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Nonsingular black hole

Aleksandar Bogojević
Institute of Physics, P.O. Box 57, Belgrade 11001, Yugoslavia

Dejan Stojković*
Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, Ohio 44106

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We present a completely integrable deformation of the CGHS dilaton gravity model in two dimensions. The solution is a singularity free black hole that at large distances asymptotically joins to the CGHS solution.

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I. INTRODUCTION

One of the fundamental unsolved problems in theoretical physics is the unification of quantum theory and gravity. Many reasons why this has proved so difficult stem from the complicated nonlinear structure of the equations of general relativity. Gravitational equations are much simpler in lower dimensions. This is the reason why there has been so much activity related to the quantization of gravity in two and three dimensions [1–12]. One of the most important results in 2D was the exactly solvable dilaton gravity model constructed by Callan, Giddings, Harvey and Strominger (CGHS). The CGHS model has 2D black hole solutions that are remarkably similar to the Schwarzschild solution of general relativity.

Of the four fundamental interactions in nature, gravity is by far the weakest. For this reason, we can hope to see quantum effects only in the vicinity of classical singularities. Penrose and Hawking [13–15] have shown that these singularities are endemic in general relativity. The general belief is that quantization will rid gravitation of singularities, just as in atomic physics it got rid of the singularity of the Coulomb potential. If this is indeed the case, then there must exist a non-singular gravitational effective action whose classical equations encode the full quantum theory. This effective action must have the Planck length L_{Planck} in it as an input parameter. For $L \gg L_{\text{Planck}}$ the effective model must be indistinguishable from the classical gravity action. In an interesting series of papers [16–19] Brandenberger, Mukhanov and their collaborators initiated a search for such effective models of 2D dilaton gravity. They investigated a procedure by which one could make models free of singularities. This parallels Landau's treatment of phase transitions in ferromagnets. Landau chose (the simplest) effective action (Gibbs potential in statistical mechanics parlance) that led to a qualitatively correct description of phase transitions.

We should mention also some problems with ''limited curvature hypothesis'' which are especially bad in higher dimensions. For example in four dimensions non-singular curvature does not imply that all curvature invariants are non-singular and it has proven very difficult to keep all of them finite. Further, we can have singular plane waves in (3+1)-dimensional general relativity that have curvature invariants zero (null singularities). In general, finite curvature is a necessary but not sufficient condition for the space-time to be either non-singular or geodesically complete.

A recent success in the field of 2D dilaton gravity has been the work of Louis-Martinez and Kunstatter [20], who reduced the solution of the general dilaton gravity model to the solution of two ordinary integrals, i.e. to two quadratures. In this paper we will use their result to construct an exactly solvable class of models—modified CGHS models. For $L \gg L_{\text{Planck}}$ these models go over into the CGHS model. Like CGHS, the modified models are exactly solvable (the two quadrature integrals can be calculated in closed form) and have black hole solutions (solutions with event horizons). Unlike CGHS, the modified models are non-singular and geodesically complete. As we shall see, the maximal curvature is proportional to $1/L_{\text{Planck}}$.

II. CGHS MODEL

The action of all dilaton gravity models can be put into the general form

$$S=\int d^2x\sqrt{-g}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi-V(\phi)+D(\phi)R\right]. \quad (1)$$

The potentials $V(\phi)$ and $D(\phi)$ classify all the possible models. Let us perform a conformal scaling of the metric

$$\tilde{g}_{\alpha\beta}=e^{-2F(\phi)}g_{\alpha\beta}, \quad (2)$$

where the scaling factor $F(\phi)$ satisfies

$$\frac{dF}{d\phi}=-\frac{1}{4}\left(\frac{dD}{d\phi}\right)^{-1}. \quad (3)$$

This puts the action into the simplified form

$$S=\int d^2x\sqrt{-\tilde{g}}[\tilde{\phi}\tilde{R}-\tilde{V}(\tilde{\phi})], \quad (4)$$

where $\tilde{R}$ is the scalar curvature corresponding to $\tilde{g}_{\alpha\beta}$, and we have introduced the new dilaton field and potential according to

$$\tilde{\phi}=D(\phi) \quad (5)$$

*Email address: dbs3@po.cwru.edu

$$\tilde{V}(\tilde{\phi}) = e^{2F(\phi)} V(\phi). \tag{6}$$

This form of the dilaton gravity action is obviously much easier to work with since we have lost the kinetic term for the dilaton field.

