge invariant string field theory [1] has been gauge fixed [2,3] in the linear gauge proposed by Siegel [4] The problem of fixing the gauge in an interacting gauge invariant string field theory is highly non-trivial due to the issue of ghost of ghosts. The gauge-fixing was accomplished by Bochicchio using the procedure developed by Batalin and Vilkovisky [5,6] Independently, plausibility arguments for the form of the gauge-fixed action were given by the Kyoto group [7,8], and by Thorn [9] The gauge-fixed action was found to be invariant under a non-linear generalization of the BRST transformations of the free theory The problem of finding a correct measure that would lead to a BRST invariant generating functional for Witten's string theory was still left open

The Feynman rules for Witten's theory were derived in the first-quantized formalism [10–12] In order to complete the proof of the equivalence of first- and second-quantized formalisms it must be shown that they lead to the same Feynman rules. To be able to do this the string field measure must be specified In ref [9] some indication was given that by using the simple measure (product of differentials of all the fields) the correct Feynman rules can be derived

In this letter we examine the string jacobian corresponding to the BRST transformation of the simple measure and find that it is identically equal to one Throughout, a parallel comparison with Yang–Mills theories is given

2 The BRST jacobian For a set of fields $\{A_a\}$ with a $\mathbb{Z}_2$ grading $(-)^{\epsilon_a}$ the jacobian of the measure $\Pi_a \mathrm{d}A_a$ with respect to an arbitrary transformation $A_a \to A'_a$ is given by

$$J = \det(M_a{}^b) ,$$

where

$$M_a{}^b = \partial A'_a / \partial A_b \quad \text{if } A_a \text{ is even} ,$$
$$= \partial A_a / \partial A'_b \quad \text{if } A_a \text{ is odd}$$

It is easy to see that if the fields of opposite grading do not mix we get back the familiar result

$$J = \det(\text{even fields}) \; [\det(\text{odd fields})]^{-1}$$

In general, a BRST transformation is of the form

$$A_a \to A'_a = A_a + \delta_\theta A_a ,$$

where $\delta_\theta A_a$ is proportional to a Grassmann parameter θ Using the fact that $\theta^2 = 0$ we find that

$$M_a{}^b = \delta_a^b + (-)^{\epsilon_a} \frac{\partial}{\partial A_b} (\delta_\theta A_a)$$

Finally, we get

$$J = \exp\left(\sum_a (-)^{\epsilon_a} \frac{\partial}{\partial A_a} (\delta_\theta A_a) \right) \tag{2 1}$$

The vanishing of the graded trace (called a super-

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trace) in the exponent above enforces BRST invariance of the simple measure

In Yang–Mills theories we are given a Lie group with generators $\{t^a\}$ such that

$$[t^a, t^b] = f^{ab}{}_c t^c . \tag{2 2}$$

Using the fact that the gauge group is semi-simple we can construct a Killing form

$$\mathrm{Tr}(t^a t^b) = g^{ab} , \tag{2 3}$$

where the generators can be chosen so that g^{ab} has 1's and -1's on the diagonal, while all the other components vanish The Killing form is used as a metric to raise and lower group indices From the BRST transformations in Yang–Mills and (2 1) we find the jacobian to be

$$J_{\mathrm{YM}} = \exp[(g-D) f^{ab}{}_a c_b \theta] , \tag{2 4}$$

$c = c_b t^b$ being the ghost field The BRST invariance of the measure follows directly from the group property (2 3) We find that for all a and b

$$f^{ab}{}_a = \frac{1}{g^{aa}} \mathrm{Tr}(t^a [t^a, t^b]) = 0 .$$

The last step follows from the cyclic symmetry of the trace. The sum in the exponential in (2 4) thus vanishes since all of its terms vanish

