Interplay between finite size scaling and aspect ratio in continuum percolating networks

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(1) Motivation

Percolation is a random process, and we can distinguish two types of the percolation problems: lattice percolation and continuum (or irregular lattice) percolation. It is widely accepted that lattice and continuum percolation belong to the same class in the sense that the latter possesses the same critical exponents as the former.

Stick percolation has not been extensively studied theoretically until now [1, 2]. Still it is an important representative of continuum percolation and interesting due to its relevance for systems consisting of conducting rodlike particles.

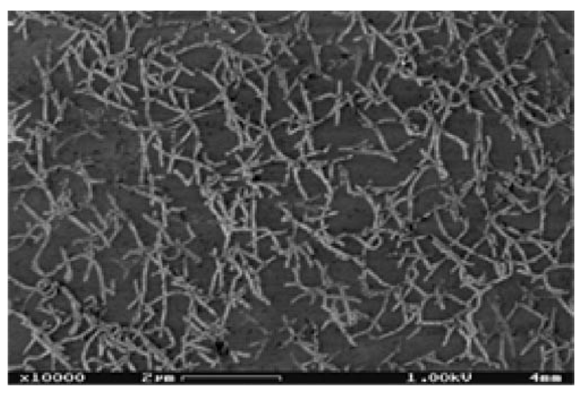
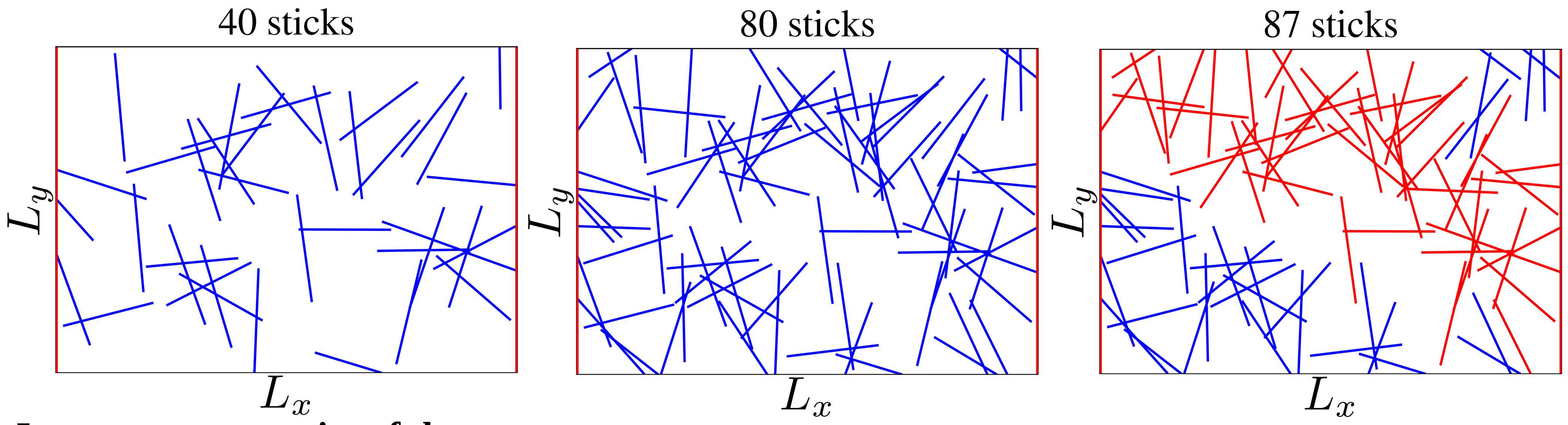


Illustration of carbon nanotube network taken from M.Y. Zavodchikova et. al., Nanotechnology 20 (2009) 085201.

(2) Monte Carlo simulations of rodlike particle percolation

In Monte Carlo simulations the widthless sticks with unity length are randomly placed between electrodes (left / right) with free boundary conditions (top / bottom). Two sticks lie in the same cluster if they intersect. The system percolates if two opposite boundaries (left and right) are connected with the same cluster.



Important properties of the system:

1) Aspect ratio $r = L_x / L_y$.
2) Normalized system size $L = \sqrt{(L_x L_y)}$.
3) Stick density n, i.e., the number of sticks per unit area $n = N / L^2$.

