Interference of small and of large quantum particles behind an asymmetric grating

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Quantum theory of interference phenomena does not take the diameter of the particle into account, since particles were much smaller than the width of the slits before the rise of molecular interferometry. In the experiments with large molecules, the diameter of the particle has approached the width of the slits. Therefore, an analytical description of these cases should include a finite particle size. An asymmetric double-slit grating seems to be very suitable for the study of the influence of a particle's size on the interference pattern. We identify three characteristic cases for the ratio of slit widths δ_1 and δ_2 and the particle diameter D: $D \ll \delta_1$ and $D \ll \delta_2$, $\delta_1 > D > \delta_2$, and $D > \delta_1 > \delta_2$. Taking into account the influence of both slits on the particle wave function, regardless of through which slit the particle passed, we treat the particle-wall interaction in a simple fashion, such that if the particle size is greater than the slit opening there is no transmission. The transverse momentum distribution is independent of the distance from the slits and the particle size, while the space distribution strongly depends on this distance and the particle size. We found that the interference is absent only when the particle's diameter is larger than both slit widths, $D > \delta_1 > \delta_2$.

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I. INTRODUCTION

Prior to the advent of atomic interferometry, quantum interference experiments were conducted with objects (photons, electrons, neutrons, protons, etc.) of size much smaller than their (de Broglie) wavelength and also much smaller than the characteristic dimensions of the diffraction structure. First, we recall data for the electron, neutron, and photon.

The value of the electron radius r_e has not been precisely established yet [1]. But, there is an agreement that it was much less than the Compton wavelength of the electron $\lambda_{Ce} = h/m_e c = 3.86 \times 10^{-13}$ m. The de Broglie wavelength λ_{Be} of a nonrelativistic electron is much larger than λ_{Ce} since $\lambda_{Be} = h/m_e v = \kappa \lambda_{Ce}$, where κ is the ratio $\kappa = c/v$. For nonrelativistic electrons κ is much greater than one. Electron diffraction and interference were demonstrated with gratings and wavelengths having the following values: Davisson-Germer experiment [2] on nickel crystal, lattice constant $d = 0.215$ nm, $\lambda_{Be} = 0.167$ nm; Jonsson's double-slit electron experiment [3], $d = 500$ nm, $\lambda_{Be} = 0.005\,36$ nm; Mollenstedt [4], $d = 100$ nm, $\lambda_{Be} = 0.2$ nm; Tonomura *et al.* [5], $d = 90$ nm, $\lambda_{Be} = 0.1$ nm.

The neutron radius is $r_n = 0.7 \times 10^{-15}$ m, its Compton wavelength equals $\lambda_{Cn} = 1.319 \times 10^{-15}$ m, and de Broglie wavelength for thermal neutrons $(\kappa = c/v \sim 10^5)$ is $\lambda_{Bn} = \kappa \lambda_{Cn} = 0.1319$ nm. The slit widths (lattice constants) in perfect crystal neutron interferometers for thermal neutrons are of the order of 1 μm (Rauch and Werner [6]). In the single- and double-slit interferometer of Zeilinger *et al.* [7] with cold neutrons, the values were $\lambda_{Bn} = 2$ nm and slit widths δ were in the range 20–100 μm.

In the frame of quantum optics and quantum electrodynamics there is no place for a notion of the diameter of a photon, but it is worth mentioning that Hunter and Wadlinger [8] studied this problem and planned a single-slit transmission experiment in order to verify their assertion that the photon diameter is equal to λ/π, where λ is photon's wavelength.

With the advent of atomic interferometry [9], the ratio of the size of a particle with respect to its de Broglie wavelength has been changed significantly. The atomic radii [10], determined from ionic crystal atomic data, show periodicity with respect to atomic number and take values in the interval 0.06 nm–0.28 nm. But atoms in Rydberg states are much larger, as shown by Fabre *et al.* [11] by measuring their transmission through micrometer size slits. "The extension of an n-state wavefunction—of the order of $2n^2 a_0$ where a_0 is the Bohr radius—falls in the micrometer range for $n = 100$ and corresponding Rydberg atom has the dimension of a biological cell or even of a manufactured object (wire, slit …)" [11].

