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BRST QUANTIZATION OF KAC-MOODY ALGEBRAS †‡

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ABSTRACT

We apply the BRST quantization technique to current algebras in two dimensions (Kac-Moody algebras with central extensions) and show that nilpotency is satisfied only at a value of the central charge that makes the theory live in a sector without a lower bound in energy. No improvement is made by semi-direct coupling to the conformal algebra (Virasoro algebra). We explain how this problem is resolved by realizing that the root of the central charge of the Kac-Moody algebra is in the chiral anomaly. Taking the effect of the chiral anomaly into account allows for problem free quantization of the algebra. We also give an example of a system where this problem is absent. In the case of the N = 2 super-Virasoro algebra, the Kac-Moody algebra is realized as an abelian O(2) subalgebra and it is the contribution of the bosonic ghost fields that accompany the supergauge transformations that gives the negative contribution to the chiral anomaly making the critical dimension of the string to be D = 2, and the Regge slope parameter be equal to zero. All this makes the O(2) string theory tachyon free and "trivial".

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1. Introduction

Kac-Moody algebras and their semi-direct sum with the Virasoro algebra and supersymmetric generalizations have generated much interest lately. They naturally appear in (super) string theories[1] and in two dimensional theories relevant to statistical systems at the critical point[2]. Therefore it seems important to learn as much as possible about these infinite algebras. Much work has been done by mathematicians on the clasification of the unitary representations of these algebras[3].

A covariant approach to the quantization of string theories has brought the BRST technique[4] under the spot light. The BRST method of quantization also known as the Fadeev-Popov quantization procedure for constrained systems was developed long before the string reneissance but it was considered to be mostly a usefull technical tool. We have learned to appriciate the full power of the BRST formalism with the rebirth of string theory as a candidate for the unification of all interactions. Given a nilpotent ($Q^2 = 0$) generator of the BRST transformation it is possible to construct a covariant string field theory[5]. Suppose the second quantized form of the Lagrangian is established in a complete form, the next problem would be to search for a true vacuum, i.e. the possibility of spontaneous compactification of the extra dimensions, which would involve difficult dynamical problems.

The symmetry algebra of a string theory defines the physical states by imposing an infinite set of gauge conditions. The String theory compactified on a certain group manifold, for example by a Frenkel-Kac mechanism, has Kac-Moody algebra as its symmetry. Therefore it is of interest, and is relevant for our better understanding of the structure of compactified strings, to study the BRST quantization of Kac-Moody algebras.

2. The BRST Formalism

In this paragraph we give a quick review of the BRST quantization procedure for a constrained system. We closely follow the prescription given by Fradkin and Vilkovisky[4].

Let $H_0, \psi_a (a = 1, \dots, k)$ be the original Hamiltonian and first class constraints satisfying the Poisson bracket relations: $\{\psi_a, \psi_b\}_{PB} = U^c_{ab}\psi_c$, $\{H_0, \psi_a\}_{PB} = V^b_a \psi_b$. (Here we confine ourselves to the case where ψ_a are either bosonic or fermionic and the structure constants U^c_{ab} and V^b_a are ordinary c-numbers). Then there exist classes of quantum Hamiltonians H invariant under the BRST transformation satisfying nilpotency:

$$H = H_0 + B_a V^a_b C^b + [Q, \Omega]_{(graded)},$$

$$Q = \psi_a C^a - \frac{(-)^{n_a}}{2} U^c_{ab} : B_c C^a C^b :,$$

$$[H, Q] = 0,$$

where Q is the BRST charge, Ω is an arbitrary gauge fixing function, (the scattering matrix is independent of the choice of Ω); n_a is called the grading and is equal to 1 for a ψ_a bosonic and -1 for ψ_a fermionic. The fields $B(C)$ are antighosts (ghosts) satisfying: $\{C_a, B^b\} = \delta^b_a$ if they are fermionic (associated to the bosonic constraints), and $[C_a, B^b] = \delta^b_a$ if they are bosonic (associated to the fermionic constraints). In order that we get a consistent quantization the BRST charge Q must be nilpotent *i.e.* $Q^2 = 0$. This requirement automatically gives us a quantum theory that obeys all the gauge symmetries. Physical states are defined by the condition: $Q|phys.>= 0$ (modulo $Q|anything>$). The semicolons : : designate normal oredering with respect to the ghost vacuum.

