Unbalanced clustering and solitary states in coupled excitable systems

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ABSTRACT

We discover the mechanisms of emergence and the link between two types of symmetry-broken states, the unbalanced periodic two-cluster states and solitary states, in coupled *excitable* systems with attractive and repulsive interactions. The prevalent solitary states in non-locally coupled arrays, whose self-organization is based on successive (order preserving) spiking of units, derive their dynamical features from the corresponding unbalanced cluster states in globally coupled networks. Apart from the states with successive spiking, we also find cluster and solitary states where the interplay of excitability and local multiscale dynamics gives rise to so-called *leap-frog* activity patterns with an alternating order of spiking between the units. We show that the noise affects the system dynamics by suppressing the multistability of cluster states and by inducing pattern homogenization, transforming solitary states into patterns of patched synchrony.

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With the remarkable discovery of chimera states, the research of self-organization in coupled oscillators witnessed a change of focus from the synchronization transition and the onset of the collective mode toward the emergence and the relationship between the states with symmetry breaking of synchrony (cluster states, chimeras, solitary states), where assemblies of indistinguishable oscillators with symmetric couplings split into groups with different dynamics. Currently, these problems remain widely open for a class of coupled excitable systems, which have a linearly stable rest state but may be triggered to oscillate by strong enough perturbations due to interactions and/or noise. We address the mechanisms of emergence and the link between two types of symmetry-broken states in coupled excitable FitzHugh-Nagumo systems, namely, the unbalanced periodic two-cluster states in globally coupled networks, characterized by an uneven partition between the clusters comprising identically

synchronized units, and solitary states in non-locally coupled arrays, where small groups of units display an average frequency distinct from the typical units forming the synchronized cluster. The prevalent solitary states, where the self-organization is based on successive (order preserving) spiking of units, are found to appear in the same parameter range as the corresponding unbalanced cluster states, inheriting the ratio of average frequencies of solitary and typical units and the form of corresponding units' orbits. Apart from the states displaying successive spiking, we also find the states involving leap-frog (leader-switching) dynamics, where the units from different clusters, or even within the same cluster, exchange their relative order of spiking. We further demonstrate that the noise reduces the multistability of cluster and solitary states by the effect of noise-induced preference of attractors, promoting the attractors with a larger basin of attraction at the expense of those with a smaller one.

I. INTRODUCTION

The discovery of chimera states^{1,2} spurred a profound change of paradigm in understanding of self-organization in assemblies of coupled oscillators. Instead of the synchronization transition and the onset of the collective mode,³ attention has shifted to states emerging via symmetry breaking of synchrony,⁴ where assemblies of indistinguishable oscillators with symmetric couplings split into groups with different dynamics. Classical examples of symmetry breaking of synchrony include cluster states,⁵⁻⁸ chimeras,⁹⁻¹² and solitary states.¹³⁻²² In contrast to low-dimensional dynamics of cluster states, where the units within each group are identically synchronized, chimeras are self-organized patterns that are comprised of coexisting domains of coherence and incoherence.¹¹ A similar coexistence of locked and unlocked units underlies solitary states, where a single or a small subset of solitary units display an average frequency different from the synchronized cluster. However, distinct to the structure of chimeras, the solitary units spread randomly instead of forming spatially localized domains. Another difference is that solitary states involve spatial chaos,²³ reflecting sensitive dependence of the dynamics on spatial coordinates, which gives rise to extensive multistability. Regardless of these differences, both chimeras and solitary states satisfy the definition of weak chimeras.²⁴

For coupled oscillators, much progress has been made in resolving the two fundamental problems, namely, the mechanisms of onset and potential links between symmetry-broken states along the path from complete coherence to incoherence. In particular, emergence of cluster states from complete synchrony has been explained by unfolding of a so-called cluster singularity, revealing cascade transitions from a synchronous state to a balanced two-cluster partition, characterized by an equal number of oscillators within each cluster, via different unbalanced cluster states.^{5,25} Also, clustering has been identified as a prerequisite for the onset of chimeras.²⁶ Self-organization of strong chimeras, where coherent domains comprise identically (in-phase) synchronized oscillators, was shown to involve stabilization of the coherent cluster by the incoherent one,²⁷ while solitary states were found to mediate transition from complete coherence to chimeras.¹⁶