(3) Results for percolation probability distribution

(A) Conversion of number of sticks N into stick density n

Convolving the discrete percolating probability function $R_{N,L,r}$ for N sticks in a system with the Poisson distribution we obtain a percolating probability function (PF) for any stick density n :

$$R_{n,L,r} = \sum_{N=0}^{\infty} \frac{(n L^2/r)^N e^{-nL^2/r}}{N!} R_{N,L,r}.$$

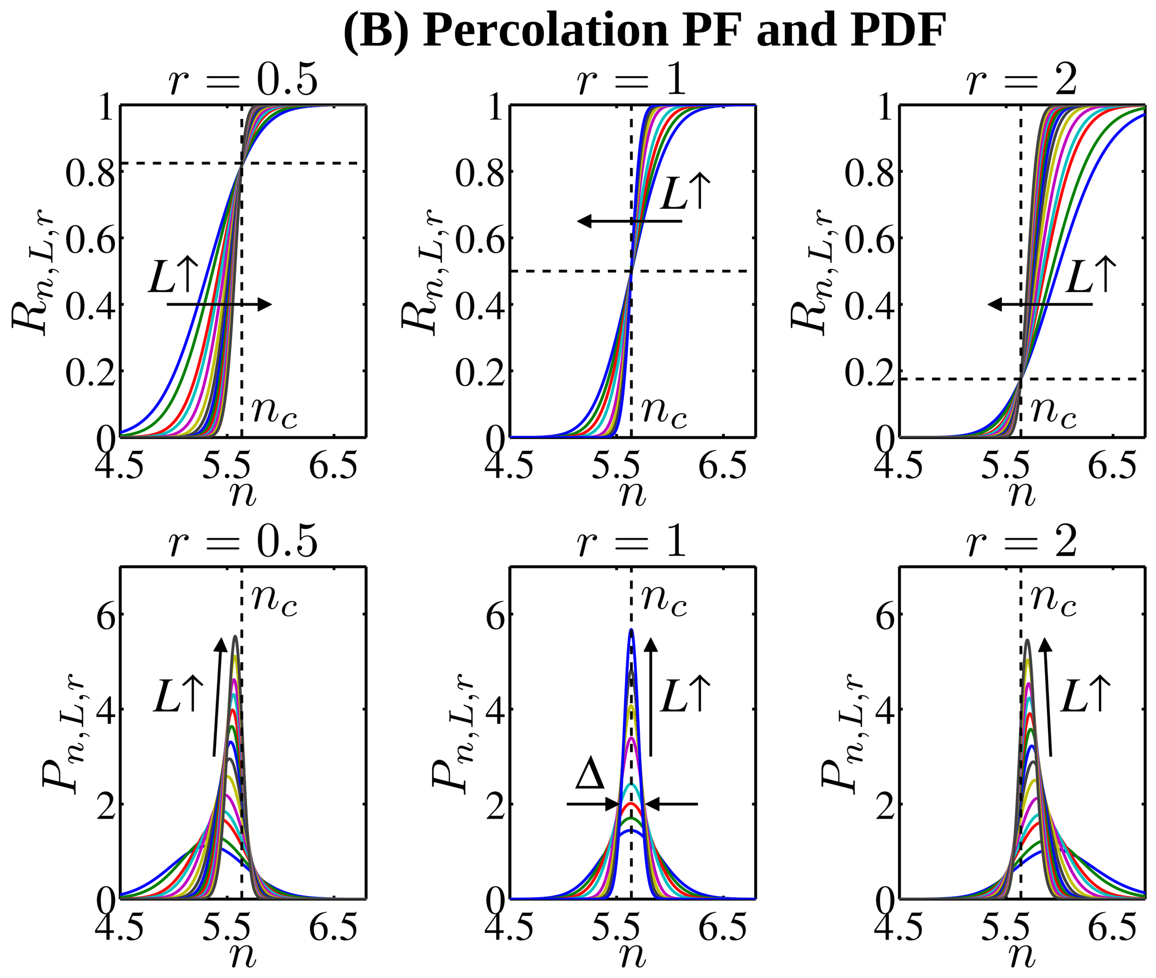
Percolation probability density function (PDF):

$$P_{n,L,r} = \frac{dR_{n,L,r}}{dn}.$$

Recently has been shown that threshold density for stick percolation should be [1]:

