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Two Field Formulation of Closed String Field Theory

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ABSTRACT

A formulation of closed string field theory is presented that is based on a two field action. It represents a generalization of Witten's Chern-Simons formulation of 3d gravity. The action contains only 3 string interactions and no string field truncations, unlike the previous non-polynomial action of Zwiebach. The two field action is found to follow from a purely cubic, background independent action similar to the one for open strings.

1. Introduction

It has been know for some time how to construct a covariant, gauge invariant theory of the open bosonic string. Witten [1] showed that the open string is to be described by an action that we write formaly as

$$S = \int (AQA + \frac{2}{3}A^3) , \tag{1.1}$$

satisfying generalized Chern-Simons axioms. The theory was put on firm footing by the explicit operator construction of Gross and Jevicki [2,3]. The subsequent work of many people [4-13] has shed light on the structure of Witten's theory. In particular, it was found [14-20] that (1.1) can be obtained from a more fundamental, background independent, purely cubic action

$$S = \frac{2}{3}\int X^3 . \tag{1.2}$$

The original Witten action is obtained from this by expanding around a particular vacuum solution.

In contradistinction to open strings progress with closed strings has been more difficult. Recently, work by Zwiebach [21-31] has re-ignited interest in closed string field theory. The theory is governed by a non-polynomial action

$$S = \int \delta(N - \tilde{N})(BQB + B^3 + B^4 + B^5 + \ldots + \hbar \quad corrections) . \tag{1.3}$$

This was derived by insisting on having a similar action to the one for open strings, and then adding extra interaction terms in such a way that one reproduces the correct tree amplitudes. It was found that each term in the interaction is associated with a regular polyhedron. The continuation of this program is in obtaining the (infinitely many) measure terms, *i.e.* $\hbar$ corrections. One can show that the interaction has no poles, hence that (1.3) can indeed be interpreted as a fundamental (rather than effective) action of the theory. The end result is a formalism that is by construction correct, yet quite difficult to work with.

The huge difference between (1.1) and (1.3) parallels a similar difference between Yang-Mills theories, described by an action of the form

$$S_{YM} = \int F \wedge *F = \int (A_\mu K_{\mu\nu} A_\nu + A^3 + A^4) , \tag{1.4}$$

and gravity. If the only thing we knew about gravity was that it describes a massless spin 2 field then we would first try to mimick the Yang-Mills action as much as possible. By following the program of self-consistent coupling to a Poincaré invariant current discovered by Feynman [32,33] we would then obtain

$$S_G = \int (h_{\mu\nu} K_{\mu\nu,\rho\sigma} h_{\rho\sigma} + h^3 + h^4 + h^5 + \ldots) , \tag{1.5}$$

i.e. another non-polynomial action. If this had been the historic way that gravitation was discovered further progress would have been exceedingly difficult. It is only with hindsight that we know that we can formaly sum the Taylor series in (1.5) in terms of a new field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and recover the Einstein-Hilbert action

$$S_G = \int R . \tag{1.6}$$

The problem with the non-polynomial approach to closed strings is that we don't know the stringy generalization of the Einstein-Hilbert action, in fact finding this is the central motivation of looking into closed string field theory in the first place. At the same time explicit calculations have been difficult even in the much simpler open string formalism. Finaly, one can't go from (1.3) to a purely cubic action, and hence to a background independent formalism.

The non-polynomial approach has been successfull in that it has uncovered the relation of closed strings to polyhedra (and their quantum relatives). This beautiful relationship certainly hints at the existence of a simpler formalism in terms of other fields. For this reason we look at a new approach to describing closed strings.

2. Relation to Chern-Simons Formulation of 3d Gravity

The fact that open strings are described by a generalized Chern-Simons action is perhaps, looking back, not too strange. Ultimately strings should be formulated in a way that is independent of background geometry and topology. Topological theories in 3 dimensions are thus natural toy models for investigating strings. The massless particle in the open string has spin 1, hence Witten based open string field theory on the generalization of

$$S_A = \int \mathrm{tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) . \tag{2.1}$$

The program of constructing a background independent formulation was then completed by the purely cubic action (1.2).