However, in a myriad of examples, from neural and cardiac tissue to chemical reactions, system components are not intrinsic oscillators, but are rather excitable units,28,29 nonlinear threshold elements that in the absence of input lie at rest, but may be triggered to oscillate by sufficiently strong perturbations. There is no reason to expect a priori that results for coupled oscillators translate to excitable systems, where even the onset of collective oscillations requires repulsive rather than attractive interactions.³⁰⁻³² Apart for theoretical relevance, resolving fundamental questions on emergence and relation between periodic cluster states, chimeras, and solitary states in coupled excitable systems may be important for applications, e.g., for treating in neuroscience the problems of cluster synchronization during information transmission and processing,33,34 localized activity associated with working memory,³⁵⁻³⁸ or inducing desynchronization to control pathological states.³⁹⁻⁴¹

In this paper, we reveal mechanisms of onset and links between different types of unbalanced periodic two-cluster states and solitary states in systems with excitable local dynamics on multiple timescales, typical for but not confined to neuroscience,^{29,42-45} and varying attractive/repulsive⁴⁶ type of interactions. We show that the prevalent solitary states in non-locally coupled arrays, having a self-organization based on successive spiking of units, derive their dynamical features, such as the frequency locking between typical and solitary units and the form of corresponding orbits, from the unbalanced two-cluster states in globally coupled networks, characterized by a permutation symmetry $\mathcal{S}_{N_A}\otimes\mathcal{S}_{N_B}$ with $N_A
eq N_B$ being the cluster sizes. However, we also discover cluster and solitary states where such a correspondence cannot be established. The self-organization of these peculiar cluster and solitary states is based on so-called *leap-frog* dynamics,⁴⁷⁻⁵¹ characterized by an alternating order of spiking (leader-switching) between the units. Leap-frog dynamics was initially observed as a near-synchrony state in models of phase oscillators or type I relaxation neural oscillators supplied by strong nonlinear couplings with finite time constants.^{48,50,51} Nevertheless, it has recently been shown that leap-frogging can also occur in repulsively coupled type II excitable systems, considering an example of binary motifs of FitzHugh-Nagumo units poised close to (above or below) the bifurcation threshold.⁴⁷ There, leap-frog solutions of different complexity emerge from a slow-fast dynamics in vicinity of a canard transition and beyond a small coupling limit represent a particular type of mixed-mode oscillations at a folded node singularity.^{52,53} Given a strong sensitivity of excitable systems to noise,²⁸ we also investigate the resilience of the observed unbalanced cluster states and solitary states to noise. While noise has already been known to facilitate spontaneous clustering⁵⁴ and emergence of chimeras^{55,56} in coupled FitzHugh-Nagumo systems, here, we demonstrate the effect of noise-induced preference of attractors,57 where the noise suppresses the system's multistability by promoting only certain types of cluster states or by favoring patched patterns at the expense of solitary states.

Our system is an array of N identical FitzHugh–Nagumo units²⁹ whose dynamics obeys

$$\varepsilon \dot{u}_{k} = u_{k} - \frac{u_{k}^{3}}{3} - v_{k} + \frac{\kappa}{2R} \sum_{l=k-R}^{k+R} [g_{uu}(u_{l} - u_{k}) + g_{uv}(v_{l} - v_{k})] + \sqrt{\varepsilon}\sigma \xi_{k}(t),$$
(1)
$$\dot{v}_{k} = u_{k} + a + \frac{\kappa}{2R} \sum_{l=k-R}^{k+R} [g_{vu}(u_{l} - u_{k}) + g_{vv}(v_{l} - v_{k})],$$