A well known property of two dimensional manifolds allows us to locally, i.e. patch by patch, choose conformally flat coordinates for which

$$\tilde{g}_{\alpha\beta} = e^{2\rho} \eta_{\alpha\beta}. \tag{7}$$

Louis-Martinez and Kunstatter [20] have shown that we can choose a coordinate system in which the solution of the general dilaton model is static and given by

$$x = -2 \int \frac{d\tilde{\phi}}{W(\tilde{\phi}) + C} \tag{8}$$

$$e^{2\rho} = -\frac{C + W(\tilde{\phi})}{4}, \tag{9}$$

where the pre-potential $W(\tilde{\phi})$ is given by $dW/d\tilde{\phi} = \tilde{V}(\tilde{\phi})$, and C is an invariant. As we can see, the solution is given in terms of two quadratures: Eqs. (3) and (8), determining $F(\phi)$ and $\tilde{\phi}(x)$ respectively. A given model is completely integrable only if we can calculate both quadratures in closed form.

The CGHS model is an example of a completely integrable dilaton gravity model. The standard form of the CGHS action is

$$S = \int d^2x \sqrt{-g}\, e^{-2\varphi} (R + 4 g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 4\lambda^2). \tag{10}$$

The simple field redefinition $\phi = \sqrt{8} e^{-\varphi}$ puts this into the general form for dilaton gravity actions given in Eq. (1). We find

$$S = \int d^2x \sqrt{-g} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \frac{1}{2} \lambda^2 \phi^2 + \frac{1}{8} R \phi^2 \right), \tag{11}$$

hence

$$V(\phi) = -\frac{1}{2} \lambda^2 \phi^2 \tag{12}$$

$$D(\phi) = \frac{1}{8} \phi^2. \tag{13}$$

The first quadrature is easily integrated and we get

$$F(\phi) = -\ln\phi. \tag{14}$$

Using this, $\tilde{\phi} = D(\phi)$, as well as the definition of $\tilde{V}(\tilde{\phi})$ we immediately find that $\tilde{V}(\tilde{\phi}) = -\frac{1}{2}\lambda^2$, and thus the pre-potential is $\tilde{W}(\tilde{\phi}) = -\frac{1}{2}\lambda^2 \tilde{\phi}$. The simplified form of the CGHS action is therefore

$$S = \int d^2x \sqrt{-\tilde{g}} \left(\tilde{\phi} \tilde{R} + \frac{1}{2} \lambda^2 \right). \tag{15}$$

The CGHS model is completely integrable. In our notation this means that the second quadrature (8) can also be solved in closed form. A trivial integration gives

$$x = \frac{4}{\lambda^2} \ln\left(\frac{1}{2} \lambda^2 \tilde{\phi} - C \right). \tag{16}$$

Inverting this we find

$$\tilde{\phi}(x) = \frac{2}{\lambda^2} (e^{(\lambda^2/4)x} + C). \tag{17}$$

According to the general prescription this gives

$$\rho(x) = -\ln 2 + \frac{\lambda^2}{8} x, \tag{18}$$

$$\phi(x) = \frac{4}{\lambda} (e^{(\lambda^2/4)x} + C)^{1/2}. \tag{19}$$

This, along with the expression for $F(\phi)$, gives us

$$F(x) = -\ln\frac{4}{\lambda} - \frac{1}{2} \ln(e^{(\lambda^2/4)x} + C). \tag{20}$$

The scalar curvature of the general model can be given in terms of $\rho(x)$ and $F(x)$. We find

$$R = -2 e^{-2(F+\rho)} \frac{d^2}{dx^2} (F + \rho). \tag{21}$$

For the CGHS model this gives

$$R = \frac{4\lambda^2 C}{e^{(\lambda^2/4)x} + C}. \tag{22}$$

Obviously R has a singularity for $C < 0$. This is the CGHS black hole solution. In fact, it can be shown that $-C$ is proportional to the mass, and hence C must be negative. From now on we will choose $C = -1$, thus putting the singularity at $x = 0$.