Applying this formulation to the algebras of string theories allows one to derive the corresponding critical dimensions and the Regge trajectory intercepts[6,7,8]: $D = 26, \alpha = -1$ for the bosonic string, $D = 10, \alpha = -1/2, (0)$ for the $N = 1$ femionic string in the Neveau-Schwarz (Ramond) sector, $D = 2, \alpha = 0$ for all three sectors of the $N = 2$, O(2) string (*cf.* section 4.)

3. BRST Quantization of Systems With a Kac-Moody Algebra

A Kac-Moody or an affine algebra $\mathcal{A}$ is a central extension of the loop algebra[3] $\tilde{\mathcal{G}} = C[t, t^{-1}] \otimes_C \mathcal{G}$ defined by commutation relations:

$$[X(m), Y(n)] = [X, Y](m+n) + k \cdot m(x|y)\delta_{m+n,0}, \tag{3.1}$$

where: $(\cdot|\cdot)$ is an invariant symmetric bilinear form on $\mathcal{G}$ normalized by the condition $(\psi|\psi) = 2$, ψ being the highest root in the Cartan subalgebra: $X(m) = t^m \otimes X$. $m \in Z$, $X \in \mathcal{G}$. In particular we will take $\mathcal{G}$ to be a compact Lie group with the generators taken to be hermitian. The symmetric bilinear form on $\mathcal{G}$ is proportional to the Killing metric $Tr(J^a_m \cdot J^b_n) = \delta_{n+m,0}\eta^{ab}$. If we write f^{abc} for the structure constants of the group $\mathcal{G}$ then the Kac-Moody algebra takes the familiar form:

$$[J^a_m, J^b_n] = if^{abc} J^c_{m+n} + m\kappa\delta_{m+n,0}\eta^{ab}, \tag{3.2}$$

where the constant κ is positive and is called a center *. We can include the noncompact group if we allow κ to be negative (keeping all other definitions the same).

We now use the BRST formalism presented in section **2.** and apply it to Kac-Moody algebras. In this particular case the BRST charge takes the form

$$Q = d + \delta + B^a_0 r^a_+ + C^a_0 r^a_0 - \frac{i}{2} f^{abc} B^a_0 C^b_0 C^c_0. \tag{3.3}$$

In writing down the BRST charge Q we have used the definition of normal ordering with the respect to the (fermionic) ghost vacuum $|0>$, such that $B^a_m|0>= C^a_m|0>= 0$, $m \in Z_+$, and $B^a_0|0>= 0$. This normal ordering prescription together with the definition $Q|phys.>= 0$ assures that physical states satisfy the gauge conditions $J^a_m|phys.>= 0$, for $m \in Z_+ \cup \{0\}$. The operators r^a_+, r^a_0 act as (part of) the generators of SU(1,1) rotation[5] among ghost fields (B^a_m, C^a_m) designated by subscripts $+, 0$. They satisfy:

$$[r^a_+, d+\delta] = [r^a_0, d+\delta] = [r^a_+, r^b_+] = 0, [r^a_+, r^b_0] = if^{abc} r^c_+.$$

* the central charge is sometimes called the second cocycle

The eperators d and δ contain no zero modes and are nilpotent, *i.e.* $d^2 = \delta^2 = 0$. The operator d creates a net ghost number when acting on states, while δ annihilates a net ghost number. They are the (co) boundary operators on the Fock space $\prod_q B_{-q} \cdot \prod_m C_{-m}|0>$. Explicitly we have:

$$\tau_+^a = -i\sum_1^\infty f^{abc} C^b_{-m} C^c_m$$

$$\tau_0^a = T_0^a + if^{abc} \sum_{m\neq 0} : C^b_{-m} B^C_m :$$

$$d = [J^c_m - if^{abc}(B^a_{-q} C^b_{q+m} + \frac{1}{2} C^a_{-q} B^b_{q+m})]C^c_{-m}$$

$$\delta = [J^c_{-m} - if^{abc}(C^a_{q-m} B^b_q + \frac{1}{2} B^a_{q-m} C^b_{-q})]B^c_m .$$

The ghost field and the BRST charge satisfy the Maurer-Cartan equation. The full generators of the symmetry are the sum of the original ones (J^n_a) and the ghost contribution ($-if^{bc} : \sum C^b_{n-m} B^c_m :$). The algebra of the full constraints (ordinary plus ghost) is supposed to be centerless.