where local slow-fast dynamics is governed by activator variables u_k and recovery variables v_k with timescale separation $\varepsilon = 0.05$. All indices are periodic modulo *N*. Local bifurcation parameter *a*, fixed to a = 1.001, mediates the transition from excitable (|a| > 1) to oscillatory regime (|a| < 1). Due to a singular character of Hopf bifurcation at a = 1, onset of oscillations is followed by a *canard transition* $(a \approx 1 - \varepsilon/8)$ from small-amplitude (subthreshold) to large-amplitude relaxation oscillations.⁶⁰ Nonlocal interactions have coupling strength $\kappa = 0.4$, with each unit coupled to *R* neighbors on both sides, yielding a coupling radius r = R/N. Impact of direct and cross-coupling terms in u_k and v_k is compactly described via a rotational coupling matrix⁶¹ $G = \begin{pmatrix} g_{uu} & g_{uv} \\ g_{vu} & g_{vv} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}$. Parameter ϕ mod-

ifies prevalence of attractive and repulsive interactions. Spiking can also emerge due to noise, which here affects the fast variables similar to synaptic noise in neuronal systems,⁶² having each unit influenced by independent Gaussian white noise $\xi_k(t)$ of intensity σ : $\langle \xi_i(t) \rangle = 0$, $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{ij} \delta(t - t')$. Note that the systems of ordinary (stochastic) differential equations were integrated by the standard adaptive ODE45 solver (SDETools toolbox available at https://github.com/horchler/SDETools).

We first focus on how the stability of unbalanced two-cluster states in globally connected networks (r = 1/2) changes with ϕ and then analyze the onset of solitary states in non-locally coupled arrays (r < 1/2) of excitable elements.

II. TWO-CLUSTER STATES IN GLOBALLY COUPLED NETWORKS

To gain insight into the structure of unbalanced periodic twocluster states, their stability domains, and underlying bifurcations, we implement a twofold approach, combining the semi-analytical method of evaporation exponents and the numerical path-following method based on introducing probe oscillators. Since our interest is in solutions where both clusters emit spikes, the splitting scenario by which clusters emerge from a collective rest state is beyond our current scope. We remark that for stable local dynamics (|a| > 1), interaction-induced destabilization of a stationary state at $\phi^* = \arccos\left(\frac{1-a^2}{2\kappa}\right)$ is a highly degenerate point where 2(N-1)Jacobian eigenvalues with real parts $1 - a^2 - 2\kappa \cos(\phi)$ simultaneously become critical, giving rise to a large number of different cluster partitions featuring subthreshold oscillations, which in an exponentially small ϕ region start to display spikes via secondary canard transitions. Stability of a stationary state is regained at $\phi = \phi^* + \pi$. Onsets of cluster instability and periodic cluster states for type I excitable units were addressed in Refs. 30 and 63.

Unlike Lyapunov exponents, evaporation exponents^{64–66} can describe perturbations that destroy cluster partitions. They characterize stability of clusters to emanation of elements, induced by perturbations transversal to invariant subspace of certain partition. Negative evaporation exponents indicate assembly's attractors, while their positive values imply instability. We consider a two-cluster state with partition parameter $p \in (0, 1)$ such that $N_A = pN$ units are in cluster A and $N_B = (1 - p)N$ units in B. Its dynamics is independent of N and is governed by the reduced system

$$\varepsilon \dot{u}_{i} = u_{i} - \frac{1}{3}u_{i}^{3} - v_{i} + \kappa w_{i}(g_{uu}(u_{j} - u_{i}) + g_{uv}(v_{j} - v_{i})),$$

$$\dot{v}_{i} = u_{i} + a + \kappa w_{i}(g_{vu}(u_{j} - u_{i}) + g_{vv}(v_{j} - v_{i})),$$
(2)

