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String Inspired Cosmology: Why Space is 3 Dimensional*

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ABSTRACT

A simple cosmological model with built-in duality $R \to 1/R$ is presented. This dynamical model is string induced, and is used to look at the universe near the Planck length. The model fixes the number of space dimensions to be three. Throughout, the model is found to be in agreement with recent work on superstring thermodynamics near the Planck scale.

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1. Introduction

In casting string theory into the role of a "Theory of Everything" we demand quite a lot from it. It should not only give the correct dynamics of all the fields (for example tell us how to quantize gravity), but should also conform to a basic paradigm of a truly unifying theory - it should be free of tunable parameters. We therefore expect strings to give us the masses and coupling constants of all particles. In cosmology the first few things we should learn from strings are why the world seems $1+3$ dimensional, why the cosmological constant is so small today, *etc.*

Having burdened strings with such high expectations it is quite remarkable that something has not come up to prove strings wrong. The recent lowering of enthusiasm has come not from failures of the theory but rather from technical difficulties that make even the simplest calculations quite formidable. We now have a more realistic feeling of how long it will take to answer some of the fundamental questions posed above. Some of the basic challenges in front of us are the following: First, from string field theory[1] we learn that strings lead to non-local interactions. Our understanding of even ordinary non-local field theories is very limited and many puzzles remain to be resolved[2]. Second: to extract relevant information from strings we need to have access to real non-perturbative physics. Third: we have not come any closer it seems to learning more about underlying inherently "stringy" symmetries of the theory.

Recently[3,4] an interesting path towards explaining some of these questions has been taken by looking at strings at very high energies. It is in this limit, after all, that strings are the most "stringy".

Still, what string theory lacks even more than a sophisticated formalism are (hopefully solvable) toy models that illuminate certain facets of the theory. At the same time, attempts to link the theory with experiments (or at least *gedanken* experiments) have to be made. The most promising place to look for an experiment with sufficiently high energy is the big bang itself.

In a recent paper Brandenberger and Vafa[5] looked into some implications of superstrings on cosmology. They focused on string thermodynamics, and showed that in order for it to make sense, strings must propagate on spaces where all the spatial directions are compact. This is a uniquely "stringy" result. Working on manifolds with toroidal compactification (*i.e.* with dual symmetry $R \to \frac{1}{R}$) and assuming equilibrium thermodynamics they calculated the dependence of temperature on the radius of the universe. As a consequence of duality there is no singularity at $R = 0$. Next, they looked into the process of annihilation of winding modes, and concluded that for more than 3 spatial dimensions the winding modes can't be annihilated; thus preventing the universe to attain its present size. Though illuminating their work is not free of controversy. To a certain extent what follows builds on this work and should also therefore be taken with a grain of salt.

Their scenario has to be "prodded up" by some dynamical calculations. The rate of expansion of the universe $\frac{\dot{R}}{R}$ has to be compared with the rate $\Gamma(w\bar{w} \to 0)$ for a winding and an antiwinding to annihilate. Even a rough approximation for $\frac{\dot{R}}{R}$ could enable us to answer the central question of whether windings equilibriate or not. In order to calculate this we need some generalization of the equations of standard cosmology, but with built in duality. This paper presents such a construction. After giving the basic model and its cosmological consequenses we show the connections of this model with strings.

How can a classical model hope to give even a rough approximation to physics at the begining of the universe? Classical results should be trusted far enough above the Planck length. As we approach Planck length (from above) we come into the quantum gravity regime. It is still possible that classical theory represents a sensible approximation for gravity up to the Planck length. Once we go to smaller distances, however, we find that we are in a ultra-quantum regime and classical results bare no relation to the real physics.

A *fundamental* consequence of duality is that it gets rid of the ultra- quantum region. Another words physics bellow the Planck length is related to physics above it. (The physical meaning of duality has been presented in [5] in terms of gedanken experiments.) We, therefore, never enter a region where a classical (dual) description of gravity is totaly inapropriate. It is still important to learn how strings give a

consistent quantization of gravity, it is just that for strings with duality (in all spatial directions) we can proceed to answer some important questions in cosmology even before we know how to implement that quantization.

We should not be too disheartened by the 20 orders of magnitude that separate Planck-length physics from the standard cosmological model (which is roughly valid up to temperatures of 100 MeV). The standard model, after all, represents a very good way of understanding some aspects of the universe across some 40 orders of magnitude of scale!