Now it is easy to compute the square of the BRST charge and check nilpotency. We have[9]:

$$Q^2 = 2(\kappa + c_2) \sum_{m=1}^\infty m C^a_{-m} C^a_m, \tag{3.4}$$

where c_2 is the second Casimir invariant of the group $\mathcal{G}$ in the adjoint representation. It is a positive number for any compact group. We can express our result in a parametrization independent way. The level of the representation of the of the Kac-Moody algebra is defined by $\mathbf{x} = 2\kappa/(\psi|\psi)$ and the dual Coxeter number is given by $\tilde{\mathbf{h}}(\mathcal{G}) = c_2/(\psi|\psi)$. The level of the highest weight representation of the Kac-Moody algebra over a compact (graded) Lie group and the dual Coxeter number of the compact semisimple group with positive metric signature are positive integers.

From equation (3.4) it is obvious that the BRST charge for the Kac-Moody algebra is nilpotent only if

$$\mathbf{x} = -2\tilde{\mathbf{h}}(\mathcal{G}) \tag{3.5}$$

holds. This condition is impossible to satisfy for any choice of the (graded) compact group. If instead one chooses to use the lowest weight representation of the Kac-Moody algebra then one must change the sign of the of the operator:

$$L_0 \propto \sum : J^a_n J^a_n : \tag{3.6}$$

in the Cartan subalgebra. However, for a physical system the operator (3.6) is the energy operator. and therefore the choice of the lowest weight representation implies that the lower bound on the energy spectrum does not exist. For some noncompact groups the center κ may be negative and then if it is equal to c_2, the BRST charge will be nilpotent.

One might hope that coupling the current algebra with the conformal algebra (semidirect sum of the Virasoro and Kac-Moody algebras) will cure the problem. If one repeats the calculation described for a combined system (conformal current algebra) one finds that no change ocured in the central moment of the Kac-Moody part. The center of the conformal part , however, changes according to the rule $D \to D - 26 - 2dim(\mathcal{G})$. This should be contrasted to the case of the pure conformal symmetry where the nilpotency of the BRST charge determines the critical dimension of the physical system. This happens to be the case in string theories.

Let us mention in passsing that supersymmetry does not cure the problem in full. In a recent work, Bilal and Gervais[10] show that if one considers the supersymmetric version of the Kac-Moody algebra, the supersymmetry, because of the presence of bosonic ghost associated with supergauge transformations, cancels the contribution of the ordinary fermionic ghost, that is the c_2 part. However, the Kac-Moody center still remains and the nilpotency condition is $\kappa = 0$. Thus, the super-Kac-Moody algebra has a nilpotent BRST charge only when it has no central extension. Also, according to the same work[10], the coupling to the superconformal algebra[11] only changes the superconformal charge according to the rule $D \to D - 10 - 2dim(\mathcal{G})$, leaving the super-current center unchanged. The situation is identical to the bosonic case.

The results given above indicate that there is a problem with the BRST quantization of systems having the current algebra as a symmetry (sub) algebra. In light of that it is even more peculiar that the BRST procedure works for $N = 2, 3, 4$ string theories*

We finish this section by presenting the resolution of the problem as it has been worked out recently by Ninomiya and Yamagishi[14] for the bosonic case. We belive that the very same method works in the supersymmetric case.