At the massless level of the closed string we find a spin 2 particle. It therefore seems natural to generalize the Chern-Simons formulation of 3 dimensional gravity [34-37] given by the action

$$S_G = \int \mathrm{tr}(e \wedge d\omega + e \wedge \omega \wedge \omega) = \int \mathrm{tr}(e \wedge F(\omega)) , \tag{2.2}$$

where e and ω may be viewed as two SU(2) valued 1-form fields. The above action is invariant under the gauge transformations

$$\begin{aligned} \delta e &= d\Lambda + [e, \Sigma] + [\omega, \Lambda] \\ \delta\omega &= d\Sigma + [\omega, \Sigma] . \end{aligned} \tag{2.3}$$

The Einstein-Hilbert action is recovered from this after we integrate out the ω field. In the following sections we shall make explicit the generalization of this to close strings.

Just as gravity has a two field formulation in 3 dimensions, so is the case with Yang-Mills theory. The appropriate action is

$$S_{YM} = \int \mathrm{tr}(B \wedge F(A)) + \int \mathrm{tr}(B \wedge *B) \ , \tag{2.4}$$

with matrix valued 1-form fields A and B, and gauge invariance under

$$\begin{aligned} \delta A &= d\Lambda + [A, \Lambda] \\ \delta B &= 0 \ . \end{aligned} \tag{2.5}$$

By integrating out B we recover (1.4). These two examples illustrate the beauty of two field formulations of gauge theories in that different theories can be cast in a very similar form. Note that (2.2) and (2.4) differ only in a "mass" term for B. It is because of this term that S_{YM} does not describe a topological theory.

Two field formulations don't just represent a simplified notation. Some times a manifestly gauge invariant one field formulation does not exist. For example look at the non-abelian antisymmetric tensor field in 4 dimensions. The Freedman-Townsend action [38] reads

$$S_{FT} = \int \mathrm{tr}(B \wedge F(A)) + \int \mathrm{tr}(A \wedge *A) \ . \tag{2.6}$$

A is a 1-form, B a 2-form field. The gauge invariance of the Freedman-Townsend action is

$$\begin{aligned} \delta A &= 0 \\ \delta B &= d\phi + [B, \phi] \ . \end{aligned} \tag{2.7}$$

If we now try and integrate out the auxilliary field A we get $A \propto *DB$. In non-abelian theory the right hand side also depends on A (through the covariant derivative D). Therefore $A(B)$ is messy, and in fact can't be written in manifestly geometric language.

3. Free Theory

As we have seen, the choice of field content, *i.e.* free theory, is very important lest we end up with a very complicated description of the interacting theory. Free field theory for the closed string was analyzed by Marcus and Sagnotti [39]. We will here present a simpler derivation of their results by the use of string projection operators. Free theory of all strings is described by the equation of motion

$$Q|X> = 0 \ , \tag{3.1}$$

where Q is the appropriate BRST charge. Nilpotence of Q in critical dimension implies the gauge invariance $\delta|X> = Q|\Lambda>$. Writting the ghost zero modes explicitly, in the case of the open bosonic string we have

$$Q = c_0 L + d + \delta - 2b_0 R \ , \tag{3.2}$$

where L is quadratic in time derivatives, d and δ linear, and R has no time derivatives. Nilpotence of the BRST charge gives the relations

$$\begin{aligned} d^2 &= \delta^2 = 0 \\ d^+ &= \delta \\ \{d, \delta\} &= -2RL \ , \end{aligned} \tag{3.3}$$

hence we see that d is a generalization of the exterior derivative, L of the Laplacian.

Similarly to the well-known transverse and longitudinal projection operators of QED we define the string projectors [40]

$$\begin{aligned} P_T &= \frac{b_0 Q}{L} \\ P_L &= \frac{Q b_0}{L} \end{aligned} \tag{3.4}$$