2. The Model

The standard Friedmann-Robertson-Walker (FRW) model in 1+3 dimensions corresponding to a closed universe (*i.e.* with k=1) is governed by the equations

$$\begin{aligned} \left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} &= \frac{8\pi G}{3}\,\rho(R) \\ 2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} &= -8\pi G\, p(R) \qquad (1) \\ \frac{d}{dt}\Big(\rho(R)R^3\Big) &= -p(R)\frac{d}{dt}R^3 \,. \end{aligned}$$

To these equations one also has to add the thermodynamic equation of state $p = p(\rho)$. To bypass the initial singularity of standard cosmology let us look at a similar model with built in duality

$$R \leftrightarrow \frac{R_0^2}{R} \,. \qquad (2)$$

If we look at how derivatives transform under duality we find

$$\begin{aligned} \left(\frac{\dot{R}}{R}\right) &\to -\left(\frac{\dot{R}}{R}\right) \\ \left(\frac{\ddot{R}}{R}\right) &\to -\left(\frac{\ddot{R}}{R}\right) + 2\left(\frac{\dot{R}}{R}\right)^2 \qquad (3) \\ &\textit{etc.} \end{aligned}$$

It follows that the only simple generalization of equation (1b), *i.e.* that preserves duality and has the same derivative terms, is

$$\frac{\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2 + A\left(\frac{1}{R^2} - \frac{R^2}{R_0^4}\right) = Bp_a(R)\ , \tag{4}$$

where A,B are constants, and the pressure-like quantity $p_a(R)$ is antisymmetric under duality. In general we might, of course, have other terms present in (4), however, this equation is the only one which is made solely of terms with counterparts in (1b), and is linear in $\ddot{R}$.

An automatic consequence of duality is that $R = R_0$ is a solution of the equations of motion. It is therefore for this reason that the self-dual point R_0 should be taken to be the Planck length (henceforth we shall write R_p instead of R_0). Linearizing equation (4) around $R = R_p$ can answer the question of stability or instability of that solution. The answer is dependent on specific properties of the model with duality.

We now present a string inspired model with built in duality. On toroidaly compact manifolds strings have not only momentum modes, but also winding modes. The first have energies $\propto \frac{1}{R}$, the second $\propto R$. Strings thus directly lead to duality (if we work on these manifolds). The conjugate variable to the momentum modes is the coordinate x (lives in a space of extension R). Similarly the conjugate to the winding modes is the coordinate $\tilde{x}$ (lives in space of extension $\frac{1}{R}$). In order to have manifest duality we should treat x and $\tilde{x}$ on an equal footing. To do that, let strings live on a $1+d+d$ dimensional manifold. The first d corresponds to usual d dimensional space with coordinates x. The x coordinates have a period R. The second d dimensional block has coordinates $\tilde{x}$ on it, whose period is $\frac{1}{R}$. We have obviously doubled the number of excitations. To get back the same physics as in our $1+d$ string we need further constraints for momentum, winding and oscillator modes:

$$\begin{aligned} p &= \tilde{w} \\ w &= \tilde{p} \\ \alpha &= \tilde{\alpha}\ . \end{aligned} \tag{5}$$

It is interesting to see if the constraints (5) can be incorporated into some purely geometrical formulation of strings - this is currently being looked into[7]. For our task,

we don't need this. We have identified the manifold on which to work, and due to the constraints, we no longer have to think of winding modes (we treat them as just momentum modes on the tilde space).

Our cosmological model corresponds to doing usual general relativity a lá FRW but on this $1+d+d$ dimensional manifold. We automatically get duality of the equations. Let us develop this model first. In the next section we shall go into a detailed discussion of its connection to strings, as well as the validity of the approximations used.

As stated, our coordinates are $x^\mu = (t, x^i, x^{\bar{i}}) = (t, x^A)$. As in standard cosmology we take our metric to be given by

$$ds^2 = dt^2 - R(t)^2 h_{ij} dx^i dx^j - \frac{R_p^4}{R(t)^2} \bar{h}_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}\ . \tag{6}$$

Therefore we have

$$\begin{aligned} g_{tt} &= 1 \\ g_{ij} &= -R(t)^2 h_{ij}(x^i) \\ g_{\bar{i}\bar{j}} &= -\frac{R_p^4}{R(t)^2} \bar{h}_{\bar{i}\bar{j}}(x^{\bar{i}}) \end{aligned} \tag{7}$$

where h_{ij} and $\bar{h}_{\bar{i}\bar{j}}$ are both metrics of *maximally symmetric* spaces of dimension d. We calculate the connections $\Gamma^\sigma_{\lambda\mu} = \frac{1}{2} g^{\sigma\nu}(\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\lambda\nu} - \partial_\nu g_{\lambda\mu})$. It is easy to see that the non-vanishing Γ's are simply

$$\begin{aligned} \Gamma^t_{ij} &= \dot{R}R\ h_{ij} \\ \Gamma^t_{\bar{i}\bar{j}} &= -\frac{R_p^4 \dot{R}}{R^3}\ \bar{h}_{\bar{i}\bar{j}} \\ \Gamma^i_{tj} &= \frac{\dot{R}}{R}\delta^i_j \\ \Gamma^{\bar{i}}_{t\bar{j}} &= -\frac{\dot{R}}{R}\delta^{\bar{i}}_{\bar{j}} \\ \Gamma^i_{jk} &= \gamma^i_{jk} \\ \Gamma^{\bar{i}}_{\bar{j}\bar{k}} &= \bar{\gamma}^{\bar{i}}_{\bar{j}\bar{k}}\ . \end{aligned} \tag{8}$$