For later convenience we write the curvature as

$$R = -\frac{32}{A}, \tag{23}$$

where we have introduced

$$A = \frac{8}{\lambda^2} (e^{(\lambda^2/4)x} - 1). \tag{24}$$

The metric for the general dilaton model, given in terms of F and ρ, is simply

$$ds^2 = e^{2(F+\rho)} (-dt^2 + dx^2). \tag{25}$$

In the case of CGHS we get

$$e^{2(F+\rho)}=\frac{\lambda^2}{64}\frac{e^{(\lambda^2/4)x}}{e^{(\lambda^2/4)x}-1}, \tag{26}$$

which vanishes for $x=-\infty$. For stationary metrics the equation $g_{00}=0$ determines the horizon. Therefore, in these coordinates the CGHS black hole has a horizon at $x=-\infty$. The curvature, on the other hand, is well behaved at this point. As with the Schwarzschild black hole one can find coordinates which are well behaved at the horizon. In this way one finally obtains information about the global character of the manifold (Sec. IV).

III. MODIFIED CGHS MODEL

In this section we will construct a new dilaton gravity model that satisfies the following requirements:

(1) It is completely integrable, i.e. both quadratures can be solved in closed form.

(2) For $x\to\infty$ it goes over into the CGHS model.

(3) It is singularity free.

As we have seen, dilaton gravity models are specified by giving the two potentials $D(\phi)$ and $V(\phi)$. It is very difficult to see how one should deform these potentials from their CGHS form in order to satisfy the above criteria. Note, however, that the models are also uniquely determined by giving $F(\phi)$ and $\tilde{V}(\tilde{\phi})$. This is much better for us since we have now untangled the two integrability requirements: $F(\phi)$ determines the first quadrature and $\tilde{V}(\tilde{\phi})$ the second. Deformations of a given model correspond to changes of both of these functions. In this paper we will look at a simpler problem. We shall keep $\tilde{V}(\tilde{\phi})$ fixed, i.e. it will have the same value as in the CGHS model

$$\tilde{V}(\tilde{\phi})=-\frac{1}{2}\lambda^2. \tag{27}$$

We will only deform $F(\phi)$. By doing this we are guaranteed that the second (and more difficult) quadrature is automatically solved. Because of this $\tilde{\phi}(x)$ and $\rho(x)$ are the same as in the CGHS model. Using the value for $\rho(x)$ we may write the scalar curvature for all the remaining models solely in terms of $F(x)$. We have

$$R=-8e^{-(\lambda^2/4)x}\left(e^{-2F}\frac{d^2F}{dx^2}\right). \tag{28}$$

Let us now choose F. From our second requirement we see that for large x the dilaton field $\phi(x)$ must be near to its CGHS form. Specifically, $x\to\infty$ corresponds to $\phi\to\infty$. Thus, our second requirement imposes that for $\phi\to\infty$ we have

$$F(\phi)\to-\ln\phi. \tag{29}$$

$F(\phi)$ must also be such that the first quadrature (3) is exactly solvable. To do this we choose

$$F(\phi)=-\frac{1}{\alpha}\ln\left(\frac{1+\beta\phi^\alpha}{\beta}\right), \tag{30}$$

with $\alpha>0$. The α and β values parametrize our class of deformations. The first quadrature now gives

$$D(\phi)=\begin{cases}\frac{1}{8}\phi^2+\frac{1}{4\beta}\ln\phi & \text{for } \alpha=2\\ \frac{1}{8}\phi^2+\frac{1}{4\beta(2-\alpha)}\phi^{2-\alpha} & \text{for } \alpha\neq 2.\end{cases} \tag{31}$$

On the other hand, the potential $V(\phi)$ is now simply

$$V(\phi)=-\frac{1}{2}\lambda^2\left(\frac{1+\beta\phi^\alpha}{\beta}\right)^{2/\alpha}. \tag{32}$$

The choice of α corresponds to a choice of explicit model, while β just sets a scale for the dilaton field. Rather than work here with the general modified model we will now concentrate on the simplest model in this class; the one corresponding to the choice $\alpha=4$. The action for this model is

$$S=\int d^2x\sqrt{-g}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\frac{1}{2}\lambda^2\left(\frac{1+\beta\phi^4}{\beta}\right)^{1/2}+\frac{1}{8}\left(\phi^2-\frac{1}{\beta\phi^2}\right)R\right). \tag{33}$$

Note that for $\beta\to\infty$ this goes over into the action of the CGHS model. As we have seen, β is just a scale for ϕ, hence, this is just a re-statement of our second requirement. From our construction we see that Eq. (33) corresponds, for each finite value of β, to a model that satisfies our first two requirements. All that is left is to check that the theory is indeed free of singularities. Being in two dimensions all that we need to check is the scalar curvature.