One easily derives the properties

$$\begin{aligned} P_T + P_L &= 1 \\ P_T^2 &= P_T \\ P_L^2 &= P_L \\ P_T P_L = P_L P_T &= 0 \\ P_T^\dagger &= P_L \ , \end{aligned} \tag{3.5}$$

as well as

$$\begin{aligned} QP_T &= Q \\ b_0 P_L &= b_0 \ . \end{aligned} \tag{3.6}$$

From this it follows that only the transverse fields are in the string equation of motion $0 = Q|X> = QP_T|X> = Q|X_T>$. Similarly $\delta|X>$ is pure longitudinal. The simplest gauge choice is thus

$$|X_L> = 0 \ , \tag{3.7}$$

and it gives us the Siegel gauge $b_0|X> = 0$. Any open string field is written as $|X> = (X_1 + c_0 X_2)|->$. In the Siegel gauge (3.7) we have $X_2 = 0$, and the equations of motion read

$$\begin{aligned} (d+\delta)X_1 &= 0 \\ LX_1 &= 0 \ . \end{aligned} \tag{3.8}$$

The field X_1 satisfies the correct equation of motion. To be physical it must have zero ghost number, so $g(|X>) = -1/2$. This is the precise value of the ghost number that makes it possible for us to write the free action for the open string as $S_0 = <X|Q|X>$.

Let us repeat this analysis for the closed string. We have $Q_{cl} = Q + \tilde{Q}$, and

if we introduce

$$\begin{aligned} c_\pm &= \frac{1}{\sqrt{2}}(c_0 \pm \tilde{c}_0) \\ b_\pm &= \frac{1}{\sqrt{2}}(b_0 \pm \tilde{b}_0) \ , \end{aligned}$$

we may write

$$Q_{cl} = \frac{1}{\sqrt{2}} c_+ K + \frac{1}{\sqrt{2}} c_- \Delta + d + \delta + \tilde{d} + \tilde{\delta} - \sqrt{2} b_+ R_+ - \sqrt{2} b_- R_- \ , \tag{3.9}$$

where $K = L + \tilde{L}$ is the Laplacian, and $\Delta = L - \tilde{L} = N - \tilde{N}$. Q_{cl} commutes with the number operators N, $\tilde{N}$, and so $[Q_{cl}, \Delta] = 0$, hence we can analyze the equations of motion in the $\Delta = 0$ and $\Delta \neq 0$ sectors separately.

In analogy with the treatment for open strings, in the $\Delta \neq 0$ sector of the closed string we define the projection operators

$$\begin{aligned} \Pi_+ &= \sqrt{2}\frac{b_- Q_{cl}}{\Delta} \\ \Pi_- &= \sqrt{2}\frac{Q_{cl} b_-}{\Delta} \ . \end{aligned} \tag{3.10}$$

As before we find that only the "+" fields appear in (3.1). Therefore we choose the gauge fixing condition

$$|X_-> = 0 \ , \tag{3.11}$$

that gives us $b_-|X> = 0$. Expanding the string field in ghost zero modes we get

$$|X> = (X_1 + c_+ X_2 + c_- X_3 + c_+ c_- X_4)|-,-> \ . \tag{3.12}$$

The gauge (3.11) sets $X_3 = X_4 = 0$, and the equations of motion become

$$\begin{aligned} (d + \delta + \tilde{d} + \tilde{\delta})X_1 - \sqrt{2} R_+ X_2 &= 0 \\ KX_1 - \sqrt{2}(d + \delta + \tilde{d} + \tilde{\delta})X_2 &= 0 \\ \Delta X_1 &= 0 \\ \Delta X_2 &= 0 \ . \end{aligned} \tag{3.13}$$

We are in the $\Delta \neq 0$ sector, so the last two equations immediately give $X_1 =$

$X_2 = 0$. The $\Delta \neq 0$ fields are not physical.

We treat the $\Delta = 0$ sector in complete parallel, by introducing the projectors

$$P_T = \sqrt{2}\frac{b_+ Q_{cl}}{K}$$
$$P_L = \sqrt{2}\frac{Q_{cl} b_+}{K} \ . \quad (3.14)$$

We gauge away the longitudinal states by $|X_L >= 0$, *i.e.* $b_+|X >= 0$, or $X_2 = X_4 = 0$. The equations of motion are

$$\begin{aligned}(d + \delta + \tilde{d} + \tilde{\delta})X_1 - \sqrt{2}R_- X_3 &= 0 \\ (d + \delta + \tilde{d} + \tilde{\delta})X_3 &= 0 \\ KX_1 &= 0 \\ KX_2 &= 0 \ .\end{aligned} \quad (3.15)$$

We see that both X_1 and X_3 satisfy the correct gauge fixed equation for a physical field. If X_1 is to be the physical field we must have $g(|X >) = -1$, while if the physical field is to reside in X_3 we must have $g(|X >) = 0$. As in the open string the ghost number assignments above directly lead us to a free action

$$S_0 =< A|Q_{cl}|B > \ . \quad (3.16)$$

In the above formula $|A >$ has ghost number -1 and $|B >$ ghost number 0. There is no truncation to the $\Delta = 0$ sector necessary.