The clue is to realize that the origin of the center of the Kac-Moody algebra is in the chiral anomaly. It is a well known fact that there is an inconsistency in the quantization of anomalous gauge theories. One way to get rid of the inconsistency is to consider a theory of Wayl fermions with equal number of left and right multiplets. Recently, however, an interesting quantization procedure has been proposed by Fadeev and Shatashvili[15]. They suggested that the anomalous term, manifesting itself in the commutator of the Gauss law constraints, may be absent if one starts from a modified Lagrangian supplemented by a one-cocycle Wess-Zumino term[16]. In two dimensions, since the one-cocycle becomes a constant[17], the resulting anomalous Gauss law commutation relation represents an affine algebra with central extension. The effect of the anomalous term is the Schwinger mechanism[18], yielding the mass term to (non) abelian gauge fields. The bosonization technique in two dimensions allows for calculation of the one-cocycle in two dimensions. Therefore, one can explicitly verify the conjecture of Fadeev and Shatashvili. The outcome is that the introduction of the one-cocycle ammounts to a simple duplication of the current J^a_m but with an opposite central charge. The total effect, after counting ghosts for both currents, cancells, rendering the BRST charge nilpotent. The negative central charge is generated for the dupplicate of the current because the change in the dynamics of the system due to the introduction of the

* the Fock space realization is only known for $N = 2$ and $N = 4$ case and the $N = 4$ case is a bad theory since it has propagating ghosts (critical dimension is $D = -2$.) We will come back to the $N = 2$, $O(2)^{10,11,12}$ string algebra (it has Kac-Moody subalgebra) case in section **4.**

one-cocycle is desribed with a field that has negative metric. This is very much like how the bosonic ghost in the supersymmetry case cancels the central charge of the fermionic ghost. In the supersymmetric case the bosonic ghost has negative metric

It would be interesting to repeat the analysis of Ninomiya and Yamagishi for the super-current and super-conformal-current algebras.

4. Application of the BRST Formalism to the O(2) String

Applying the BRST quantization technique to string theory is a first step toward a formulation of the covariant string field theory[5]. Nowadays, we know that the BRST charge is crucial for the costruction of a sensible interacting string field theory[19,20]. Our motivation for constructing the BRST charge for the O(2) String theory is, however, primarily based on the issues discussed in the previous section. Using the O(2) string theory as an example, we want to illustrate that the super-conformal algebra with the Kac-Moody subalgebra need not have the problem that is faced in the naive BRST quantization of the current algebra in 2-dimensions. The possibility of the construction of string field theory is exciting as well. On the other hand one shouldn't expect great surprises since it is well known that the critical dimension of the O(2) model is $D = 2$ [12,21]. Because the critical dimension is $D = 2$, the string theory is trivial (in two dimensions there is no room for transverse oscillations) since there is one state (ground) in each of the three sectors of the theory.

The O(2) string theory is a $N = 2$ supergravity theory in two dimensions coupled to D scalar matter multiplets.[13,12] There are three sectors in the theory* and an interesting question is whether there is a space-time supersymmetry. The answer to this question we leave for a further report.

The string theory is completely determined by giving the algebra of constraints and a

* to be explained later

set of gauge conditions that define the physical states. For the O(2) string the algebra of constraints contains four infinite sets of operators. Together they close the $N = 2$ super-Virasoro algebra with an abelian O(2) symmetry realized as a Kac-Moody algebra. The gauge conditions defining the physical states are:

$$(L_n - \alpha\delta_{n,0})|phys.> = T_n|phys.> = G^i_r|phys> = 0, (n, r \geq 0, i = 1, 2), \tag{4.1}$$

where the L's are the generators of the conformal symmetry, T's of the O(2) symmetry and G^i's of supersymmetry. In expression (4.1) the values that the subscripts n and r take, depend on the sector. Because the theory is defined on the cilynder (a circle in a complex plane), and because the algebra of the constraints (to be explicitly written bellow) can be realized by a set of four real fields, there are three different ways of choosing the boundary conditions. The sectors are named after the boundary conditions: 'periodic' (Ramond sector), 'antiperiodic' (Neveau-Schwarz sector) and 'twisted'. L being a generator of the conformal transformations has an integer valued index in all three sectors. In the periodic sector the boundary condition is periodic and therefore the T and G^i generators are integer valued. In the antiperiodic sector T is integer valued and G^i's are half integer valued. In the twisted sector G^1 is integer valued and T and G^2 are half integer valued.