From Eqs. (5) and (31) for $\alpha=4$ we find the connection between ϕ and $\tilde{\phi}$

$$\tilde{\phi}=\frac{1}{8}\left(\phi^2-\frac{1}{\beta\phi^2}\right). \tag{34}$$

On the other hand, as we have seen, $\tilde{\phi}(x)$ is the same as in the CGHS model, so that Eq. (17) holds. Combining with Eq. (34) we find $\phi^2-1/\beta\phi^2=2A$, where we have again taken $C=-1$ [$-C$ is still the Arnowitt-Deser-Misner (ADM) mass [21] because the asymptotic form of the modified solution as $x\to\infty$ is the same as the asymptotic form of the CGHS solution]. Equivalently, $\phi^4-2A\phi^2-1/\beta=0$. This is easily solved—that is what makes the choice $\alpha=4$ so nice. We find

$$\phi^2=A+\sqrt{\frac{1}{\beta}+A^2}, \tag{35}$$

where we chose the solution of the quadratic equation that allowed ϕ to go over to ϕ_{cghs} in the $\beta\to\infty$ limit.

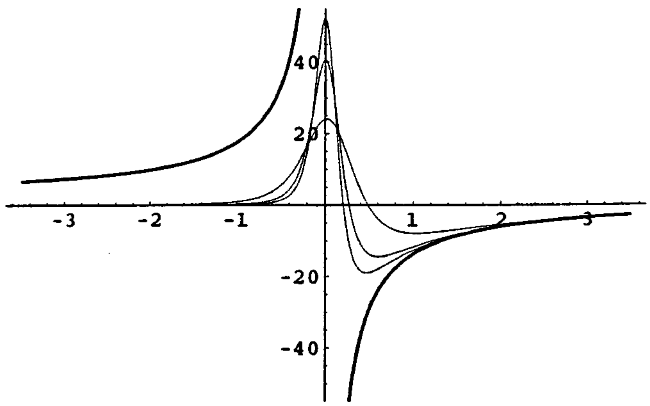

FIG. 1. $R(x)$ for the CGHS model (thick line) and its deformation for $\beta=1$, 3 and 5. As β increases the deformations look more and more like the CGHS result (for $x>0$). The graphs have been plotted for $\lambda^2=1$.

Calculating the scalar curvature is now just a matter of plugging this into Eq. (28). A simple but tedious calculation now gives

$$R=\sqrt{2}\lambda^2\left(\frac{1}{\beta}+A^2\right)^{-7/4}\left(A+\sqrt{\frac{1}{\beta}+A^2}\right)^{1/2}$$
$$\times\left\{\frac{16}{\beta\lambda^2}+\frac{3}{\beta}A-\frac{8}{\lambda^2}A^2+\left(\frac{1}{\beta}-\frac{8}{\lambda^2}A\right)\sqrt{\frac{1}{\beta}+A^2}\right\}, \tag{36}$$

where $A(x)$ was given in Eq. (24). For $\beta\to\infty$ we indeed find that

$$R\to-\frac{32}{A}, \tag{37}$$

which is the CGHS result. From Eq. (36) we see that the curvature of the modified CGHS model is indeed not singular.

As may be seen in Fig. 1, the modified model has maximal curvature at $x=0$. Its value is

$$R_{\max}=\sqrt{2}(16\beta^{1/2}+\lambda^2). \tag{38}$$

At right infinity ($x\to+\infty$) the modified model tends to the CGHS result. On the other hand, at left infinity ($x\to-\infty$) both the CGHS model and its deformation tend to a de Sitter space $R=\Lambda$. However, for CGHS we have $\Lambda=4\lambda^2$, while for the modified model the constant is a complicated function of β and λ. Rather than writing it out let us only give the result for large β when we have $\Lambda=2^{-10}\lambda^8\beta^{-3/2}$. We have just determined that

$$\lim_{x\to-\infty}\lim_{\beta\to\infty}R\neq\lim_{\beta\to\infty}\lim_{x\to-\infty}R. \tag{39}$$

Put another way: imposing that our model joins to CGHS at right infinity does not automatically guarantee a similar joining at left infinity.