If we are willing to allow the kinetic term to have insertions then there is another possibility for the free action

$$S_0 =< B|b_-\delta(\Delta)Q_{cl}|B > \ . \quad (3.17)$$

This is the free action used by Zwiebach. Note that the price for using a formulation in terms of one field has been the introduction of the truncation to $\Delta = 0$

states, without which this action is not gauge invariant. At the level of the free theory there is no advantage to (3.16) as opposed to (3.17) because the auxilliary ($\Delta \neq 0$) fields do not mix with the physical ($\Delta = 0$) fields. In fact, the two field action leads to a doubling of physical states, one copy residing in A and one in B. This is not going to be a problem once we introduce interactions. If we looked at just the quadratic piece of the gravity action (2.2) we would also conclude that there is a doubling of physical fields. The neglected interaction piece, however, plays a crucial role. It is what allows us to differentiate between the vierbein e and the connection ω. There is no doubling of physical states. Similarly the fact that (3.16) is not bounded from bellow does not represent a problem in the interacting theory. Again there is a complete parallel with (2.2).

Once we introduce interactions we will couple auxilliary and physical states. The fact that the two field formulation doesn't use truncations will prove to be crucial in keeping it polynomial, in fact cubic.

4. Cubic Theory

As we have seen, the most fundamental formulation of open strings comes from the purely cubic action (1.2). It was argued by Strominger [17.18] that on-shell closed string states exist in the non-associative extension of the space of open strings. This very interesting proposal has not proven useful since one doesn't know how to treat off-shell states. Rather, we will be here interested in a closed string analogue of (1.2) directly based on closed string states. To do this let us try and generalize a bit the structure of cubic theories.

The central object is the string derivative defined by

$$D_X B = XB - (-)^B BX \ . \quad (4.1)$$

If the string field X is odd, satisfies $X^2 = 0$, and the string product is associative then one easily shows that D_X satisfies the properties

$$\begin{aligned} D_X(BC) &= D_X BC + (-)^B B D_X C \\ \int D_X B &= 0 \\ D_X^2 &= 0 \, , \end{aligned} \tag{4.2}$$

and so indeed is a derivation. Two actions lead to $X^2 = 0$ as an equation of motion. The usual action is

$$S_1 = \frac{2}{3} \int X^3 \, , \tag{4.3}$$

and the other possibility is

$$S_2 = \int A X^2 \, . \tag{4.4}$$

By shifting around the vacuum X_0 (*i.e.* $X = X_0 + B$) we pick up kinetic terms. For (4.3) we get

$$S_1 = \int (B D_{X_0} B + \frac{2}{3} B^3) \, , \tag{4.5}$$

invariant under

$$\delta B = D_{X_0} \Lambda + [B, \Lambda] \, . \tag{4.6}$$

Similarly, for the two field cubic action (4.4) we get

$$S_2 = \int (A D_{X_0} B + A B^2) \, , \tag{4.7}$$

which is invariant under gauge transformation

$$\begin{aligned} \delta A &= D_{X_0} \Sigma + \{B, \Sigma\} + [A, \Lambda] \\ \delta B &= D_{X_0} \Lambda + [B, \Lambda] \, . \end{aligned} \tag{4.8}$$

We see that S_1 is a string generalization of Chern-Simons theory for Yang-Mills. This is the generalization appropriate to open strings. On the other hand S_2 represents a string generalization of Einstein gravity (2.2). It is therefore our proposal to take S_2 as the closed string action. Note that S_1 and S_2 are the only two possible cubic action, just as strings can only be either open or closed.