Here γ's and $\bar{\gamma}$'s denote the connections of the d dimensional spaces with metrics h

and $\tilde{h}$. The Riemann tensor is in our notation

$$\mathcal{R}^{\mu}{}_{\alpha\nu\beta} = \partial_\nu \Gamma^{\mu}_{\alpha\beta} - \partial_\beta \Gamma^{\mu}_{\alpha\nu} + \Gamma^{\mu}_{\sigma\nu}\Gamma^{\sigma}_{\alpha\beta} - \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\alpha\nu} ,$$

so that its contraction the Ricci tensor is

$$\begin{aligned} \mathcal{R}_{\alpha\beta} \equiv \mathcal{R}^{\mu}{}_{\alpha\mu\beta} = \partial_t \Gamma^{t}_{\alpha\beta} + \partial_i \Gamma^{i}_{\alpha\beta} + \partial_{\bar{i}} \Gamma^{\bar{i}}_{\alpha\beta} - \partial_\beta \Gamma^{i}_{\alpha i} - \\ - \partial_\beta \Gamma^{\bar{i}}_{\alpha\bar{i}} + \Gamma^{i}_{ti}\Gamma^{t}_{\alpha\beta} + \Gamma^{i}_{ji}\Gamma^{j}_{\alpha\beta} + \\ + \Gamma^{\bar{i}}_{t\bar{i}}\Gamma^{t}_{\alpha\beta} + \Gamma^{\bar{i}}_{\bar{j}\bar{i}}\Gamma^{\bar{j}}_{\alpha\beta} - \\ - \Gamma^{t}_{i\beta}\Gamma^{i}_{\alpha t} - \Gamma^{t}_{\bar{i}\beta}\Gamma^{\bar{i}}_{\alpha t} - \Gamma^{i}_{t\beta}\Gamma^{t}_{\alpha i} - \Gamma^{i}_{j\beta}\Gamma^{j}_{\alpha i} - \\ - \Gamma^{\bar{i}}_{t\beta}\Gamma^{t}_{\alpha\bar{i}} - \Gamma^{\bar{i}}_{\bar{j}\beta}\Gamma^{\bar{j}}_{\alpha\bar{i}} . \end{aligned} \tag{9}$$

Straightforward calculation gives us the only non-vanishing $\mathcal{R}_{\alpha\beta}$'s to be

$$\begin{aligned} \mathcal{R}_{tt} &= -2d\left(\frac{\dot{R}}{R}\right)^2 \\ \mathcal{R}_{ii} &= \rho_{ii} + \left(\frac{\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2\right) R^2 h_{ii} \\ \mathcal{R}_{\bar{i}\bar{i}} &= \rho_{\bar{i}\bar{i}} - \left(\frac{\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2\right) \frac{R_p^4}{R^2} \tilde{h}_{\bar{i}\bar{i}} , \end{aligned} \tag{10}$$

where ρ and $\tilde{\rho}$ are Ricci tensors corresponding to h and $\tilde{h}$. The fact that h and $\tilde{h}$ are maximally symmetric spaces allows us to calculate $\rho, \tilde{\rho}$ directly. Due to the symmetry, the curvature must be the same at all points, so ρ_{ijkl} must be constructed out of constant tensors h_{ij}, $\epsilon_{i_1 \ldots i_d}$. It is not difficult to show that the only combination of these that is consistent with the symmetries of ρ_{ijkl} is

$$\rho_{ijkl} = \lambda(h_{ik}h_{jl} - h_{il}h_{jk}) \tag{11}$$

where λ is some constant. λ is dimension-independent (dimension comes only out of contractions of h_{ij}'s). Contracting (11) twice we get the scalar curvature $\rho = \lambda d(d-1)$.