We are now in the position of trying to interpret the meaning of our modified CGHS model. Obviously, one possibility is to think of Eq. (33) as the classical action of a model with scale $1/\beta$. However, it seems more natural to interpret our model as an effective action. $1/\beta$ then naturaly comes about from quantization, while $\beta\to\infty$ corresponds to the semiclassical limit. Our model should thus be the effective action corresponding to the quantization of the CGHS model. Quantization gives $S\sim\hbar$, and essentially dimensional analysis (in units $G=c=1$) gives $\phi^2\sim\hbar$, as well as $1/\beta\sim\hbar^2$. Therefore, if we are to interpret our model as an effective action then $\beta=\kappa\hbar^{-2}$, where κ is a constant of the order of unity. We see then that the maximal curvature (38) is proportional to $1/\hbar$, i.e. represents a non-perturbative effect. Expanding our model in $\hbar$ we find

$$S_{\rm eff}=S_{\rm cghs}-\frac{1}{8\kappa}\hbar^2\int d^2x\sqrt{-g}(R-2\lambda^2)\phi^{-2}+o(\hbar^4). \tag{40}$$

The leading correction to CGHS is of the form of the Jackiw-Teitelboim action for 2D gravity. It would be very interesting to get this result by quantizing some fundamental 2D theory. To do this we would need to start from the CGHS model coupled to some matter fields. We would then have to integrate out the matter. The last step would be to calculate the effective action. It is probably impossible to do this exactly, however, we could hope to do this perturbatively and compare with Eq. (40).

IV. GLOBAL PROPERTIES OF THE SOLUTION

To support the claim that the solution is non-singular we are going to discuss global properties of the solution and show that, in Kruskal coordinates, it is nonsingular everywhere and geodesically complete. For this purpose we write the CGHS metric in the form

$$ds^2=e^{2(F+\rho)}(-dt^2+dx^2) \tag{41}$$

where

$$e^{2(F+\rho)}=\frac{\lambda^2}{64}\frac{e^{(\lambda^2/4)x}}{e^{(\lambda^2/4)x}-1.} \tag{42}$$

In the light-cone coordinates $z^+=x+t$, $z^-=x-t$ metric (41) is

$$ds^2=\frac{\lambda^2}{64}\frac{e^{(\lambda^2/8)(z^++z^-)}}{e^{(\lambda^2/8)(z^++z^-)}-1}dz^+dz^-. \tag{43}$$

We define new coordinates $\xi^\pm$ by $\xi^\pm\equiv e^{(\lambda^2/8)z^\pm}$ in which metric (43) becomes

$$ds^2=\frac{1}{\lambda^2}\frac{d\xi^+d\xi^-}{\xi^+\xi^--1}. \tag{44}$$

We set $\lambda^2=1$. Introducing the so-called Kruskal coordinates X and T, $\xi^+=X+T$ and $\xi^-=X-T$ we have

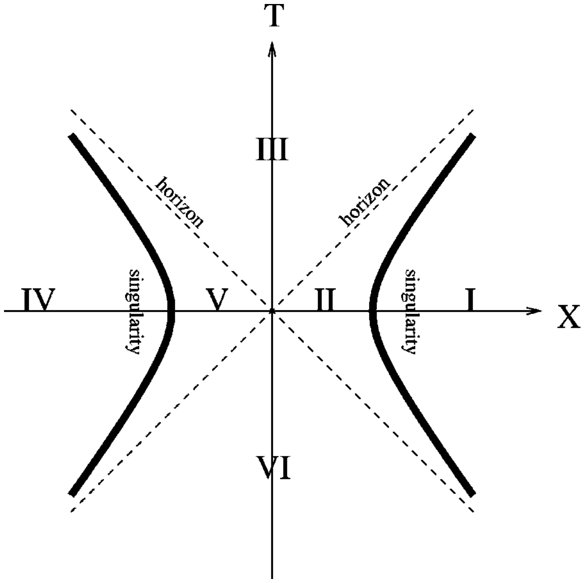

FIG. 2. CGHS space-time in Kruskal coordinates.

$$ds^2 = \frac{1}{X^2 - T^2 - 1}(-dT^2 + dX^2). \tag{45}$$

The relationship between the old coordinates x and t and the new ones is given by

$$X = e^{x/8}\cosh\left(\frac{t}{8}\right) \tag{46}$$

$$T = e^{x/8}\sinh\left(\frac{t}{8}\right). \tag{47}$$

In Kruskal coordinates X and T, the metric is always conformally flat and there is no coordinate singularity at horizon. Figure 2 shows Kruskal extension of the CGHS model.