The algebra of constraints is:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{4}(m^3 - m)\delta_{m+n,0}$$

$$\{G^i_r, G^j_s\} = 2\delta^{ij}L_{r+s} + 2i\epsilon^{ij}(r-s)T_{r+s} + \frac{D}{4}(4r^2 - 1)\delta_{r+s,0}$$

$$[L_n, G^i_r] = (\frac{n}{2} - r)G^i_{n+r}$$

$$[T_m, T_n] = \frac{D}{4}n\delta_{m+n,0} \tag{4.2}$$

$$[L_m, T_n] = -nT_{m+n}$$

$$[T_m, G^i_r] = \frac{i}{2}\epsilon^{ij}G^j_{n+r}$$

We see from the algebra that G^i's anticommute and that L's and T's commute. According to the BRST prescription this uniquely (up to zero modes which depend on the

sector) determines the ghost structure. We need two sets of commuting ghosts (associated with G^i's) and two sets of anticommuting ghosts (associated with L's and T's respectivly) as well as the corresponding antighosts. We choose:

$$\{\eta_n, \bar{\eta}_m\} = \{\lambda_n, \bar{\lambda}_m\} = \delta_{n+m}$$

$$[\xi^i_r, \bar{\xi}^j_s] = \delta^{ij}\delta_{r+s}.$$

Note that our convention for ghosts is such that $\eta, \bar{\eta}, \lambda, \bar{\lambda}$ and ξ^i are hermitian and $\bar{\xi}^i$ is antihermitian. We also define the ghost vacuum by:

$$(ghost)_n|0> = (antighost)_n|0> = 0, \qquad (n > 0).$$

We also choose that the antighost zero modes (whenever there are any) annihilate the vacuum. This last requirement, together with the definition of physical states $Q|phys.> = 0$ (according to the BRST quantization prescription) assures that the gauge conditions (4.1) are satisfied.

For simplicity we will display the expression for the BRST charge in the antiperiodic sector only. The other sectors will differ only in the structure of the zero modes and can be easily infered from the general expression:

$$Q = \sum_n (L_n - \alpha\delta_{n,0})\eta_{-n} + \sum_n T_n\lambda_{-n} + \sum_r \sum_{i=1}^{2} G^i_r\xi^i_{-r} + \frac{1}{2}\sum_{n,m}(n-m) : \bar{\eta}_{n+m}\eta_{-m}\eta_{-n} :$$
$$- \sum_{n,m} m : \bar{\lambda}_{n+m}\lambda_{-m}\eta_{-n} : + \sum_{n,r}(\frac{n}{2} - r) : \bar{\xi}^i_{n+r}\xi^i_{-r}\eta_{-n} : - \frac{i}{4}\sum_{n,r}\epsilon^{ij} : \bar{\xi}^i_{n+r}\xi^j_{-r}\lambda_{-n} :$$
$$- \sum_{r,s} : \bar{\eta}_{r+s}\xi^i_{-r}\xi^i_{-s} : + i\sum_{r,s}(r-s)\epsilon^{ij} : \bar{\lambda}_{r+s}\xi^i_{-r}\xi^j_{-s} : .$$

In the antiperiodic sector (Neveau-Schwarz) the expression above can be written as:

$$Q = d + \delta + \mathcal{L}\eta_0 + \mathcal{T}\lambda_0 - 2\Delta\bar{\eta}_0 - 2\Xi\bar{\lambda}_0, \tag{4.3}$$

where:

$$\mathcal{L} = L_0 + \sum_{n>0} n(\eta_{-n}\bar{\eta}_n + \bar{\eta}_{-n}\eta_n + \lambda_{-n}\bar{\lambda}_n + \bar{\lambda}_{-n}\lambda_n) + \sum_{r>0} r(\bar{\xi}^i_{-r}\xi^i_r - \xi^i_{-r}\bar{\xi}^i_r),$$

$$\mathcal{T} = T_0 + \frac{i}{4}\sum_{r>0} \epsilon^{ij}(\xi^i_{-r}\bar{\xi}^j_r - \bar{\xi}^i_{-r}\xi^j_r),$$