Explicit calculation tells us that for $d = 3$ we have $\rho = 6$, so that $\lambda = 1$. The Ricci curvature is thus

$$\rho_{ij} = (d-1)h_{ij} . \tag{12}$$

A similar formula is valid for $\tilde{\rho}_{\bar{i}\bar{j}}$. We can now calculate the Einstein tensor $G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$. We find (no summation implied)

$$\begin{aligned} G^t_t &= -d\left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{2}d(d-1)\left(\frac{1}{R^2} + \frac{R^2}{R_p^4}\right) \\ G^i_i &= -\left(\frac{\ddot{R}}{R} - (1+d)\left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{2}d(d-1)\frac{R^2}{R_p^4} + \right. \\ &\qquad \left. + \left(1 - \frac{d}{2}\right)(d-1)\frac{1}{R^2}\right) \\ G^{\bar{i}}_{\bar{i}} &= \frac{\ddot{R}}{R} - (1-d)\left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{2}d(d-1)\frac{1}{R^2} - \\ &\qquad - \left(1 - \frac{d}{2}\right)(d-1)\frac{R^2}{R_p^4} , \end{aligned} \tag{13}$$

all other components vanish. It is easy to check that under duality we have

$$\begin{aligned} G_{tt} &\to G_{tt} \\ G_{ii} &\to G_{\bar{i}\bar{i}} \\ G_{\bar{i}\bar{i}} &\to G_{ii} . \end{aligned} \tag{14}$$

The Einstein equation reads

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} , \tag{15}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter. From (14) and (15) we see that $T_{\mu\nu}$ must be diagonal, that it has three sectors t, d and $\tilde{d}$, and that it transforms under duality as

$$\begin{aligned} T_{tt} &\to T_{tt} \\ T_{ii} &\to T_{\bar{i}\bar{i}} \\ T_{\bar{i}\bar{i}} &\to T_{ii} . \end{aligned} \tag{16}$$

We therefore take T to be

$$T^{\mu}_{\nu} = \begin{bmatrix} \rho(R) & & \\ & -p(R)\,\mathbb{1} & \\ & & -p\left(\frac{R_p^2}{R}\right)\,\mathbb{1} \end{bmatrix} , \tag{17}$$

where $\mathbb{1}$ denotes a d dimensional unit matrix, and the energy density is symmetric under duality: $\rho(R) = \rho(\frac{R_p^2}{R})$. As we see the form of T here is in close parallel with the usual ideal gas energy-momentum tensor of the FRW model.

Bianchi identities give us $G^{\mu\nu}{}_{;\nu} = 0$, therefore due to (15) the energy-momentum of matter must be covariantly conserved, *i.e.*

$$T^{\mu\sigma}{}_{;\sigma} \equiv \partial_\sigma T^{\mu\sigma} + \Gamma^{\mu}_{\sigma\nu} T^{\nu\sigma} + \Gamma^{\sigma}_{\sigma\nu} T_{\mu\nu} = 0 \ . \tag{18}$$

The time component of this equation gives us

$$\dot\rho = -d\Big(p(R) - p\Big(\frac{R_p^2}{R}\Big)\Big)\frac{\dot R}{R} , \tag{19}$$

which is the analogue of (1c) in the FRW model. The space components of (18) are identically satisfied.

We can now substitute the values (13) into Einstein's equation (15) to get the following three equations (corresponding to the t, d and $\tilde d$ sectors).

$$\begin{aligned} \Big(\frac{\dot R}{R}\Big)^2 - \frac{d-1}{2}\Big(\frac{1}{R^2} + \frac{R^2}{R_p^4}\Big) &= -\frac{8\pi G}{d}\rho(R) \\ \frac{\ddot R}{R} - (1+d)\Big(\frac{\dot R}{R}\Big)^2 - \frac{1}{2}d(d-1)\frac{R^2}{R_p^4} + \Big(1-\frac{d}{2}\Big)(d-1)\frac{1}{R^2} &= 8\pi G p(R) \\ \frac{\ddot R}{R} - (1-d)\Big(\frac{\dot R}{R}\Big)^2 + \frac{1}{2}d(d-1)\frac{1}{R^2} - \Big(1-\frac{d}{2}\Big)(d-1)\frac{R^2}{R_p^4} &= -8\pi G p\Big(\frac{R_p^2}{R}\Big) \ . \end{aligned} \tag{20}$$

By adding equations (20b) and (20c) we get

$$\frac{\ddot R}{R} - \Big(\frac{\dot R}{R}\Big)^2 + \frac{d-1}{2}\Big(\frac{1}{R^2} - \frac{R^2}{R_p^4}\Big) = 4\pi G\Big(p(R) - p\Big(\frac{R_p^2}{R}\Big)\Big) \ . \tag{21}$$

As we see, our model indeed leads to an equation of the form (4), with

$$p_a(R) \equiv p(R) - p\Big(\frac{R_p^2}{R}\Big) \ . \tag{22}$$

We shall call this the *total pressure.* We see that the momentum modes in the d sector give a pressure $p(R)$, while the momentum modes in the $\tilde d$ sector (or equivalently the winding modes in d) contribute a pressure of the opposite sign *i.e.* $-p\Big(\frac{R_p^2}{R}\Big)$ to the total pressure. This is just what we would get from thermodynamics.