3 The BRST jacobian for strings Witten's gauge invariant string field action [1] is given by

$$S_{\mathrm{Witten}} = \frac{1}{2} \int A * QA + \frac{1}{3} \int A * A * A$$

The string operations $\int$ and $*$ satisfy the following axioms

$$A * (B * C) = (A * B) * C ,$$
$$\int A * B = (-)^{AB} \int B * A ,$$
$$Q(A * B) = QA * B + (-)^A A * QB ,$$
$$\int QA = 0 \quad \text{for all } A ,$$
$$g(A * B) = g(A) + g(B) + \tfrac{3}{2} ,$$
$$\int A * B = 0 \quad \text{unless } g(A) + g(B) = 0 ,$$

where g is the first-quantized ghost number operator

$$g = \sum_n c_{-n} b_n - \tfrac{1}{2}$$

The gauge invariance of S_{Witten} is thus

$$\delta_\Lambda A = Q\Lambda + [A, \Lambda]$$

Imposing the Siegel gauge $b_0 A = 0$, the authors of refs [2,3,9] derived the gauge-fixed action

$$S = \frac{1}{2} \int \Phi * Q\Phi + \frac{1}{3} \int \Phi * \Phi * \Phi - \int b_0 \beta * \Phi ,$$

invariant under the BRST transformations

$$\delta_\theta \Phi_+ = (b_0 \beta)_+ \ \theta ,$$
$$\delta_\theta \Phi_- = (Q\Phi + \Phi * \Phi)_- \ \theta ,$$
$$\delta_\theta \beta = 0 \tag{3 1}$$

Φ is just a sum of string fields of all ghost numbers, i.e $\Phi = \sum_g \Phi_g$ In Φ_+ the sum is restricted to positive g, and in Φ_- to negative g It is easy to show that the associated (second-quantized) BRST charge $\boldsymbol{Q}$ is nilpotent only on-shell

The operator g can be used to classify states in the Fock space spanned by the α, c and b. We write a basis in this space as

$$\{ | t^i_g \rangle \ g \in \mathbb{Z} - \tfrac{1}{2} \}$$

(i represents all other quantum numbers) The string fields are written as general kets in this Fock space

$$\Phi = | \Phi \rangle = \sum_{g\,i} | t^i_g \rangle A_{g\,i}$$

There is a simple relation between the second-quantized ghost number G and g

$$G = -\tfrac{1}{2} - g .$$

The easiest way to see this is to look at the free theory gauge-fixing procedure (this has been nicely reviewed in ref. [13], where the original references are also listed). As we see from this $\Phi_{-1/2} = A$

String fields are taken to be overall odd, thus the ghost number g induces the following $\mathbb{Z}_2$ gradings

$| t_g \rangle$ has grading $(-)^{g-1/2}$,

A_g has grading $(-)^{g+1/2}$

From the Fock space expressions for the interaction vertices of Gross and Jevicki [14,15] we see that $\int$ carries a $(-)$ and $*$ a $(+)$ $\mathbb{Z}_2$ grading

The BRST jacobian for strings follows from these gradings, eq (3 11), as well as the general result (2 1)

$$J_{\text{string}}=\exp\left(\sum_{g<0}\sum_{i}(-)^{g+1/2}\frac{\partial}{\partial A_{g\,i}}\delta_\theta A_{g\,i}\right) \tag{3 2}$$

To every basis state $|t_g^i\rangle$ we associate its $\tilde{\ }$dual state $|\tilde{t}_{g\,i}\rangle$

$$|t_g^i\rangle\to{}^{\sim}|t_g^i\rangle=|\tilde{t}_{g\,i}\rangle\,,$$

$$|\tilde{t}_{g\,i}\rangle\to{}^{\sim}|\tilde{t}_{g\,i}\rangle=|t_g^i\rangle\,,$$

defined so that

$$\int|\tilde{t}_{g\,j}\rangle*|t_g^i\rangle=\delta_j^i \tag{3 3}$$

Using the $\langle V_2|$ vertex of refs [14,15] it is possible to explicitly calculate the $\tilde{t}$. We find that, up to a normalization constant, $\tilde{\ }$duality, in the oscillator basis, just corresponds to the interchange of the b and c (as well as the $|+\rangle$ and $|-\rangle$ vacua) As a consequence of this $g(\tilde{t}_g)=-g$