$$n_c = 5.63726 \pm 0.00002.$$

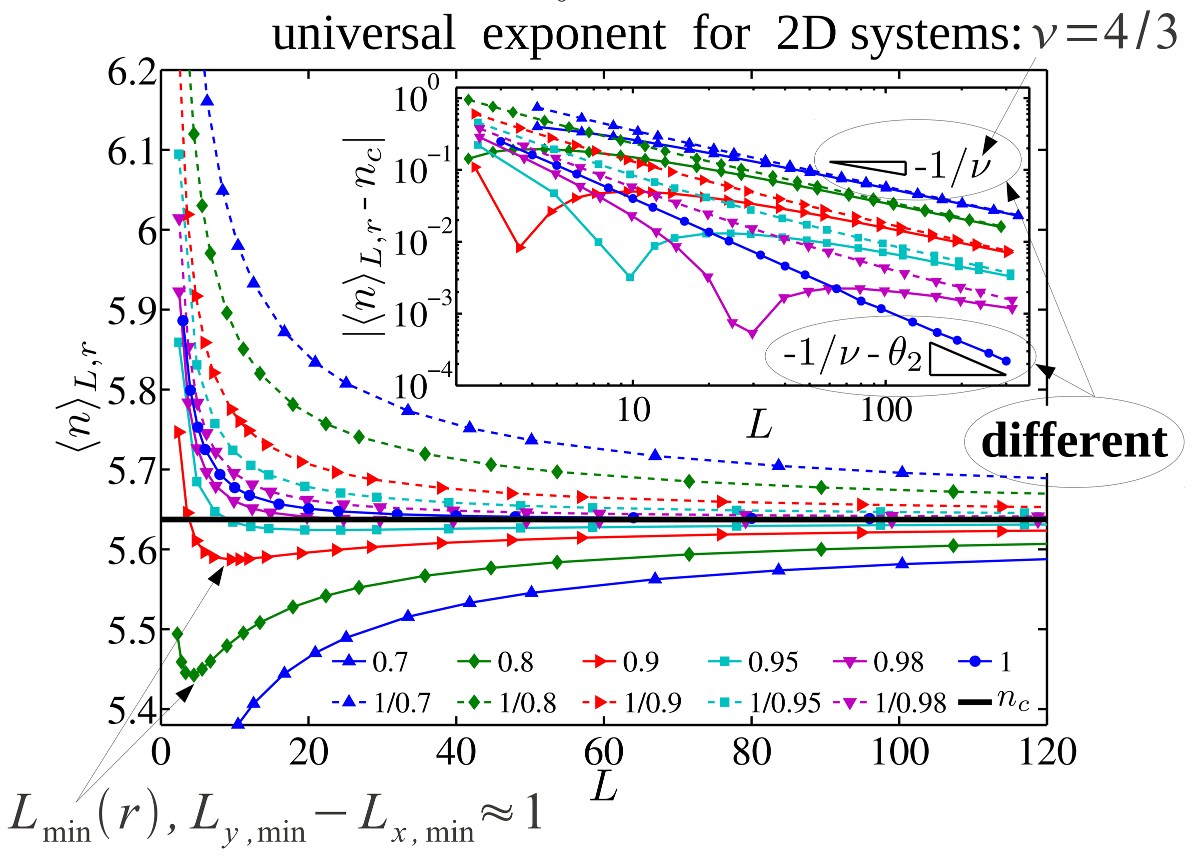
(B) Percolation PF and PDF



(4) Average stick percolation density

$$\langle n \rangle_{L,r} = \int_0^{\infty} n P_{n,L,r} \, \mathrm{d}n$$

universal exponent for 2D systems: $\nu = 4/3$



$L_{\min}(r),\ L_{y,\min} - L_{x,\min} \approx 1$