It was demonstrated in molecular interference experiments of Schmiedmayer *et al.* [12] and Chapman *et al.* [13] that interference fringes can be observed when the size of a particle ($\sim$0.6 nm for Na_2 molecule) is considerably larger than both its de Broglie wavelength (0.016 nm) and its coherence length (typically 0.1 nm). Based on these results the following fundamental question was raised by Schmiedmayer *et al.* [14]: "What limits do the size and complexity of particles place on the ability of their center-of-mass motion to exhibit interference effects?" This and related questions [15,16] stimulated Arndt *et al.* [17], Nairz *et al.* [18], Brezger *et al.* [19], and Hackermuller *et al.* [20] to perform experiments with objects of larger mass and diameter, including macromolecules.

Arndt *et al.* [17] demonstrated that a C_{60} molecule interferes despite its complexity. They report the observation of

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de Broglie wave interference of C_{60} molecules by diffraction at a material absorption grating having 50-nm-wide slits with 100-nm period and stated: "This molecule is the most massive and complex object in which wave behavior has been observed. Of particular interest is the fact that C_{60} is almost a classical body, because of its many excited internal degrees of freedom …" [17]. In addition, de Broglie wavelength of the interfering fullerenes, $\lambda_{Bf}=2.5$ pm, is already smaller than their diameter ($\sim$1 nm) by a factor of almost 400 and Arndt *et al.* [17] pointed out that "it would be certainly interesting to investigate the interference of objects the size of which is equal or even bigger than the diffraction structure."

In the standard quantum theory particles are implicitly treated as material points, so their size has not been taken into account in the probabilistic interpretation of the modulus square of a wave function. However, in light of the molecular interference experiments, it is necessary to clarify the notion of the size of a quantum particle, as well as to reconsider the quantum theory of interference by taking into account the size of a particle. A full quantum description of interference assuming that the characteristic sizes of the diffraction structure are of the order of the diameter of a particle is rather complex. It requires that particle-wall interaction be taken into account. However, the work presented in this paper treats the particle-wall interaction in a simplified way. At the same time it takes properly into account the influence of both slits on a particle wave function, regardless of through which slit the particle passed.

We study the dependence of the interference pattern on particle size using a theoretical approach developed earlier [21–23]. The approach that explains interference phenomena as a process of accumulation of individual particle arrivals to the screen is briefly summarized in Secs. II–IV. An asymmetric double-slit interferometer with slit widths δ_1 and δ_2 is the generic case for this study [24]. One may consider three options for the ratio of slit widths to the diameter of a particle, D. In Sec. V we discuss the following cases: D is negligible with respect to the widths δ_1 and δ_2, $\delta_1 > D > \delta_2$, and $D > \delta_1 > \delta_2$. In Sec. VI we discuss results for all three cases.

II. WAVE FUNCTION OF A PARTICLE BEHIND AN *N*-SLITS GRATING

By measuring the time of arrival of atoms on a screen at the distance y behind an n-slit grating, Kurtsiefer *et al.* [25] found good agreement of their experimental results with the modulus square of the time-dependent wave function $\psi(x,t)$ of atomic transverse (along x axis) motion. The function was evaluated numerically from the space-dependent part $\varphi(x,y)$ of the stationary solution of the two-dimensional Schrödinger equation. The function $\varphi(x,y)$ satisfies the Helmholtz equation and is written in the form of the Fresnel-Kirchhoff integral [26]. Then, the following two important assumptions were made: (1) the longitudinal motion of particles behind the slits was classical-like, satisfying the relation $y=vt$; (2) the time-dependent transverse wave function is independent of the initial longitudinal velocity of an atom.