Photon traveling towards future reaches singularity in finite proper time. Actually, in $d=2$ photons do not exist but scalar particles moving with the speed of light can take their role. In Kruskal coordinates horizon is not singular, so we can extend our space-time. Besides regions I and II we need region III in order to complete the manifold in future. Now, if we want geodesically complete manifold we have to add regions IV, V, and VI. Singularity divides the manifold in three pieces: I, IV and the piece between them. Motivation to have them all in one picture will come from our modified CGHS model.

We can perform the same analysis in the modified CGHS model. For this purpose we write the metric (41) in the form

$$ds^2 = 4e^{2F}e^{(\lambda^2/4)x}(-dt^2 + dx^2) \tag{48}$$

where we used Eq. (18) for $\rho = \rho(x)$. Difference between the modified and the original CGHS models is marely in the factor e^{2F} and due to the fact that this factor is always nonzero in both models (in the modified model is also nonsingular), horizon is the same for both models, $x = -\infty$. In the light-cone coordinates $z^{\pm}$ the metric (48) is

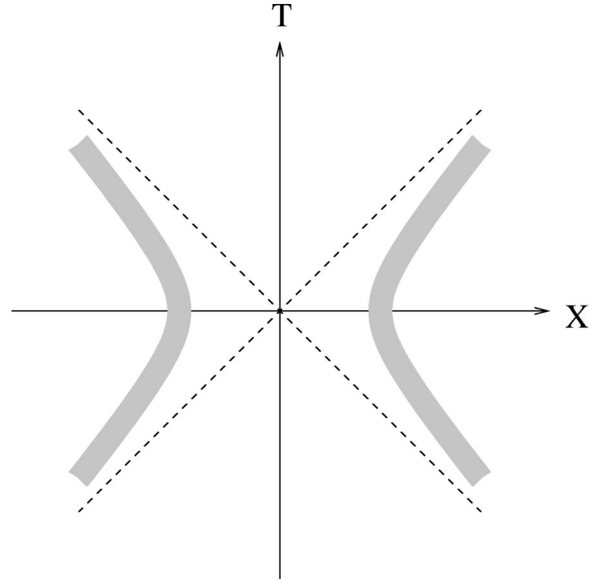

FIG. 3. Space-time of the modified non-singular CGHS model in Kruskal coordinates. Shaded surface is the region of the strong gravitational fields.

$$ds^2 = 4e^{2F}e^{(\lambda^2/8)(z^+ + z^-)}dz^+ dz^-. \tag{49}$$

Following the procedure done in the CGHS case, we choose new coordinates $\xi^{\pm} \equiv e^{(\lambda^2/8)z^{\pm}}$ which gives

$$ds^2 = e^{2F}\frac{256}{\lambda^4}d\xi^+ d\xi^-. \tag{50}$$

We set again $\lambda^2 = 1$. Transformation to Kruskal coordinates is identical as in the CGHS case, $\xi^+ = X + T$ and $\xi^- = X - T$, so we write

$$ds^2 = 256e^{2F}(-dT^2 + dX^2). \tag{51}$$

Global structure of the non-singular modified CGHS model is given in Fig. 3.

Diagram is almost the same as in the CGHS case with the important difference that the whole space-time is non-singular. In the place where singularity was present (shaded surface), now we have a region of the strong gravitational fields (big space-time curvature).

V. CONCLUSION

We have constructed a class of exactly solvable 2D gravity models that represent deformations of the CGHS dilaton gravity model. In the semi-classical limit these effective theories go over into the CGHS model. The modified CGHS models lead to non-singular black hole solutions—i.e. horizons without singularities.

It will be interesting to apply this method to non-singular 2D cosmology models. A further avenue of research is to consider modified models for dilaton gravity in the presence of matter.

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