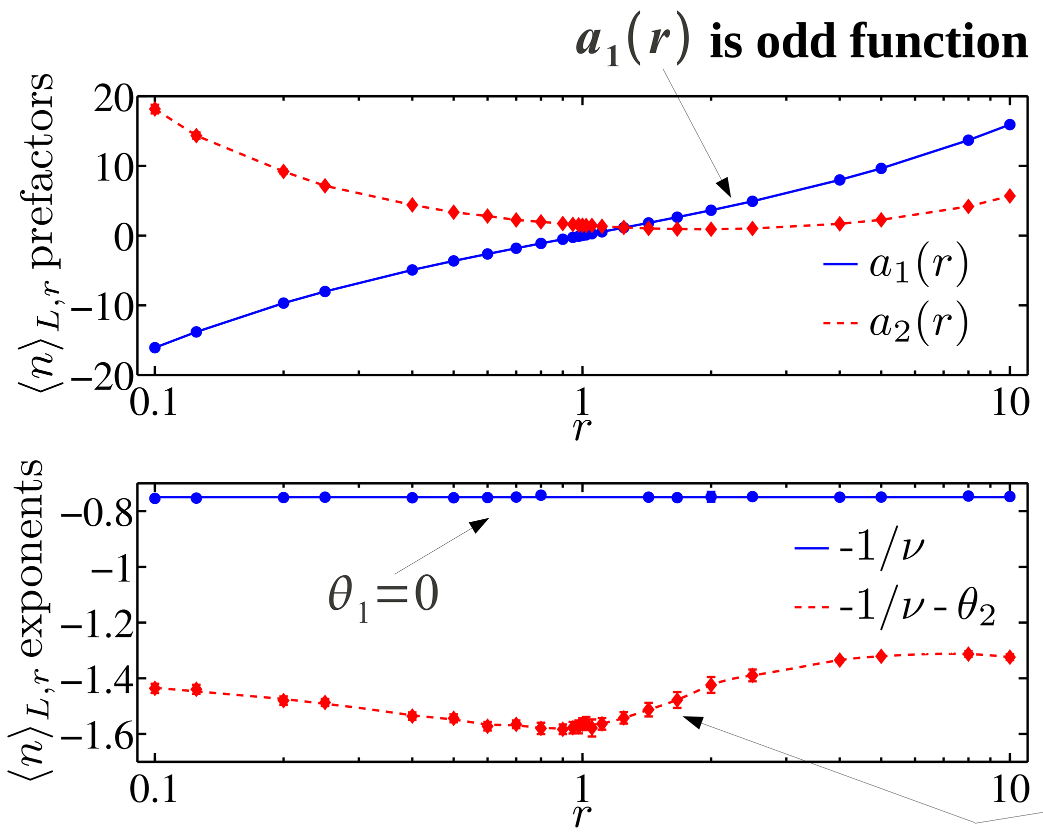
(5) Generalized moment scaling functions

$$\langle n \rangle_{L,r} = n_c + L^{-1/\nu} \sum_{i=1}^{\infty} a_i(r) L^{-\theta_i}.$$

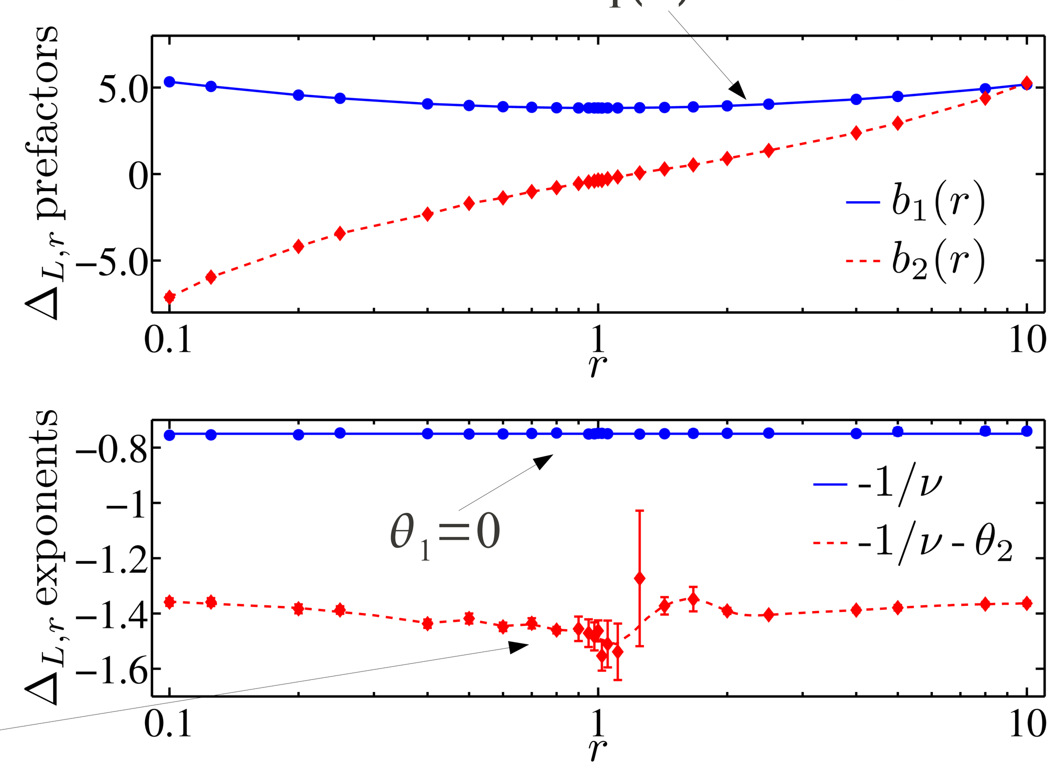
$$\Delta_{L,r} = L^{-1/\nu} \sum_{i=1}^{\infty} b_i(r) L^{-\theta_i}.$$