with $i, j \in \{A, B\}, i \neq j$, and $(w_A, w_B) = (1 - p, p)$ being additional coupling weights derived from particular partition. For $p \neq 1/2$, system (2) is equivalent to a pair of nonidentical excitable units. Different *p* values specify invariant subspaces in complete phase space that intersect only in the full synchrony plane. To introduce evaporation exponents, we consider symmetric small perturbations to two units, 1 and 2, in each cluster: $u_{i,1/2} = u_i \pm \delta u_i, v_{i,1/2} = v_i \pm \delta v_i$. Due to permutation symmetry, they can be applied to an arbitrary

pair of elements, leaving the cluster mean-fields unchanged. Linearized equations for deviations $[\delta u_i(t), \delta v_i(t)]$ transversal to cluster dynamics read

$$\varepsilon \delta u_i = (1 - u_i^2 - \kappa g_{uu}) \delta u_i - (1 + \kappa g_{uv}) \delta v_i,$$

$$\dot{\delta v_i} = (1 - \kappa g_{vu}) \delta u_i - \kappa g_{vv} \delta v_i.$$
(3)

Evaporation exponents $\lambda_{ev,i} = \lim_{T \to \infty} \frac{1}{2} \ln \frac{\delta u_i^2(T) + \delta v_i^2(T)}{\delta u_i^2(0) + \delta v_i^2(0)}$ are obtained by integrating the system (2) and (3).

Bifurcations of particular cluster states are determined by numerical continuation using probe oscillators, indicating whether a unit added to the cluster asymptotically remains in it or leaves it. Probes are introduced at the cluster coordinates without affecting the mean-fields such that their dynamics $[\tilde{u}_i(t), \tilde{v}_i(t)]$ obeys

$$\varepsilon \tilde{\dot{u}}_{i} = \tilde{u}_{i} - \tilde{u}_{i}^{3} - \tilde{v}_{i} + \kappa [w_{i}(g_{uu}(u_{i} - \tilde{u}_{i}) + g_{uv}(v_{i} - \tilde{v}_{i})) + w_{j}(g_{uu}(u_{j} - \tilde{u}_{i}) + g_{uv}(v_{j} - \tilde{v}_{i}))],$$

$$\dot{\tilde{v}}_{i} = \tilde{u}_{i} + a + \kappa [w_{i}(g_{vu}(u_{i} - \tilde{u}_{i}) + g_{vv}(v_{i} - \tilde{v}_{i})) + w_{j}(g_{vu}(u_{j} - \tilde{u}_{i}) + g_{vu}(v_{j} - \tilde{v}_{i}))].$$
(4)

Numerical continuation of solutions of (2) together with (4) was performed by the software package AUTO.⁶⁷

Figure 1(a) shows the stability diagram for system (2) and (3) in the (ϕ, p) plane, combining the results obtained by methods of evaporation exponents and probe oscillators. Regions supporting stable solutions are indicated in orange, with black and green lines at their boundaries denoting period-doubling bifurcations and curves of branching points, respectively. The latter are typically pitchfork bifurcations of the reduced system but correspond to unfolding of highly degenerate bifurcation points⁵ of system (1), where p becomes a solution parameter. System (2) supports six characteristic regimes with 1:1 (regions IV, V, and VI), 1:2 (I, II), or 2:3 (III) frequency locking, all conforming to mixed-mode oscillations^{52,53} with interspersed large- and small-amplitude oscillations; cf. Figs. 1(b)-1(e). Note that the partition parameter *p* for certain types of solutions can become small but still does not approach zero, indicating that only those periodic two-cluster states with a sufficiently balanced partition can exist. This is similar to the scenario recently described for coupled type I excitable systems.³⁰ Nevertheless, these results for coupled excitable systems are different from those for globally coupled networks of Kuramoto oscillators with inertia,¹⁹ where a stable existence of solitary states with a single or just few oscillators isolated from the synchronized cluster has been reported. Also note that some authors tend to refer to states characterized by a finite fraction of units (up to p = 1/2) split from the synchronized bulk cluster as solitary states.^{19,68} Nevertheless, here, we prefer to call the states with an uneven partition to two groups of *identically* synchronized units unbalanced cluster states, as opposed to the solitary states described later on for non-locally arrays, where the units are split into majority and minority groups of *frequency locked* but not identically synchronized units.