Using (3 1) we find for $g<0$ that

$$\delta_\theta A_{g\,i}=\int|\tilde{t}_{g\,i}\rangle*(\Phi_g*\Phi_{-3/2}+\Phi_{-3/2}*\Phi_g)\;\theta$$

$$+(\text{pieces without }\Phi_g)$$

Differentiating this expression with respect to $A_{g\,i}$ we get

$$\frac{\partial}{\partial A_{g\,i}}\delta_\theta A_{g\,i}$$

$$=(1+\delta_{g\,-3/2})\int|\tilde{t}_{g\,i}\rangle*[\,|t_g^i\rangle,\Phi_{-3/2}\}\;\theta\,, \tag{3 4}$$

where [, } represents the usual graded commutator Using (3 2), (3 4), as well as the (anti) cyclic property of the $\int$ given in the axioms, we find the string BRST jacobian to be equal to

$$J_{\text{string}}=\exp\Bigg(\sum_{g<0}\sum_{i}(-)^{g+1/2}\;(1+\delta_{g\,-3/2})$$

$$\times\int[\,|\tilde{t}_{g\,i}\rangle,|t_g^i\rangle]*\Phi_{-3/2}\;\theta\Bigg) \tag{3 5}$$

The above formula can also be found in ref. [3] Let us note here the formal similarity of this result with eq (2 4) for Yang–Mills theories In both cases only the ghost fields are present in the jacobian There is, however, an important difference between the two

In Yang–Mills theories the gauge algebra is a semi-simple Lie algebra, and this directly led us to BRST invariance of the measure The string algebra is different, it can be written as

$$[t_g^i,t_h^j]=f^{ij}{}_k(g,h)\;t^k_{g+h+3/2}$$

For the string algebra, the integration $\int$ serves as a generalization of the trace, as can be seen from the string overlap conditions [1,14,15] through which it is defined The natural analogue of (2 3) would be to use $\int t_g^i*t_h^j$, however, this object always has zeros on the diagonal, and thus is no good This can be fixed, as we can see from eq (3 3), by instead using

$$\int{}^{\sim}(t_g^i)*t_h^j \tag{3 6}$$

Here, in fact, is the most important difference between the gauge algebras of strings and Yang–Mills theories To be able to use the object introduced above to show BRST invariance of the string measure, we would have to be able to move the tilde operator ($\sim$) under the integral in (3 5) from one of the t to $\Phi_{-3/2}$ We are not able to perform this kind of "partial integration" since

$$\langle V|{}^{\sim\,\text{total}}\neq0\,,$$

where $^{\sim\,\text{total}}$ is the sum of $\sim$'s acting in all the corresponding string spaces As a consequence, in string theory we cannot find as elegant a proof of BRST invariance of the measure as was the case in Yang–Mills theories

4 *BRST invariance* The $*$ algebra has an identity element $\mathscr{I}$, i e a string field such that

$$\mathscr{I}*X=X*\mathscr{I}=X\quad\text{for all string fields }X$$

From the axioms and the assigned gradings we see that $\mathscr{I}$ is an even, $g=-\frac{3}{2}$ string field. The Fock space realization of $\mathscr{I}$ can be found in refs [14,15], where it is called the integration vertex (this in fact proves the existence and uniqueness of $\mathscr{I}$) From (3 5) we get

$$J(\lambda\mathscr{I})=1\,,$$

where λ is an arbitrary Grassmann number This is an indication of the BRST invariance of the string measure

The simplest way for this invariance to hold (and the one most reminiscent of Yang–Mills theories) would be to have vanishing of integrals of the form

$$\int [\,|\tilde{t}_{g,i}\rangle, |t^{i}_{g}\rangle] * |t^{j}_{-3/2}\rangle\,, \qquad (4.1)$$

for all $g<0$, i and j. The way to calculate these integrals is to take the t to be elements of the oscillator basis, and to use the three-string vertex of Gross and Jevicki [14,15].

Unfortunately, it is rather easy to construct a counter-example to the above claim. Let us take $|t^{i}_{g}\rangle = |-\rangle$ as well as $|t^{j}_{-3/2}\rangle = b_{-n}|-\rangle$. The ~ dual of $|-\rangle$ is $|+\rangle$, so we have

$$\int [\,|+\rangle, |-\rangle] * b_{-n}|-\rangle$$

$$= \langle V_3 | b^{3}_{-n} | +, -, -\rangle - (1 \leftrightarrow 2)$$

$$= (-n\tilde{N}^{31}_{n0}) - (1 \leftrightarrow 2)$$

Instead of using the explicit values of the Neumann coefficients $\tilde{N}$ we will use the following symmetries

$$\tilde{N}^{rs} = \tilde{N}^{r+1\,s+1}\,, \quad \tilde{N}^{12}_{nm} = (-)^{n+m}\tilde{N}^{13}_{nm}$$

We see that the above integral does indeed vanish for even, but not for odd values of n.