The space-dependent part $\varphi(x,y,z)$ of the wave function

$$\Psi(x,y,z,t) = e^{-i\omega t}\varphi(x,y,z) \tag{1}$$

can be written in another form [21–23] which has some advantages in comparison to the Fresnel-Kirchhoff form. In order to encompass more general cases the solutions of the three-dimensional Schrödinger equation are considered. With approximations valid for small diffraction angles the function $\varphi(x,y,z)$ can be written in the form

$$\varphi(x,y,z) = \frac{1}{2\pi}e^{iky}\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} dk_x dk_z c(k_x,k_z) \times e^{ik_x x}e^{ik_z z}e^{-ik_x^2 y/2k}e^{-ik_z^2 y/2k}, \tag{2}$$

where $c(k_x,k_z)$ is the double Fourier transform of the function $\varphi(x,y,z)$ on the aperture, $\varphi(x,0,z)$.

$$c(k_x,k_z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\int_{-\infty}^{+\infty} dz\varphi(x,0,z)e^{-ik_x x}e^{-ik_z z}. \tag{3}$$

The solution given in Eq. (2) is equivalent to the Fresnel-Kirchhoff solution, if the function $c(k_x,k_z)$ is determined from the same boundary conditions at the grating as are the ones used in deriving the Fresnel-Kirchhoff integral. This equivalence was first shown numerically [21–23] and later analytically [27]. The proof of the analogous equivalence for an infinite periodic grating was given by Dubetsky and Berman [28].

The time-dependent wave function of the transverse motion is then defined by substituting y under the integral sign in Eq. (2) by vt. Here, v denotes the initial velocity of the particle, assumed to be along the y axis. The usual justification for such a substitution is that in reality the motion of the particle along the longitudinal axis looks like the motion of a classical particle, whereas the transverse motion is quantum. With this substitution, and using the de Broglie relation $mv=\hbar k$, two exponents of two y-dependent exponentials in the integral are transformed into the following time-dependent expressions:

$$k_x^2\frac{y}{2k} = k_x^2\underbrace{\frac{\hbar}{2m}}_{\omega_x}t\,, \qquad k_z^2\frac{y}{2k} = k_z^2\underbrace{\frac{\hbar}{2m}}_{\omega_z}t\,. \tag{4}$$

With the aid of the latter substitution the wave function $\Psi(x,y,z,t)$ takes the form of a product of two time-dependent functions. The first function is the stationary plane wave along the y axis with the initial energy $\hbar\omega$. The second function has the form of the nonstationary solution of the two-dimensional Schrödinger equation in the x-z plane:

$$\Psi(x,y,z,t) = \frac{1}{2\pi}e^{iky}e^{-i\omega t}\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} dk_x dk_z c(k_x,k_z) \times e^{ik_x x}e^{ik_z z}e^{-ik_x^2\hbar t/2m}e^{-ik_z^2\hbar t/2m}, \tag{5}$$

where $\omega_x=k_x^2\hbar/2m$, $\omega_z=k_z^2\hbar/2m$. The latter function is denoted by $\psi(x,z,t)$ and is called the transverse wave function or the time-dependent wave function of the transverse motion:

$$\psi(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} dk_x dk_z c(k_x,k_z) e^{ik_x x} e^{ik_z z} e^{-i\omega_x t} e^{-i\omega_z t}. \tag{6}$$

The wave function $\Psi(x,y,z,t)$ from Eqs. (1) and (5) is expressed through $\psi(x,z,t)$ as

$$\Psi(x,y,z,t) = e^{iky} e^{-i\omega t} \psi(x,z,t). \tag{7}$$

From Eq. (6) we see that Kurtsiefer *et al.* [25] were right in assuming that the transverse wave function $\psi(x,z,t)$ is independent of the particle's initial momentum $p=\hbar k$.