Apart from solutions I–III and V with a successive spiking between clusters, where the spiking order of clusters is preserved, one also observes mixed-mode solutions IV characterized by leap-frog dynamics^{47–50} of clusters. There, switching of leadership



FIG. 1. Unbalanced periodic two-cluster states. (a) Stability diagram in the (ϕ, p) plane. Stable and unstable solutions are indicated in orange and gray, respectively. Black solid lines: period-doubling bifurcations. Green lines: curves of branching points. Black dashed lines: destabilization of rest state ($\phi = \phi^* \approx 1.573$) and its reappearance ($\phi = \bar{\phi} \approx 4.715$). (b)–(e) Time traces $u_i(t), i \in \{A, B\}$ and phase portraits corresponding to (ϕ, p) values (blue squares) from (a).

between the clusters occurs via subthreshold oscillations such that the current leader performs an extra small oscillation allowing it to be overtaken by the lagging cluster; see the arrows in Fig. 1(d). Leapfrog solutions at p = 1/2 may acquire additional antiphase symmetry $u_A(t) = u_B(t + P/2)$, $v_A(t) = v_B(t + P/2)$, where *P* denotes the oscillation period. Note that different types of leap-frog patterns and their underlying mechanisms in binary motifs of repulsively coupled FitzHugh–Nagumo units were shown to be a consequence of phase-sensitive excitability of periodic orbits,⁴⁷ a recently introduced concept⁶⁹ referring to a non-uniform sensitivity to perturbations of both relaxation and subthreshold oscillations in the FitzHugh–Nagumo system.

Evaporation exponents can also be used to approximate impact of small noise to stability of two-cluster partitions. For (1) with $r = 1/2, \sigma > 0$, we find that the noise may cause transition to another type of two-cluster state or may reorganize the state's structure by inducing migration of units between clusters without qualitatively affecting their mean-fields. Reorganization process eventually settles to a partition where the net transport between the



FIG. 2. Persistence of unbalanced two-cluster states under noise. (a) Quantity $d(\phi, p)$ distinguishes between four cases: cluster states reorganize to the partition with smaller (blue, $\lambda_{\sigma r,A} > \lambda_{\sigma r,B}$) or larger p values (red, $\lambda_{\sigma r,B} > \lambda_{\sigma r,A}$); two-cluster states are unstable (gray, d = 4); only synchronous stationary state is stable (white, d = 1). (b)–(e) Examples of evolution of partition parameter p(t) under noise. Left to right: $\phi = 2, 4.4, 2.3, 5.0$ and $\sigma = 5, 0.6, 0.6, 5 \times 10^{-3}$, respectively.

clusters reaches a dynamical balance so that the partition parameter p(t) becomes stationary. Splitting of a unit from a cluster and migration to another cluster may involve nonlinear effects of perturbations that cannot be captured by methods involving linearization around a certain solution, such as evaporation exponents. Still, at the linear level, "potential barrier" that has to be overcome when a unit leaves the cluster is proportional to $\lambda_{ev,i}$. This is used to characterize resilience of two-cluster states to noise in Fig. 2(a). We distinguish between the cases where noise is more likely to shift a two-cluster partition toward smaller ($0 > \lambda_{ev,A} > \lambda_{ev,B}$; blue regions) or a larger *p* value ($0 > \lambda_{ev,B} > \lambda_{ev,A}$; red regions), depending on the dominant stable exponent. There are also domains where unbalanced twocluster states are unstable ($\lambda_{ev,A/B} > 0$, shown gray) or where only synchronous stationary state is stable (white). For convenience, each case is assigned with a discrete variable $d \in \{1, 2, 3, 4\}$. Evolution of cluster partition p(t) under noise is illustrated in Figs. 2(b)-2(e) for solutions from regions I to VI. While states from I and V display persistence under noise, representative state from II migrates to region III. Interestingly, asymmetric leap-frog solution from IV evolves toward balanced partition p = 1/2.