From a few examples like this, it soon becomes evident that viewed in this basis BRST invariance (if it is to exist at all) comes about through a very complicated (infinite) cancellation procedure of integrals (4.1) involving (all powers of) the corresponding Neumann coefficients. So rather than have each integral (4.1) vanish, we must try to prove that

$$\sum_{i} \int [\,|\tilde{t}_{g,i}\rangle, |t^{i}_{g}\rangle] * |\Phi_{-3/2}\rangle = 0\,, \qquad (4.2)$$

for all $g<0$, and arbitrary $\Phi_{-3/2}$. At first hand there seems to be little hope of showing this in the oscillator basis. Eq. (4.2) is a kind of trace, so we may look for a basis in which it takes the simplest form.

The string integration takes on its simplest form in the coordinate representation. The basis vectors are

$$|X(\sigma)\rangle \equiv \prod_{\mu=0}^{25} |x^{\mu}(\sigma)\rangle\,|\phi(\sigma)\rangle\,,$$

where $\phi(\sigma) = x^{26}(\sigma)$ is the bosonized ghost coordinate [1,14,15]. Witten's interaction was first formulated in this basis and is represented by a product of delta functions enforcing the correct string overlaps. Ignoring mid-point insertions, for the moment, we have

$$\int |X\rangle * |Y\rangle$$

$$= \prod_{0<\sigma<\pi/2} \delta[X(\sigma) - Y(\pi-\sigma)]\ \delta[Y(\sigma) - X(\pi-\sigma)]$$

$$= \prod_{0<\sigma<\pi} \delta[X(\sigma) - Y(\pi-\sigma)]\,.$$

Thus,

$$\sim|X(\sigma)\rangle = |X(\pi-\sigma)\rangle \qquad (4.3)$$

Using the appropriate mode expansions for $x^{\mu}(\sigma)$ and $\phi(\sigma)$, and treating the zero modes in the standard way, gives us

$$|X(\sigma)\rangle = |x^{\mu}_{n}, \phi_{n}\rangle\,|p^{\mu}, g\rangle$$

Eq. (4.3) can now be written as

$$\sim|X(\sigma)\rangle = |(-)^{n}x^{\mu}_{n}, (-)^{n}\phi_{n}\rangle\,|p^{\mu}, -g\rangle$$

In this basis (4.2) is just

$$\int \prod_{n=1}^{\infty}\prod_{\mu=0}^{25} [\mathrm{d}x^{\mu}_{n}][\mathrm{d}\phi_{n}] \times \int [\,|(-)^{n}x^{\mu}_{n}, (-)^{n}\phi_{n}\rangle, |x^{\mu}_{n}, \phi_{n}\rangle] * |\Phi_{-3/2}\rangle \qquad (4.4)$$

This integral vanishes since

$$\int_{-\infty}^{\infty} \mathrm{d}x\, f(x) f(-x) = \int_{-\infty}^{\infty} \mathrm{d}x\, f(-x) f(x)\,,$$

even if $f(x)$ and $f(-x)$ do not commute.

Eq. (4.4) implies invariance of the measure. It represents a "naive" proof in the sense that the vertex is expressed in terms of delta functions, and because zero modes should be treated more carefully.

We now give an exact proof of BRST invariance using the vertex overlap equations. Let us first note that the two-string vertex V_2 has the property

$$\langle V_2 | t\rangle = \langle t|\,,$$

for our whole basis. This just means that V_2 gives us the inner product. Using this we find

$$\langle V_2 | \tilde{t}_{g,i}\rangle_1 |t_h{}^{j}\rangle_2 = \langle \tilde{t}_{g,i} | t^{j}_{h}\rangle = \delta_{gh}\delta^{j}_{i}$$

However, we also have

$$\left[\sum_{fk}\langle\tilde{t}_{fk}|_2\langle t_f^k|_1\right]|\tilde{t}_{gi}\rangle_1|t_h^i\rangle_2=\delta_{gh}\delta_i^j$$