In the operator language

$$S_1 = \frac{2}{3} < X| < X| < X|V > \, , \tag{4.9}$$

where $|V>$ is the 3-string vertex constructed in [2,3] from the Witten open string overlaps. Strominger [17,18] has shown explicitly that $X_0 = Q_L I$ is a vacuum solution with the property $D_{X_0} = Q$. This established the connection of the cubic formulation to that of Witten. In the case of closed strings we have

$$S_2 = < A| < X| < X|V_{cl} > \, . \tag{4.10}$$

As we have seen X must have ghost number either -1 or 0. It also has to be a string field with odd grading so that properties (4.2) hold. It follows that $g(|X>) = 0$, and thus $g(|A>) = -1$. Physical closed 3 string amplitudes factor [41] into a product of open string amplitudes for left and right movers

$$A_3^{cl} = A_3 \tilde{A}_3 \, . \tag{4.11}$$

Therefore, in terms of interaction vertices we have

$$|V_{cl}> = \mathcal{O}|V> |\tilde{V}> \, , \tag{4.12}$$

where $\mathcal{O}$ is an insertion made of ghosts only. From ghost number assignments it follows that $g(\mathcal{O}) = -2$.

As we have seen, physical states in $|A>$ lie on the $|-,->$ vacuum, and in $|X>$ on the $c_-|-,->$ vacuum. We therefore find that the physical 3 string amplitudes have a common factor coming from the ghosts equal to

$$< -,-|_1 < -,-|_2 < -,-|_3 \, c_-^2 c_-^3 \mathcal{O}|+,+>_1 |+,+>_2 |+,+>_3 \, .$$

This must be equal to 1, so $\mathcal{O}$ must have in it $b_-^2 b_-^3$. The cyclic closed string vertex is obtained from the insertion

$$\mathcal{O} = b^1_- b^2_- + b^2_- b^3_- + b^3_- b^1_- \ . \tag{4.13}$$

This completely specifies the closed string vertex. If there exists a vacuum X_0 such that $D_{X_0} = Q_{cl}$, then we can arrive at the closed string analogue of Witten's open string action. The existence of X_0 is guaranteed from the vertex factorization property (4.12). For example, if we disregard the ghost zero-mode insertion $\mathcal{O}$, then it is easy to show that this vacuum is simply $X_0 = Q^{cl}_L I \tilde{I}$, where $Q^{cl}_L = Q_L + \tilde{Q}_L$ is again the integral of j_{BRST} over half the string. A more careful treatment that keeps track of the insertion $\mathcal{O}$ is presently being worked on.

We have arrived at the closed string action

$$S = < A|Q_{cl}|B > + < A| < X| < X|V_{cl} > \ . \tag{4.14}$$

This is the main result of this paper. As promised this action is a straightforward generalization of the action in the Chern-Simons formulation of 3d gravity. The above action is derivable from a purely cubic, background independent action given in (4.10).

It is important to make precise the relationship of this new action to the nonpolynomial one. This is currently being looked into. We can see that by integrating out the auxilliary ($\Delta \neq 0$) fields we pick up quartic and all higher interaction terms. Since the auxilliary fields have a constant for the propogator (*i.e.* $\frac{1}{\Delta}$) it follows that the additional interaction terms will have no poles in them, as is the case with Zwiebach's action. Similarly, integrating out the auxilliary fields also gives rise to $\hbar$ corrections.

5. Conclusion

We have presented a derivation of a two field action for closed strings. As with open strings, this action is seen to represent a generalization to strings of a particular Chern-Simons theory - 3d gravity. In this formulation the closed strings interact only through 3 string overlaps. It has been shown that a background independent purely cubic action exists. It is very similar to the one discovered for open strings. In this formulation the field theory for closed strings is seen to be no more complicated than for open strings, as one would have expected from first quantization. The connection to the non-polynomial action lies in integrating out auxilliary fields (fields that satisfy $N - \tilde{N} \neq 0$). This brings in extra interactions as well as measure terms. It is important that further work establishes in detail the connection to the non-polynomial theory, and in particular to polhedra. This investigation is currently in progress.

The established connection to 3d gravity may make it possible to establish the string generalization of the gravitation equivallence principle.

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