Equation (5) has an important property. One can directly verify that it satisfies exactly the time-dependent free particle Schrödinger equation. [This looks curious and unexpected if one has in mind that Eq. (5) was obtained by substituting the relation $y=vt$ into the approximate solution of the stationary Schrödinger equation.] As such, it could be a wave function of a particle behind a grating, if it were to satisfy the boundary conditions at the grating. But, it satisfies the boundary conditions at $y=0$ only for $t=0$. This could be remedied by substituting the plane wave along the y axis (in the initial wave function as well as in the wave function behind a grating) by a wave packet with a well-defined wave front. Such a wave packet was constructed by Gottfried [29].

From Eq. (3) it is evident that the probability amplitude of transverse momenta $\overline{c}(p_x,p_z)=c(k_x,k_z)/\hbar=c(p_x/\hbar,p_z/\hbar)/\hbar$ is determined by the boundary conditions at the grating. It is independent of the initial longitudinal momentum as well. In addition, it is independent of time. Its modulus squared determines the distribution of the particle's momenta. So, one is forced to conclude that particles with zero component of transverse momentum acquire a small component of transverse momentum in passing through the grating. The distribution of momenta acquired at the grating does not change during the free evolution behind the grating.

For large values of y the function $\varphi(x,y,z)$ in Eq. (2) is approximated by [22]

$$\varphi(x,y,z) = \frac{k}{2\pi y} e^{-i(\pi/2)} e^{iky} e^{ikx^2/2y} e^{ikz^2/2y} \times \int_{\mathcal{A}} \varphi(x'',0,z'') e^{-ikxx''/y} e^{-ikzz''/y} dx'' dz'', \tag{8}$$

where $\mathcal{A}$ is the union of points (x'',z'') at the apertures. Taking Eq. (3) into account, Eq. (8) takes the form

$$\varphi(x,y,z) = e^{iky}\frac{k}{y} e^{-i(\pi/2)} e^{ikx^2/2y} e^{ikz^2/2y} c(kx/y,kz/y). \tag{9}$$

We see that the variables $K_x=kx/y$ and $K_z=kz/y$ play the role of k_x and k_z.

Since K_x and K_z are proportional to x/y and z/y, respectively, functions $|\varphi(x,y,z)|=\text{const}$ are a family of functions of x and z spreading along the x and z axis as y increases. In fact, for each value of $|\varphi|$, in the far field there exists the straight line with origin at the center of the grating along which this particular value of $|\varphi|$ propagates.

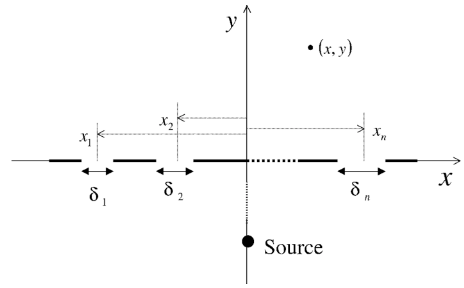


FIG. 1. Illustration of a grating with n slits of various widths.

Using the approximate Eq. (9) and having in mind the definition (6) of the transverse wave function, we may write the approximate form of this function which is valid in the far field. It is

$$\psi(x,z,t) = \frac{m}{\hbar t} e^{-i(\pi/2)} e^{i(mx^2/2\hbar t)} e^{i(mz^2/2\hbar t)} c(mx/\hbar t, mz/\hbar t). \tag{10}$$

The wave function $\psi(x,z,t)$ is proportional to the probability amplitude $c(mx/\hbar t, mz/\hbar t)$, where $mx/\hbar t$ and $mz/\hbar t$ play the role of k_x and k_z, respectively.