$$R_{n_c,L,r} = R_{n_c,L\to\infty,r} + \sum_{i=1}^{\infty} c_i(r) L^{-i}.$$

$a_1(r)$ is odd function

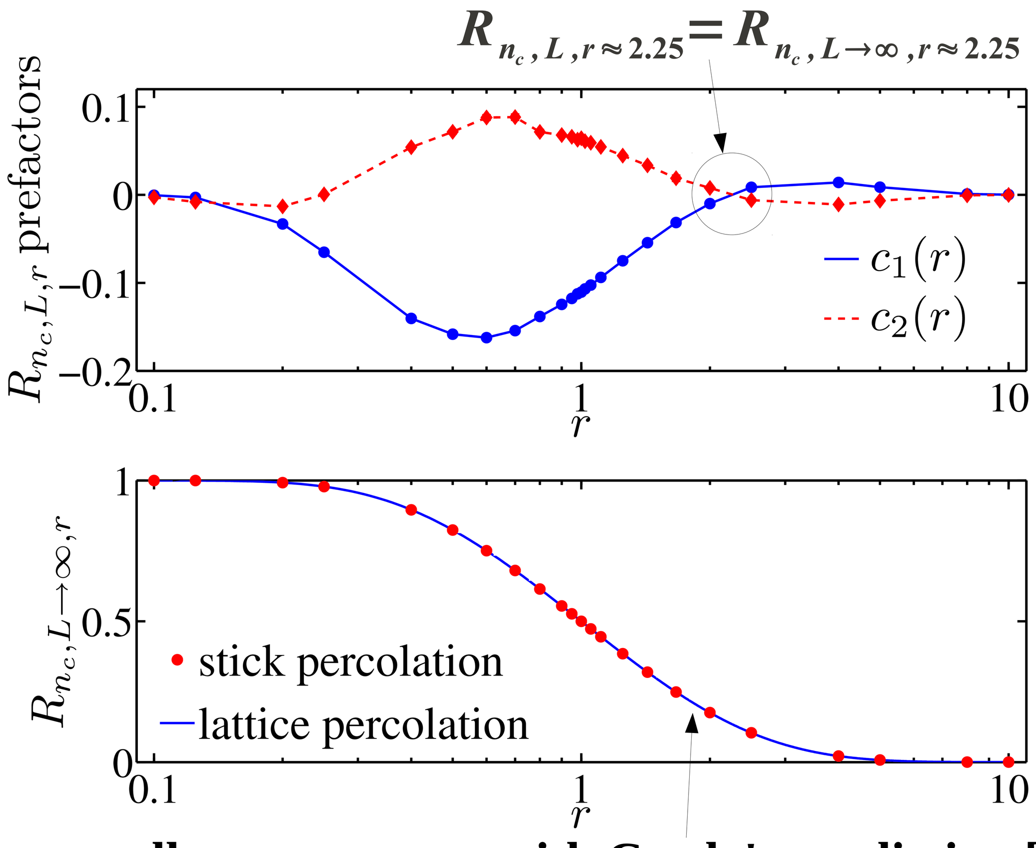


$b_1(r)$ is even function



the same shape

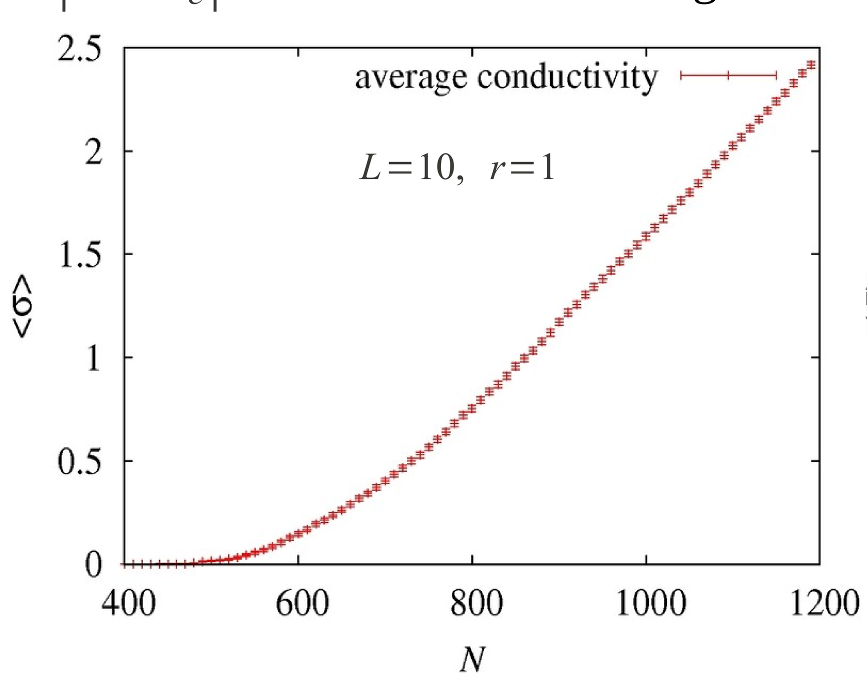
$R_{n_c,L,r\approx 2.25} = R_{n_c,L\to\infty,r\approx 2.25}$

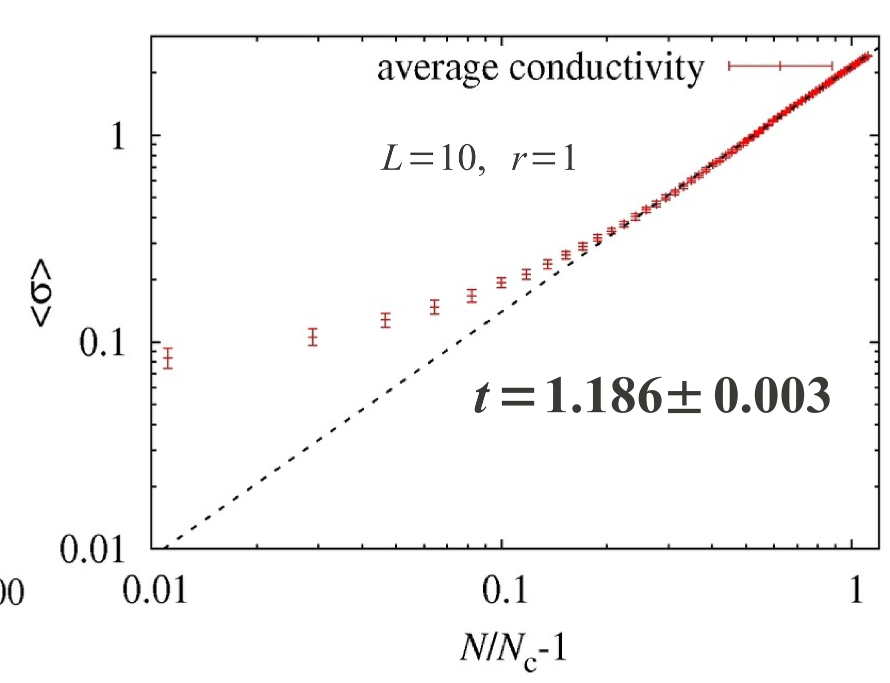


excellent agreement with Cardy's prediction [3]

(6) Conductivity exponent in stick percolation

Average conductivity $\langle \sigma \rangle = (n - n_c)^t$, $L \gg \xi$.
$\xi \propto |n - n_c|^{-\nu}$ is the correlation length.





References:

[1] J. Li and S.-L. Zhang, Phys. Rev. E **79**, 155434 (2009).
[2] J. Li and S.-L. Zhang, Phys. Rev. E **79**, 021120 (2010).
[3] J. L. Cardy, J. Phys. A **25**, L201 (1992).
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SEVENTH FRAMEWORK PROGRAMME