Thus

$$\langle V_2|_{12}=\sum_{fk}\langle\tilde{t}_{fk}|_2\langle t_f^k|_1 \tag{4.5}$$

Taking the projection of this equation with $\delta_{p^1_{26}\,f}$, where $p^1_{26}=g^1$ is just the ghost number of the first string, we find

$$\langle V_2^f|_{12}\equiv\langle V_2|_{12}\delta_{p^1_{26}\,f}=\sum_k\langle\tilde{t}_{fk}|_2\langle t_f^k|_1\,, \tag{4.6}$$

as well as

$$\langle V_2^{-f}|_{12}=\sum_k\langle t_f^k|_2\langle\tilde{t}_{fk}|_1 \tag{4.7}$$

Using this we find

$$\sum_i\int[\,|\tilde{t}_{gi}\rangle,\,|t_g^i\rangle\,]*|\Phi_{-3/2}\rangle$$

$$=\langle\Phi_{-3/2}|_3[\langle V_2^{-g}|_{12}-\langle V_2^g|_{12}]\,|V_3\rangle_{123}$$

The requirement of BRST invariance can now be cast in the compact form

$$(\langle V_2^{-g}|_{12}-\langle V_2^g|_{12})\,|V_3\rangle_{123}=0\,,\quad\text{for all } g \tag{4.8}$$

The two- and three-string vertices of Witten's theory satisfy the following overlaps (in the spaces of strings 1 and 2)

$$\langle V_2|[x^1(\sigma)-x^2(\pi-\sigma)]=0$$
$$\text{for all } \sigma\in[0,\pi]\,, \tag{4.9}$$

$$\langle V_3|[x^1(\sigma)-x^2(\pi-\sigma)]=0$$
$$\text{for all } \sigma\in[0,\pi/2] \tag{4.10}$$

The g-projection of the two-string vertex is

$$\langle V_2^g|=\langle V_2|\delta_{p^1_{26}\,g}$$

$$=\int_{-\pi}^{\pi}\frac{d\lambda}{2\pi}\exp(-i\lambda g)\langle V_2|\exp(i\lambda p^1_{26})\,. \tag{4.11}$$

For convenience we define

$$\langle V_2^\lambda|\equiv\langle V_2|\exp(i\lambda p^1_{26})$$

This obviously satisfies the overlap

$$\langle V_2^\lambda|[x^1_\mu(\sigma)-x^2_\mu(\pi-\sigma)-\lambda\eta_{\mu\,26}]=0$$
$$\text{for all } \sigma\in[0,\pi] \tag{4.12}$$

We now look at the following matrix element

$$\langle V_2^\lambda|[x^1_\mu(\sigma)-x^2_\mu(\pi-\sigma)]\,|V_3\rangle$$

Choosing $\mu=26$ as well as any $\sigma\in[0,\pi/2]$, and using the overlaps for V_3 and V_2^λ we find

$$\lambda\langle V_2^\lambda|V_3\rangle=0\quad\text{for all }\lambda$$

Thus

$$\langle V_2^\lambda|V_3\rangle\propto\mathcal{N}\,\delta(\lambda)\,, \tag{4.13}$$

giving us

$$\langle V_2^g|V_3\rangle=\int_{-\pi}^{\pi}\frac{d\lambda}{2\pi}\exp(-i\lambda g)\,\mathcal{N}\,\delta(\lambda)=\frac{\mathcal{N}}{2\pi}$$

Since this does not depend on g we have proven eq (4.8), and thus established that $J_{\text{string}}=1$, i e that the measure

$$\prod_{gi}[dA_{gi}]$$

is invariant under the BRST transformations (3 1)

We point out a few subtleties The constant $\mathcal{N}$ in eq (4 13) is infinite We get a $\delta(\lambda)$ in $\langle V_2^\lambda|V_3\rangle$ for every $\sigma\in[0,\pi/2]$, so

$$\langle V_2^\lambda|V_3\rangle\propto\prod_{\sigma\in[0,\pi/2]}\delta(\lambda)=\delta(\lambda)\prod\delta(0)$$

$$\rightarrow\mathcal{N}\propto\prod\delta(0)$$

This does not affect our result since $\mathcal{N}$ does not depend on g Another point that should be mentioned is the effect of the mid-point ghost insertion It is easy to see that it does not alter our result since to get the g-dependence of $\langle V_2^g|V_3\rangle$ we used any point in $[0,\pi/2]$ We can thus safely avoid the mid-point and the insertion in it