III. WAVE FUNCTION AND PROBABILITY AMPLITUDE OF THE PARTICLE TRANSVERSE MOMENTUM BEHIND VARIOUS ONE-DIMENSIONAL GRATINGS

We shall look for a transverse wave function behind a one-dimensional grating [22],

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} c(k_x) e^{i(k_x x - \omega_x t)} dk_x, \tag{11}$$

where

$$c(k_x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx \psi(x,0) e^{-ik_x x}. \tag{12}$$

From Eq. (12) we see that the probability amplitude of transverse momentum $\overline{c}(p_x)=c(k_x)/\sqrt{\hbar}=c(p_x/\hbar)/\sqrt{\hbar}$ behind a one-dimensional grating (see Fig. 1) is determined by the function $\varphi(x,y)=e^{iky}\psi(x,y/v)$ at $y=0$ (boundary conditions). We are choosing the simplest ones: $\psi(x,y=0)=\text{const}$ for x at the openings, $\psi(x,y=0)=0$ for x outside the openings. The constant has to be determined from the normalization condition $\int_{-\infty}^{+\infty}|\psi(x,0)|^2dx=1$. So the boundary conditions are

$$\psi(x,0)=\begin{cases}\dfrac{1}{\sqrt{\delta_1+\delta_2+\cdots+\delta_n}}, & x\in\mathcal{A},\quad \mathcal{A}=\cup\left(x_j-\dfrac{\delta_j}{2},x_j+\dfrac{\delta_j}{2}\right)\\ 0, & x\notin\mathcal{A}.\end{cases} \tag{13}$$

For the grating with one slit of width δ_j, centered at point x_j, the probability amplitude of transverse momentum is given by

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{\sqrt{2}}{\sqrt{\pi\delta_j}}\frac{\sin\frac{k_x\delta_j}{2}}{k_x}e^{-ik_x x_j}. \tag{14}$$

For the grating with n slits of widths $\delta_1,\delta_2,\ldots,\delta_n$ centered at $x_1,x_2,\ldots,x_n$, we found

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{\sqrt{2}}{\sqrt{\pi(\delta_1+\cdots+\delta_n)}}\frac{1}{k_x}\Sigma\sin\frac{k_x\delta_j}{2}e^{-ik_x x_j}. \tag{15}$$

For a grating with even number n of slits of equal width, $\delta_j=\delta$, centered at points $x_j=-(n/2)d-(d/2)+jd$, where $j=1,\ldots,n$, the latter expression reduces to

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{\sqrt{2}}{\sqrt{\pi n\delta}}\frac{\sin\frac{k_x\delta}{2}}{k_x}\Sigma e^{-ik_x x_j}=\frac{\sqrt{2}}{\sqrt{\pi n\delta}}\frac{\sin\frac{k_x\delta}{2}}{k_x}\frac{\sin\frac{k_x dn}{2}}{\sin\frac{k_x d}{2}}. \tag{16}$$

In order to understand how a one-slit wave function changes with the increase of the width of the second slit, and how the size of particles might influence the interference patterns, it was proposed recently to study interference behind an asymmetric double-slit interferometer [24]. Young's double-slit grating is symmetric since the widths of both slits are equal ($\delta_1=\delta_2$). The transverse wave function in the momentum representation behind a grating with two slits of widths δ_1 and δ_2 centered at $x_1=-d/2$ and $x_2=d/2$ is

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{\sqrt{2}}{\sqrt{\pi(\delta_1+\delta_2)}}\frac{1}{k_x}\left[e^{ik_x d/2}\sin\frac{k_x\delta_1}{2}+e^{-ik_x d/2}\sin\frac{k_x\delta_2}{2}\right]. \tag{17}$$

It is easy to see that in the cases $\delta_2=0$, $\delta_1>0$, $x_1=-d/2$ and $\delta_2=\delta_1$, $x_1=-d/2$, $x_2=d/2$, Eq. (15) reduces to the expressions valid [21–23] for $n=1$,

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{\sqrt{2}}{\sqrt{\pi\delta_1}}\frac{\sin\frac{k_x\delta_1}{2}}{k_x}e^{ik_x(d/2)} \tag{18}$$

and for $n=2$

$$\sqrt{\hbar}\overline{c}(\hbar k_x)=c(k_x)=\frac{2}{\sqrt{\pi\delta_1}}\frac{\sin\frac{k_x\delta_1}{2}}{k_x}\cos\frac{k_x d}{2}. \tag{19}$$