$$\Delta = \sum_{n>0} n\eta_{-n}\eta_n + \sum_{r>0} \xi^i_{-r}\xi^i_r,$$

$$\Xi = \sum_{n>0} n(\eta_{-n}\lambda_n + \lambda_{-n}\eta_n - 2i\epsilon^{ij}\sum_{r>0} r\xi^i_{-r}\xi^j_r,$$

$$d = \sum_{n>0}[(L_n + \ldots)\eta_{-n} - (T_n + \ldots)\lambda_{-n}],$$

$$\delta = \sum_{n>0}[(L_{-n} + \ldots)\eta_n - (T_{-n} + \ldots)\lambda_n].$$

The operators defined above $(d, \delta, \mathcal{L}, \mathcal{T}, \delta, \Xi)$ are usefull in the construction of the field theory[5,9]. They satisfy a number of relations that are only valid in the critical dimension $D = 2$ with $\alpha = 0$: d and δ are nilpotent and they anicommute with $\mathcal{L}, \mathcal{T}, \Delta$ and Ξ. $\mathcal{L}$ commutes with Δ; $\mathcal{T}$ commutes with Ξ and $d\delta + \delta d + \mathcal{L}\Delta - \mathcal{T}\Xi = 0$. (Note that in this sector there are no bosonic (anti) ghost zero modes). In other than the critical dimension a long but a straightforward calculation of the square of the BRST charge Q gives:

$$\frac{1}{4}\sum_{n>0}[(D-2)n^3 - (D-2-8\alpha)n]\,\eta_{-n}\eta_n + \frac{1}{4}\sum_{r>0}[(D-2)4r^2$$
$$-(D-2-8\alpha)]\cdot\sum_{i=1,2}\xi^i_{-r}\xi^i_r + \frac{1}{4}(D-2)\sum_{n>0} n\lambda_{-n}\lambda_n.$$

From the expression for Q^2 we see that the BRST charge is nilpotent only if the crtical dimension is $D = 2$ and if the Regge slope parameter is $\alpha = 0$. When this is satisfied the model is free from conformal, supersymmetry and gauge anomalies. A similar calculation in the other two sectors gives the same result. This is a well known result established before by different methods[12,21]. What is more interesting is the coefficient $(D-2)$ in front of the last term in the expression for Q^2. This last term represents a contribution of the Kac-Moody subalgebra (D proportional part) and the canceling mechanism ('-2' proportional term). An inspection of the the equation (4.3) reveals that it is the coupling of the supersymmetry to the O(2) algebra, a $\bar{\xi}\xi\lambda$ term that contributes a '-2' proportional term to the Kac-Moody part, thus allowing the cancelation oh the chiral anomaly in the critical dimension. This is

the type of the cancelation mechanism expected from the work of Fradkin and Tsetylin[21]. Next, one can proceed and construct the field theory of the model[9].

A calculation similar to the one described here in section **4.** for the N=2, O(2) string, when applied to the $N = 3$, $O(3)$ and $N = 4$, $SU(2)$ string theories finds that the BRST charge is nilpotent in the critical dimension[22]. Therefore, the BRST quantization procedure does not require the special cure when applied to string theories*.

5. Conclusion

In this talk we have described how to quantize consistently (super) current algebras with central extension and their semi-direct sum with the superconformal algebras, using the BRST quantization procedure. It is worth pointing out once again, that in the general case special care must be taken with the chiral anomaly in order to have a nilpotent BRST charge. In the case of the string theory algebras, such attention is not needed. This tells us that the String algebras are unique. Even though the string algebras with $N > 1$ don't seem to be of great interest for physics** because their critical dimensions are $D = 2, 0$, and -2 respectivly, it would be of interest to understand what makes them unique among the infinite dimensional algebras.

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* Ramond and Schwarz[23] found under fairly general assumptions that the only Virasoro algebras that are consistent with the Jacobi identities are the algebras with $N = 0, 1, 2, 3$, supersymmetries and (O(0), O(1), O(2), O(3)) innternal symmetries respectively and $N = 4$ with SU(2) internal symmetry.