Subtracting (20c) from (20b) and using equation (20a) to get rid of the $\left(\frac{\dot R}{R}\right)^2$ term gives us

$$\rho(R) - p_s(R) = \frac{(d-1)(2d-1)}{16\pi G}\Big(\frac{1}{R^2} + \frac{R^2}{R_0^4}\Big) , \tag{23}$$

where we have introduced an auxiliary pressure quantity

$$p_s(R) \equiv \frac{1}{2}\Big(p(R) + p\Big(\frac{R_p^2}{R}\Big)\Big)$$

that is symmetric under duality.

Apart from the thermodynamic equation of state $p = p(\rho)$ relations (19), (20a), (21) and (23) represent a complete set of equations for our model. From (20a) it immediately follows that

$$\rho(R) \le \rho_{max}(R) \equiv \frac{d(d-1)}{16\pi G}\Big(\frac{1}{R^2} + \frac{R^2}{R_0^4}\Big) \ . \tag{24}$$

Positivity of $\rho(R)$ then further implies that the number of space dimensions is greater then one. Both of these consequences have no correspondence in theories without duality.

Equation (23) is another remarkable consequence of duality. It represents a very strong constraint on the thermodynamic equation of state that links ρ and p. Models without duality leave us essentially total freedom in choosing the equation of state. The strong constraint of duality on the equation of state is a very nice thing since we have no experience with thermodynamics near the Planck scale.

The approximations that lead to our model are valid near the Planck length. Linearizing either (19) or (23) around R_p gives

$$\rho(R_p + r) = \rho(R_p) + o(r^2) \,, \tag{25}$$

i.e. the curve $\rho(R)$ has a small plateau near $R = R_p$. Both this result, and the fact that $\rho(R_p)$ can't be arbitrarily large, have been obtained by Brandenberger and Vafa[8] through purely thermodynamic arguments.

Linearizing equation (21) we may address the question of stability of the solution $R = R_p$. We get

$$\ddot{r} - \frac{2}{R_p^2} K r = 0 \,, \tag{26}$$

where $K \equiv d - 1 + 4\pi G R_p^3 \left(\frac{dp}{dR}\right)_{R=R_p}$. The sign of K determines stability. To compute K we first have to find the equation of state; we therefore turn to equation (23).

In the standard model at high temperatures we use the ultra-relativistic equation of state $p = \frac{1}{d}\,\rho$. This is just the statement that masses are small compared to the kinetic energies. If anywhere, the universe is hot at the Planck scale, so we take our equation of state to be

$$p_s(R) = C\,\frac{1}{d}\,\rho(R) \,, \tag{27}$$

where C is a constant still to be determined. We will come back to this equation of state at the end, and discuss its connection to strings. Since $\rho(R)$ is symmetric under duality it can only be proportional to the auxiliary pressure quantity $p_s(R)$ and not the total pressure $p_a(R)$. (We should note that if we had $p(R) \propto \rho(R)$ that would imply that the total pressure vanishes. It would also give us $p_s(R) \propto \rho(R)$, *i.e.* this is just a special case of (27).)

The constraint (23) now completely determines $\rho(R)$ to be

$$\rho(R) = \frac{2d-1}{d-C}\rho_{max}(R) \,. \tag{28}$$

As we have seen $\rho(R)$ is bounded on both sides by $0 < \rho(R) \le \rho_{max}(R)$, which implies

(since we had $d > 1$) that

$$\begin{aligned} d &\le 1 - C \\ d &> C \\ d &> 1 \,. \end{aligned} \tag{29}$$

The allowed d, C values are therefore confined to the wedge bounded by the curves $d = 1$ and $d = 1 - C$, and satisfying $C < 0$ (see Fig. 1). We should note that this implies that $p_s(R) < 0$ for all R; this is not a problem since p_s is not the total pressure but, as we have stressed, an auxiliary quantity.