We have proven eq (4 8) in yet another way utilizing the formula for the vacuum expectation values of products of exponentials of quadratic forms in the oscillators [17]

$$\langle 0|\exp[\tfrac{1}{2}(a|M_2|a)+(a|L_2)]$$
$$\times\exp[\tfrac{1}{2}(a^\dagger|M_1|a^\dagger)+(a^\dagger|L_1)]|0\rangle$$
$$=[\det(1-M_2M_1)]^{-N/2}$$
$$\times\exp\left[\frac{1}{2}\left(L_1\left|\frac{1}{1-M_2M_1}M_2\right|L_1\right)\right.$$
$$+\frac{1}{2}\left(L_2\left|M_1\frac{1}{1-M_2M_1}\right|L_2\right)$$
$$\left.+\left(L_1\left|\frac{1}{1-M_2M_1}\right|L_2\right)\right],$$

where N is equal to the number of values the index μ takes, so $N=D+1$ due to the bosonized ghost coordinate We will not give this proof here, but will just note that the crucial $\delta(\lambda)$ dependence of $\langle V_2^\lambda|V_3\rangle$ comes here through expressions of the form

$$\lim_{\epsilon\to 0+}\frac{1}{\sqrt{\epsilon}}\exp(-\lambda^2/\epsilon),$$

where ϵ is a regularization parameter needed since

$$\det(1-M_2M_1)=0$$

Finally, let us establish a connection with the formalism of Batalin and Vilkovisky In this formalism [5,6], the requirement of BRST invariance of the generating functional leads to the equation

$$(W,W)=i\hbar\Delta W,$$

where the "full action" W acts as a generator of BRST transformations The Poisson-like bracket (,), as well as the "laplacian" Δ are defined in ref. [5] in terms of all the fields A_a, as well as the so-called anti-fields A^*_a Writing $W=S+\hbar W_1+\hbar^2W_2+$, we obtain the "master equation" for the action S

$$(S,S)=0,$$

as well as a set of equations for the W_i, which can be thought as contributing to the measure. Our gauge-fixed string action was constructed so that the master equation is satisfied The solution $W=S$ (which corresponds to using the simple measure) is possible only if

$$\Delta S=\sum_{g\,i}\frac{\partial}{\partial A_{g\,i}}\delta_\theta A_{g\,i}=0,$$

which is precisely what was shown above

5 Conclusion To recapitulate, we have shown that the simple string field measure, represented by the product of differentials of all the fields, is invariant under the same BRST transformations as the gauge-fixed action of Bochicchio The invariance is thus maintained at the quantum level

As has been pointed out by Bochicchio [3] the lagrangian formalism (unlike the hamiltonian) does not uniquely determine the measure of the generating functional. BRST invariance is not enough We could obtain another candidate for the measure by multiplying the simple measure by a manifestly BRST invariant expression For every such measure, we would have to check unitarity order by order in perturbation theory Luckily, in string theory we know the correct Feynman rules from the first-quantized formalism [10–12]. Together with the work done by Thorn [9], this indicates that the simple measure is the correct one in string field theory

The main difference between Yang–Mills theories and strings that has been seen in this work, comes about due to a basic difference in their gauge algebras Further study of the properties of the string gauge algebra is of central importance, especially in looking for non-perturbative solutions The search for these types of solutions is the prime motivation for constructing a field theory of strings in the first place Perturbative solutions are of interest mainly in showing equivalence of first- and second-quantized formalism.

The BRST invariance discussed in this letter works only on-shell, and the extension to an off-shell invariance is not as trivial as was the case in Yang–Mills theories, and is not yet known In fact, the full interpretation of the gauge-fixing procedure of Witten's theory is not yet fully understood. This (and some related problems) is presently being looked into [18] The resolution of these problems is interesting in its own right, and is of central importance for understanding the connection with the light-cone formalism of Kaku and Kikawa [19,20], as well as the work of Strominger [21,22] on closed string field theory, centering around the cubic action [23]

At the end let us also mention the work of ref [24] where the BRST invariance of the measure was looked into in the case of the light-cone-like string field theory

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