** The $N = 2$, O(2) string might be an exception since it can give some insight into the compactification of the $N = 1$, $D = 10$ string theories[24].

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References

[1] For a lengthier selection of references on the subject see J. H. Schwarz, ed. "Superstrings", World Scientific Publishing Co., Singapore 1986;
P. Goddard and D. Olive. "Kac-Moody and Virasoro Algebras in Relation to Quantum Physics", University of Cambridge preprint DAMTP 86.

[2] V. G. Knizhnik and A. B. Zamolodchikov, *Nucl. Phys.* **B 427**, (1983). 83;
D. Friedan, Z. Qiu and S. Shenker, *Phys. Rev. Lett.* **52**, (1984), 1575.

[3] V. G. Kac, "Infinite Dimensional Lie Algebras", Birkhäuser Boston, Inc., Boston 1983.

[4] E. S. Fradkin and G. A. Vilkoviski, *Phys. Lett.* **B 55**, (1975), 224;
I. A. Batalin and G. A. Vilkoviski, *Phys. Lett.* **B 69**, (1977), 309.

[5] W. Siegel, *Phys. Lett.* **B 151**, (1985), 391-396.

[6] M. Kato and K. Ogawa, *Nucl. Phys.* **B 212**, (1983), 663;
S. Hwang, *Phys. Rev.* **D 25**, (1983), 2614.

[7] N. Ohta, *Phys. Rev.* **D 33**, (1986), 1681;
H. Terrao and S. Uehara, *Phys. Lett.* **168 B**, (1986), 70.

[8] A. R. Bogojević and Z. Hlousek, "The BRST Quantization of the O(2) String", Brown University preprint, BROWN-HET-573 (1986), to be published in Phys. Lett. B.

[9] Z. Hlousek and K. Yamagishi, *Phys. Lett.* **B 173**, (1986), 65.

[10] A. Bilal and J. L. Gervais. "BRST Analysis of Super Kac-Moody and Superconformal Current Algebras", l'Ecole Normale Supérieure, preprint LPTENS 86/12 (1986).

[11] V. G. Kac and I. T. Todorov, *Comm. Math. Phys.* **102**, (1985), 337.

[12] M. Ademollo *et al.*, *Phys. Lett* **62 B**, (1976), 105; *Nucl. Phys.* **B 111**. (1976). 73.

[13] L. Brink and J. H. Schwarz, *Nucl. Phys.* **B 121**, (1977), 285.

[14] M. Ninomiya and K. Yamagishi, "Anomaly Free of Wess-Zumino Lagrangian and Strings on Group Manifold", Brown University preprint BROWN-HET-581 (1986).

[15] L. D. Faddeev, *Phys. Lett.* **145 B**, (1984), 81;
L. D. Faddeev and S. L. Shatashvili, *Theor. Mate. Fiz.* **60**, (1984), 206.

[16] J. Wess and B. Zumino, *Phys. Lett.* **37 B**, (1971), 95.

[17] R. Jackiw, in "Lectures on Current Algebra and its Applications", S. Trieman, R. Jackiw and D. Gross, eds. Princeton University Press, Princeton N.J., 1972

[18] J. Schwinger, *Phys. Rev.* **128**, (1962), 2425

[19] E. Witten, *Nucl. Phys.* **B 268**, (1986), 253; "Interacting Field Theory of Open Superstrings" Priceton preprint, March 1986.

[20] D. J. Gross and A. Jevicki, "Operator Formulation of Interacting String Field Theory", Princeton preprint May 1986; "Operator Formulation of Interacting String Field Theory II", Brown University preprint, 1986.

[21] E. S. Fradkin and A. A. Tseytlin, *Phys. Lett.* **106 B**, (1981), 63.

[22] D. Chan and A. Kumar, "BRST Quanization of Super Conformal Theories", Maryland preprint 1986.

[23] P. Ramond and J. H. Schwarz, *Phys. Lett.* **64 B**, (1976), 75.

[24] W. Bocher, D. Friedan and A. Kent, "Determinant and Unitarity for the N=2 Superconformal Algebras in Two Dimensions or Exact Results on String Compactification", Chicago preprint, EFI 86-14 (1986).