The momentum distribution behind one slit, and asymmetric and symmetric double slits are given by, for $n=1$,

$$\hbar|\overline{c}(\hbar k_x)|^2=|c(k_x)|^2=\frac{2}{\pi\delta_1}\frac{\sin^2\frac{k_x\delta_1}{2}}{k_x^2}, \tag{20}$$

and for $n=2$ and $\delta_1\neq\delta_2$,

$$\hbar|\overline{c}(\hbar k_x)|^2=|c(k_x)|^2=\frac{2}{\pi(\delta_1+\delta_2)}\frac{1}{k_x^2}\left[\sin^2\frac{k_x\delta_1}{2}+\sin^2\frac{k_x\delta_2}{2}+2\sin\frac{k_x\delta_1}{2}\sin\frac{k_x\delta_2}{2}\cos(k_x d)\right], \tag{21}$$

for $n=2$ and $\delta_1=\delta_2$,

$$\hbar|\overline{c}(\hbar k_x)|^2=|c(k_x)|^2=\frac{2}{\pi\delta_1}\frac{\sin^2\frac{k_x\delta_1}{2}}{k_x^2}[1+\cos(k_x d)]. \tag{22}$$

These distributions are graphically represented in Fig. 2 for a chosen set of parameters. By comparing Eqs. (18)–(20), as well as the curves in Fig. 2, we may understand how the increase of the width of the second slit influences the one-slit curve. The effect of increasing the second slit width leads to Fig. 2(c), which corresponds to a symmetric double slit. The second slit induces oscillations of the one-slit curve. The period of oscillations is determined by the mutual distance of slits d. By applying an elementary trigonometric formula, one easily shows that the oscillations are bounded by two envelopes, the upper and the lower one, which are determined by the following expressions:

$$|c(k_x)|^2\leqslant|c(k_x)|_u^2=\frac{8}{\pi(k_x)^2(\delta_1+\delta_2)}\sin^2\frac{k_x(\delta_1+\delta_2)}{4}\cos^2\frac{k_x(\delta_1-\delta_2)}{4}, \tag{23}$$