III. SOLITARY STATES IN NON-LOCALLY COUPLED ARRAYS

The intrinsic dynamics of the prevalent solitary states in nonlocally coupled arrays, called SS1 and SS2, is based on successive spiking of units. In the following, we show that they derive their dynamical features from the corresponding unbalanced cluster states from Fig. 1(a). In particular, state SS1 in Fig. 3 is a dynamical counterpart of a two-cluster state from region I, whereas SS2



FIG. 3. Solitary state SS1 ($N = 100, \phi = 1.85, r = 0.2$). (a) Spatial profile of ω_k ; (b) red and blue: two snapshots of local variables (u_k, v_k), black: nullclines of isolated unit; (c) spatiotemporal evolution of $u_k(t)$; (d) phase portraits [$u_k(t), v_k(t)$] for solitary (k = 84) and typical unit (k = 60); (e) cross-correlation matrix C_{kl} ; and (f) time traces $u_k(t)$ for two units from (d).

(not shown) derives from the cluster state from region V. These solitary states occur within the same ϕ intervals as the corresponding cluster states and preserve the respective frequency locking of clusters, but due to nonlocal interactions and associated fluctuations of the local mean-fields, clusters of solitary and typical units are fuzzy 13,16 rather than exact; see Fig. 3(b). In other words, introducing a nonlocal coupling r < 1/2 results in breaking of the permutation symmetry of the unbalanced cluster states observed for r = 1/2 such that the solitary and typical clusters consist of frequency locked but not identically synchronized units. The spatial profile of the average local frequencies $\omega_k = 2\pi M_k / \Delta$, where M_k is the spike count within interval Δ , shows a 2:1 frequency ratio between solitary and typical units. The analogy with the two-cluster state from region I in Fig. 1(a) in terms of local phase portraits and time traces $u_k(t)$ is illustrated in Figs. 3(d) and 3(f). Intrinsic dynamics of SS1 is characterized by a cross-correlation matrix $C_{kl} = \frac{(\hat{u}_k(t)\hat{u}_l(t))_T}{\sqrt{\langle \hat{u}_k(t)^2 \rangle_T \langle \hat{u}_l(t)^2 \rangle_T \langle$ where $\langle \cdot \rangle_T$ denotes time averaging, while $\hat{u}_k(t) = u_k(t) - \langle u_k(t) \rangle_T$ are deviations of $u_k(t)$ from their means; cf. Fig. 3(e).

Nevertheless, we also find solitary states without two-cluster state counterparts. A typical example is a state SS3 illustrated in Fig. 4, which, unlike SS1 and SS2, is maintained by leap-frog dynamics of pairs of solitary-typical, only solitary or only typical units; cf. Figs. 4(b), 4(d), and 4(f). States of SS3 type emerge due to non-local interactions, which induce self-localized excitations⁷¹ at interfaces separating domains with distinct dynamics. Frequency profile



FIG. 4. Solitary state SS3 (N = 100, $\phi = 1.788$, r = 0.2). (a) Spatial profile of ω_k . (b) Time traces $u_k(t)$ for solitary units k = 75 and k = 76. (c) Spatiotemporal evolution of $u_k(t)$. (d) Time traces $u_k(t)$ for solitary unit k = 76 and typical unit k = 20. (e) Cross-correlation matrix C_{kl} . (f) Time traces $u_k(t)$ show leap-frog dynamics within majority cluster (units k = 20 and k = 40).

