Homogeneous Bose-Einstein Condensate with Weak Disorder

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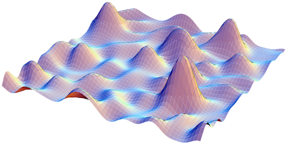
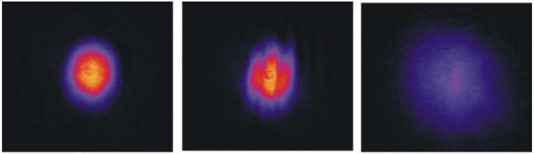


Method

- Huang, Meng, PRL **69**, 644, 1992: Bogolyubov theory for dirty bosons
- quantum fluctuations and random potential fluctuations decouple at lowest order
- for disorder effects mean-field approach sufficient
- at T=0 solve GP equation and perform then disorder ensemble average
- homogeneous disorder potential, correlation function:

$$\begin{aligned} \langle U(\mathbf{x}) \rangle &= 0 \;, \\ \big\langle U(\mathbf{x}) U(\mathbf{x}') \big\rangle &= R(\mathbf{x} - \mathbf{x}') \end{aligned}$$

Model



- R. Graham, A. Pelster, Int. J. Bif. Chaos **19**, 2745, 2009
- superfluid order parameter:
  $$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \langle \psi(\mathbf{x})\psi(\mathbf{x}') \rangle = n_0$$
- Bose-glass (Edwards-Anderson like):
  $$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \langle \psi(\mathbf{x})^2\psi(\mathbf{x}')^2 \rangle = (n_0 + q)^2$$

System at Rest

- time-independent Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + U(\mathbf{x}) - \mu + g|\psi(\mathbf{x})|^2\right]\psi(\mathbf{x}) = 0$$

- particle density: $n = \left\langle \psi(\mathbf{x})^2 \right\rangle$
- condensate density: $n_0 = \langle \psi(\mathbf{x}) \rangle^2$
- perturbatively solve equation for different orders in U

$$\psi(\mathbf{x}) = \psi_0 + \psi_1(\mathbf{x}) + \psi_2(\mathbf{x}) + \psi_3(\mathbf{x}) + \dots$$

- disorder-induced condensate depletion

$$q = n - n_0 = \underbrace{\left\langle \psi_1(\mathbf{x})^2 \right\rangle}_{\text{Huang-Meng}} + \underbrace{\left\langle \psi_2(\mathbf{x})^2 \right\rangle - \langle \psi_2(\mathbf{x}) \rangle^2 + 2\left\langle \psi_1(\mathbf{x})\psi_3(\mathbf{x}) \right\rangle + \dots}_{\text{beyond Huang-Meng}}$$

System at Rest(dimensionless form)

$$\frac{q}{n} = \frac{1}{\pi^2}\int d^3k \frac{r(\mathbf{k})}{(\mathbf{k}^2+1)^2}$$
$$+\frac{1}{\pi^4}\int d^3k d^3k' r(\mathbf{k}) r(\mathbf{k}') \left\{ -\frac{\mathbf{k}^2+2\mathbf{k}'^2+1}{(\mathbf{k}^2+1)^3(\mathbf{k}'^2+1)^2} \right.$$
$$+\frac{2\left(2\mathbf{k}^2-1\right)\left(\mathbf{k}'^2-2\right)}{\left(\mathbf{k}^2+1\right)^3\left(\mathbf{k}'^2+1\right)^2\left[\left(\mathbf{k}-\mathbf{k}'\right)^2+1\right]}$$
$$\left. +\frac{\mathbf{k}^2(\mathbf{k}^2+\mathbf{k}'^2-3)+\mathbf{k}'^2+\frac{1}{2}}{\left(\mathbf{k}^2+1\right)^2\left(\mathbf{k}'^2+1\right)^2\left[\left(\mathbf{k}-\mathbf{k}'\right)^2+1\right]^2} \right\} + \dots$$

System with Momentum

- modified Gross-Pitaevskii equation for the system with a momentum

$$\left[-\frac{\hbar^2}{2m}\Delta + U(\mathbf{x}) - \mu_{eff} + \frac{\hbar}{i}\left(\mathbf{v_n} - \mathbf{v_s}\right)\nabla + g|\psi(\mathbf{x})|^2\right]\psi(\mathbf{x}) = 0$$

  where $\mu_{eff} = \mu + m\mathbf{v_n}\mathbf{v_s} - \frac{1}{2}m\mathbf{v_s}^2$
- system momentum: $\mathbf{p} = mnV\mathbf{v_s} - \int d^3x\psi(\mathbf{x})^*\frac{\hbar}{i}\nabla\psi(\mathbf{x})$
- superfluid depletion: $q_s = \frac{1}{mV}\left.\frac{\partial\mathbf{p}}{\partial\mathbf{v_n}}\right|_{\mathbf{v_n}=0,\mathbf{v_s}=0}$

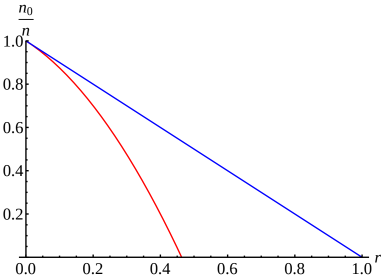
$$\frac{q_s}{n} = \underbrace{\frac{4}{3\pi^2}\int d^3k\frac{r(|\mathbf{k}|)}{(\mathbf{k}^2+1)^2}}_{\text{Huang-Meng}} + \dots$$

Results

$$R(\mathbf{x}) = R\delta(\mathbf{x}), \qquad r = R\sqrt{\frac{\pi}{2gn}}\left(\frac{m}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

At Rest

$$\frac{n_0}{n} = 1 - r - \frac{5}{2}r^2 + \dots$$

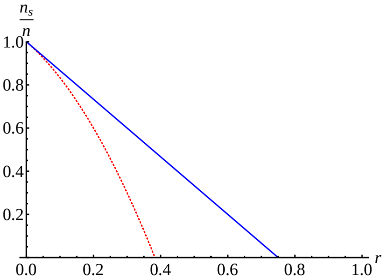


$$r_{c_1} = 1$$

$$r_{c_2} = \frac{1}{5}\left(-1 + \sqrt{11}\right) \approx 0.46$$

With Momentum

$$\frac{n_s}{n} = 1 - \frac{4}{3}r - ?r^2 + \dots$$



$$r^s_{c_1} = \frac{3}{4}$$

Outlook

- finishing calculation for q_s to the second order in disorder
- equation of state
- sound velocity $c = \sqrt{\frac{n_s}{m}\frac{\partial \mu}{\partial n}}$
- speculation on quantum phase transition
- harmonically trapped system