The parameter C is determined by initial conditions. From (28) and the definition of $\rho_{max}(R)$ we get

$$\rho(R_p) = \frac{2d-1}{d-C} \cdot \frac{d(d-1)}{8\pi G R_p^2} \,. \tag{30}$$

Using the fact that $G = M_p^{-1} R_p^{d-2}$, as well as that the volume of a sphere of radius R in d dimensions is $V(R,d) = \frac{R^d \pi^{d/2}}{\Gamma(d/2+1)}$, we may write the initial energy density as

$$\rho(R_p) = N(d,C) \cdot \frac{M_p}{V(R_p,d)} \,, \tag{31}$$

where we have defined N to be

$$N(d,C) \equiv \frac{1}{8}\,\frac{d(d-1)(2d-1)\pi^{d/2-1}}{(d-C)\Gamma(d/2+1)} \,. \tag{32}$$

N is roughly the initial number of strings in the universe, since M_p is a typical string energy at Planck length. Fig. 2 depicts the allowed wedge in the d, C plane along with the curves corresponding to $N = 1, 2, 3, 4$ and 10 strings at the begining of the universe.

We learn several things from this graph. First: N is restricted to be less then 10, *i.e.* initialy there is a very small number of strings in the universe. This follows since for $N \ge 10$ the curves of fixed N never enter the allowed region. Second: the $N = 1$ curve intersects the allowed region at (exactly!) $d = 3$. Further, $N \ge 1$ implies that we must have $d \geq 3$. Note that $N < 1$ is not allowed since the least we can have is one string in the ground state. It is important to stress here that we are assuming that there is no cosmological constant present.

We now look at the speed of expansion of the universe near the Planck length, and make contact with the work in ref.[5]. If $\frac{\dot{R}}{R} \ll \Gamma(w\bar{w} \to 0)$, then winding modes equilibriate, and the Boltzman factor $\exp(-\beta E)$ gets rid of the winding modes (since for windings $E \propto R$). On the other hand if $\frac{\dot{R}}{R} \gg \Gamma$ then windings would fall out of equilibrium, *i.e.* they would remain in the universe and (through their negative pressure) would stop the expansion long before the universe reached its present size. The (small neighborhoods of the) zeros of $\dot{R}(d)$ are thus of special interest. $\dot{R}$ at R_p has been plotted in Fig. 3. For example the zeros of the $N = 1$ curve are at $d = 3$ and near $d = 18$. Since Γ *decreases* with d (there is more space for the strings to miss each other) we should look only at the smallest zero. Also note that the $\dot{R}$ curves are almost vertical near the zeros, therefore, the interesting neighborhoods around the zeros are extremely small.

Brandenberger and Vafa determined that $d \leq 3$ using the usual $2 + 2 = 3 + 1$ argument. Here, since $N = 1$ corresponds to the minimum initial ammount of energy in the universe, we get the requirement that $d \geq 3$. We find that only 3 spatial dimensions are consistent with duality and the present size of the universe. We also see that initially there is only one string present in the universe. This kind of initial condition has already come up for different reasons[8].

Strings, however, live in $d = 9$, so how can our model be string induced? Strictly speaking it can't. However a similar model with two radiuses can be constructed. We have found that the above arguments still apply: there are 3 large directions and 6 small (Planck size) directions. The details have been worked out and will be presented in a separate publication[7].

Finally let us say a bit more about the assumtions and approximation used in this model. A detailed analysis will be given in [7]. First: the treatment is *classical* and for it to make any sense, even with duality, we need to take the extreme view that classical results remain meaningfull (to an order of magnitude) up to the Planck

length. Second: All fields except the graviton and its dual are treated as free. The favored status of gravity comes about from the fact that gravitons are massless particles. There is no *a priori* reason to believe that interactions of the other modes are not important. However the fact that our model predicts $N = 1$ seems encouraging. The number of strings grows rapidly after the big bang, and so we should expect our model to have a very limited domain of validity centering at R_p. This is quite unfortunate since it means that by simply taking $R \to \infty$ in our model we won't recover the equations of standard cosmology. In fact *no model* with dual symmetry will directly go over into the standard model, as is evident from the arguments leading to equation (4). At low temperatures (compared to Planck) the dual symmetry of the $R(t)$ equations is broken.

Also, since strings have modes with arbitrarily large masses one would assume that it is not correct to use an ultra relativistic equation of state, which assumes that the masses are small compared to the kinetic energies. Again, the severe limitation on the inital energy in the universe that arises as a consequence of duality permits us to get around this problem.

The final major assumption made was that like the Einstein equations our string induced equations will be second order in derivatives. String field theory tells us that strings have non-local interactions of the form e^{∂^2}. The true equation for the condensate $R(t)$ should then also be non-local. In the neighborhood of R_p using the restrictions imposed by duality, and assuming non-locality of the form e^{∂^2} it is possible to construct a non-local generalization of our equations. We looked at the stability of the solution R_p, and have found that the higher derivatives do not have an important effect. Hopefully this would indicate that non-localities are not important near R_p. Again, if we stray too far from R_p we find the effect of the non-localities to be very important. An thorough investigation of the effect of (non-local) string interactions on string thermodynamics is currently in progress[9]. A new detailed look at free string thermodynamics (and the connection between microcanonical and canonical ensembles) is given in [10].