 ω_k shows two clusters with a frequency ratio distinct from SS1; cf. Fig. 3(a). The difference in ω_k derives from events where solitary units emit two successive spikes rather than a spike followed by subthreshold oscillation; see the arrow in Fig. 4(d). SS3 involves a more complex correlation structure compared to SS1, cf. Figs. 4(e) and 3(e), and the corresponding maximal Lyapunov exponent⁷⁰ is 1.78×10^{-5} .

Contrasting with locally coupled excitable systems where the noise may strongly influence pattern formation by inducing, enhancing or controlling wave propagation, spiral dynamics, and pacemaking,²⁸ the deterministic dynamics of non-locally coupled arrays here features extensive multistability, and the main impact of noise is qualitatively different. We find that the noise reduces system's multistability, suppressing solitary states. This reflects the effect called noise-induced preference of attractors,⁵⁷ which may be understood as follows: in highly multistable systems, stability boundaries of attractors become smeared by noise, and only those with sufficiently large basins of attraction remain visible. This may be seen as a highly biased switching^{72,73} to a coexisting state without returning to the initial one. For small noise, unbalanced splitting into frequency clusters is preserved, but the preferred spatial distribution of minority units is *localized* rather than random. This gives rise to patched patterns with 1:2 subharmonic frequency locking. A typical example is shown in Fig. 5, where an initial SS1 state transforms under small noise into a state of patched synchrony.⁶¹ Note

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FIG. 5. Transformation of an SS1 state under noise. (a) Typical SS1 dynamics without noise. (b) Patched pattern developed from SS1 at $\sigma = 0.0011$. Parameters are N = 200, $\phi = 2.0$, r = 0.2.

that introducing intermediate noise favors rotating waves instead of patched synchrony, while an even larger noise leads to turbulence.

IV. CONCLUSION AND OUTLOOK

We have discovered the mechanisms of onset and links between unbalanced periodic two-cluster states and solitary states, as a form of weak chimeras, in coupled excitable systems. The fact that the prevalent solitary states SS1 and SS2 in non-locally coupled arrays, characterized by self-organization based on successive spiking of units, derive their dynamical features from unbalanced cluster states in globally coupled networks is to a certain extent qualitatively similar to the finding for globally coupled Stuart-Landau oscillators, where clustering has been identified as a symmetry-breaking step required for emergence of chimeras.²⁶ Distinct from the physical picture reported for the systems of Kuramoto oscillators with inertia,¹⁹ we have not observed states with a single or just a few units split from the synchronized cluster that can be continued for an arbitrary range of couplings from global via nonlocal to local. A peculiar finding associated with the interplay of local excitability and nonlocal interactions concerns the solitary states SS3 that have no cluster states counterparts and whose structure involves leap-frog activity patterns. Leap-frog dynamics derives from multiscale character of the system, and, in particular, the phase-sensitive excitability of relaxation oscillations, underlying their strong sensitivity to perturbations in the vicinity of a canard transition.^{47,69} Current results, together with Refs. 47 and 74, indicate the importance of this concept to pattern formation in multiscale systems, both in regard to coupled type II excitable units and oscillators. A question that remains open is whether a similar type of unbalanced cluster states and solitary states based on leap-frog dynamics can be observed in coupled type I excitable systems. Since leap-frogging in type I neural oscillators has so far only been found in the presence of strong nonlinear couplings,^{50,51} we suspect that they are also required for the onset of leap-frog states in type I excitable systems.

Regarding the impact of noise, we have found that it affects the cluster and solitary states by suppressing the multistability of system dynamics. This is a manifestation of noise-induced preference of attractors, an effect previously corroborated in coupled oscillators,⁵⁸ Hénon maps,⁷⁵ and multistable fiber lasers.⁵⁹ An additional subtlety is that the small noise influences pattern formation by promoting homogeneous patched patterns at the expense of solitary states. Since solitary states in coupled oscillators may mediate the transition from complete synchrony to chimeras,¹⁶ it would be interesting to investigate whether a similar scenario applies to coupled excitable systems.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts of interest to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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