3. Conclusion

A simple (string inspired) cosmological model with duality has been presented. The model is found to correctly determine the dimensionality of space. The assumption on which the model is built are valid only very near the Planck length. There the model is found to be in agreement with the thermodynamic arguments that have been presented by Brandenberger and Vafa. Another consequence of the model is that the initial number of strings in the universe is 1. Further connections with strings are being looked into. This, along with a more realistic two radius model, will be given in a separate publication.

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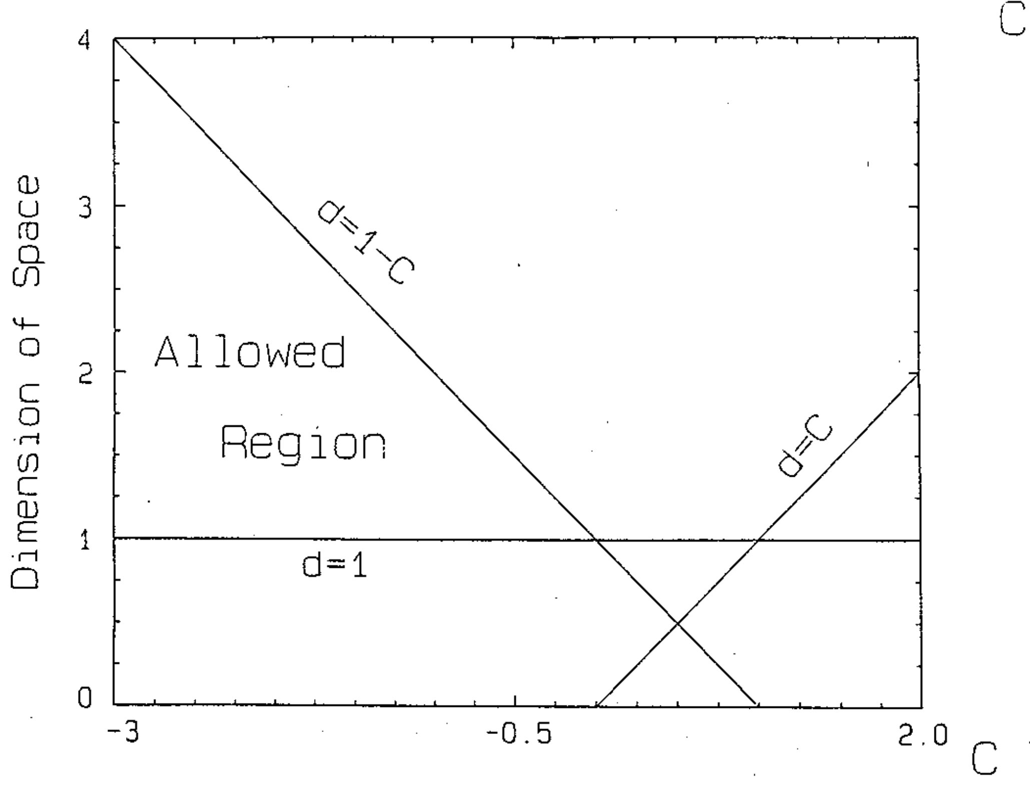

Figure 1.

The allowed region in the d, C plane bounded by the critical curves $\rho(R) = 0$ (*i.e.* $d = 1$) and $\rho(R) = \rho_{MAX}(R)$ $(d = 1 - C)$.

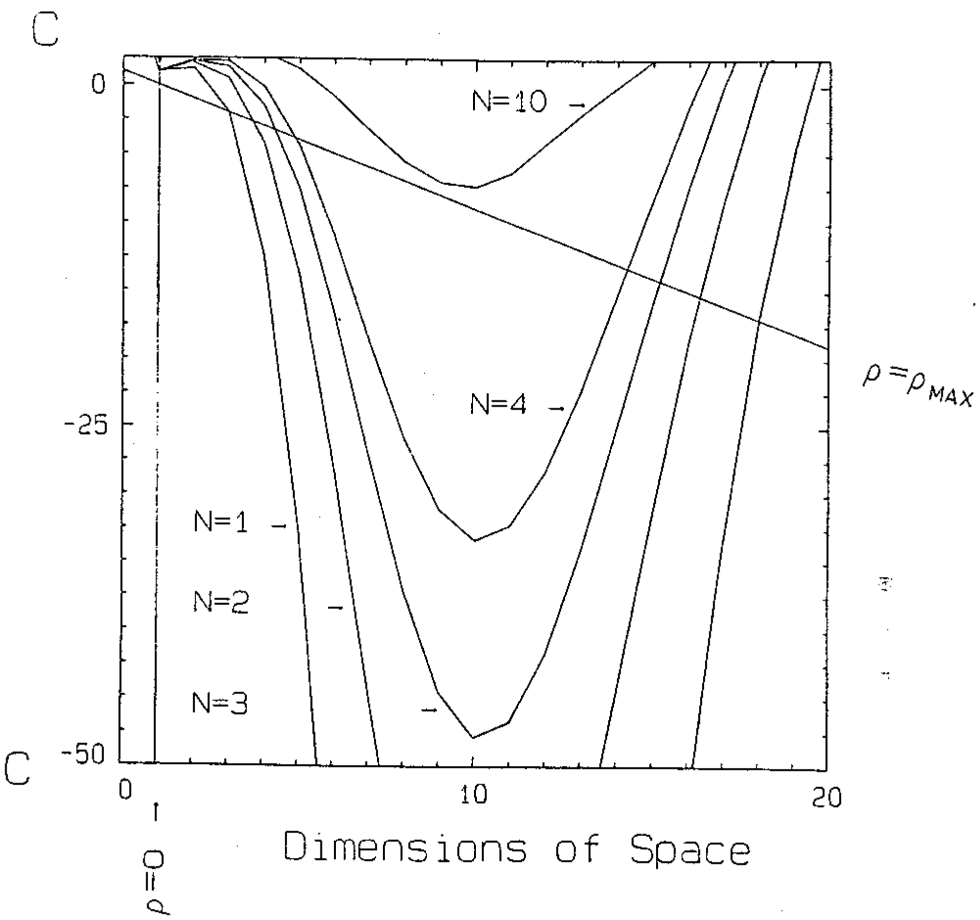

Figure 2.

Curves corresponding to $N = 1, 2, 3, 4, \ldots, 10$ strings in the universe at Plank length, and their relation to the allowed region.

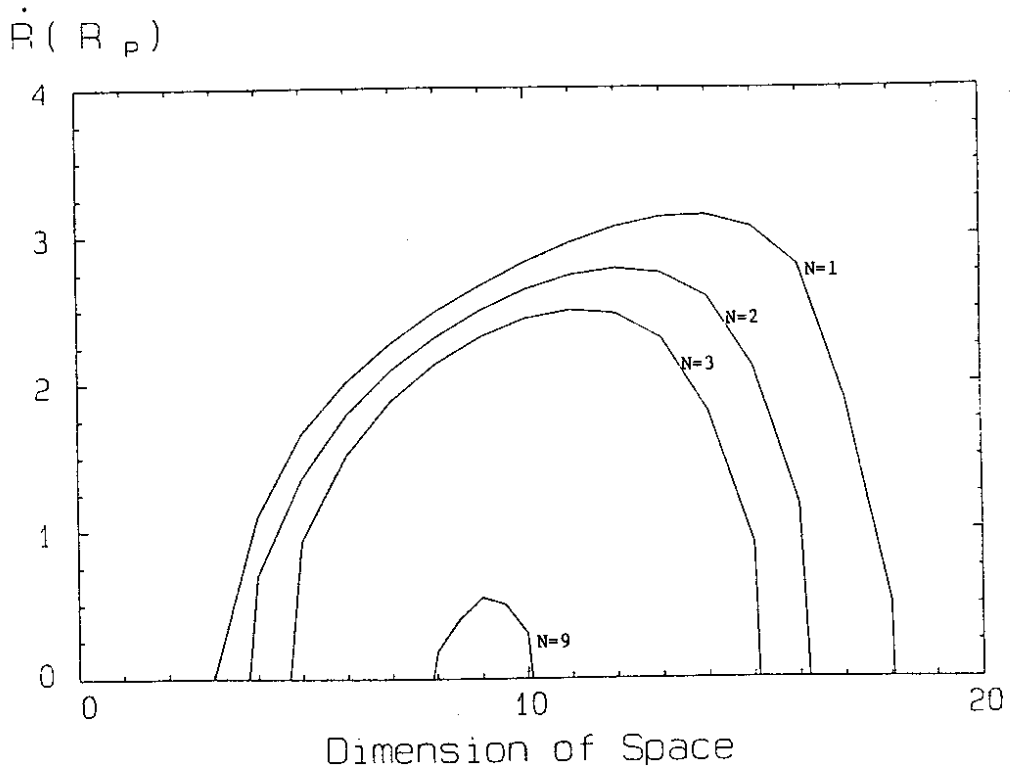

Figure 3.

Speed of expansion of the universe at the Plank length as a function of the dimension of space and the initial number N of strings in the universe.