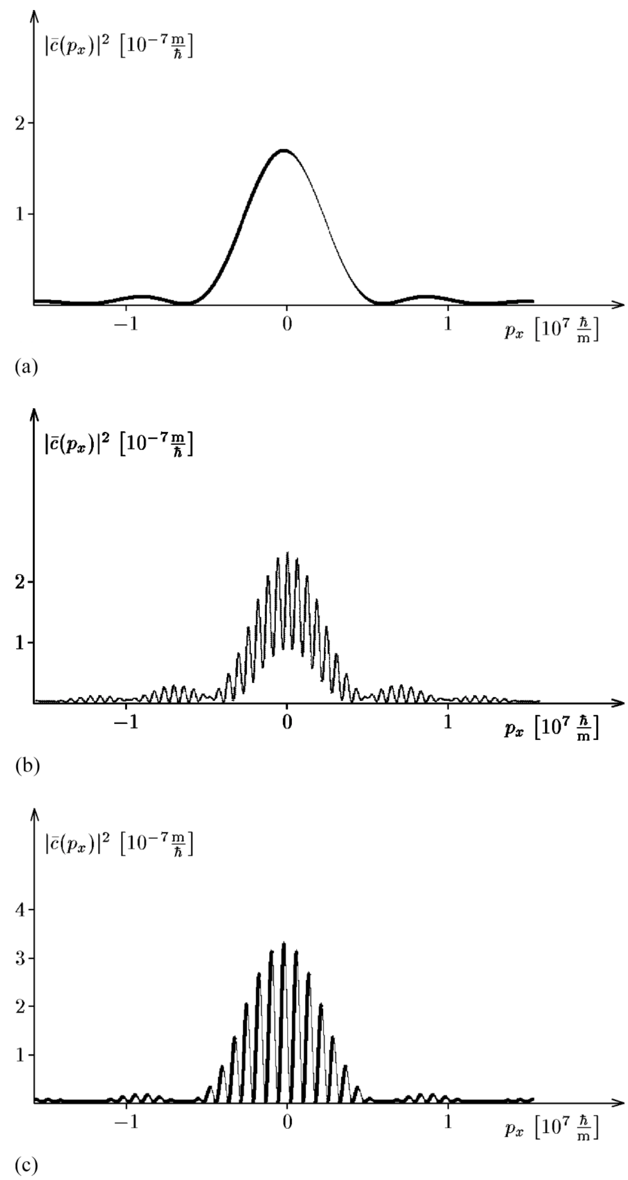


FIG. 2. The particle transverse momentum distribution $|\overline{c}(p_x)|^2$ behind: (a) one-slit grating ($\delta_1=1\ \mu$m), (b) asymmetric double-slit grating
($\delta_1=1\ \mu$m, $\delta_2=0.25\ \mu$m, $d=8\ \mu$m), and (c) symmetric double-slit grating ($\delta_1=\delta_2=1\ \mu$m, $d=8\ \mu$m).

$$|c(k_x)|^2 \geqslant |c(k_x)|_l^2 = \frac{8}{\pi(k_x)^2(\delta_1+\delta_2)}\sin^2\frac{k_x(\delta_1-\delta_2)}{4}\cos^2\frac{k_x(\delta_1+\delta_2)}{4}. \qquad (24)$$

From Eq. (23) one concludes that for $\delta_2 \ll \delta_1$ the upper envelope is *very close* to $|c(k_x)|^2$ for one slit. From Eq. (24) one sees that for $\delta_2=\delta_1$ the lower envelope reduces to the k_x axis, as it should be. One sees also that in this case the upper envelope is equal to $2|c(k_x)|^2$ for one slit.

A study of a wave function behind a grating with two different slits ($\delta_1 \neq \delta_2$) is useful because one may see how a one-slit wave function changes with the increase of the width of the second slit from zero to δ_1. The time and space particle distribution behind an asymmetric double-slit grating is shown in Fig. 3. By comparing this distribution with the distributions behind a one-slit grating and behind a two-slit symmetric grating [21–23], one sees how the presence of the second slit and the increase of its width influence and change the one-slit distribution.

As in the case of momentum distribution, the effect of the very narrow second slit is to induce oscillations of the one-slit $|\psi(x,t)|^2$ curve. With increasing width of the second slit, these oscillations would become larger, and the main peaks would move towards the places corresponding to the symmetric double slit.

IV. STATISTICAL EXPLANATION OF THE SPACE DISTRIBUTION USING TRANSVERSE MOMENTUM DISTRIBUTION

Based on the above factorization of the wave function $\Psi(x,y,t)$ and the properties of its factors summarized above, we proposed [22] the new expression for the probability density $\widetilde{P}(x,t)$ for the particle's arrival at time t to a point $(x,y=vt)$, which is far from the grating:

$$\widetilde{P}\left(x,\frac{y}{v}\right) = \widetilde{P}(x,t) \equiv \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dx'' \, |c(k_x)|^2 |\psi(x'',0)|^2 \times \delta\left(x - x'' - \frac{\hbar k_x t}{m}\right). \qquad (25)$$

Particles emerge from different points $(x'',0)$ at the aperture. That is the reason for integration over x''. The contribution of each point at the aperture is proportional to $|\psi(x'',0)|^2$. The integration over dk_x and the function $|c(k_x)|^2$ reflects the contribution of various angles/momenta in diffraction. Finally, the δ function assumes straight trajectory from a point $(x'',0)$ at the slits to the point (x,y) and leads to the simplified form

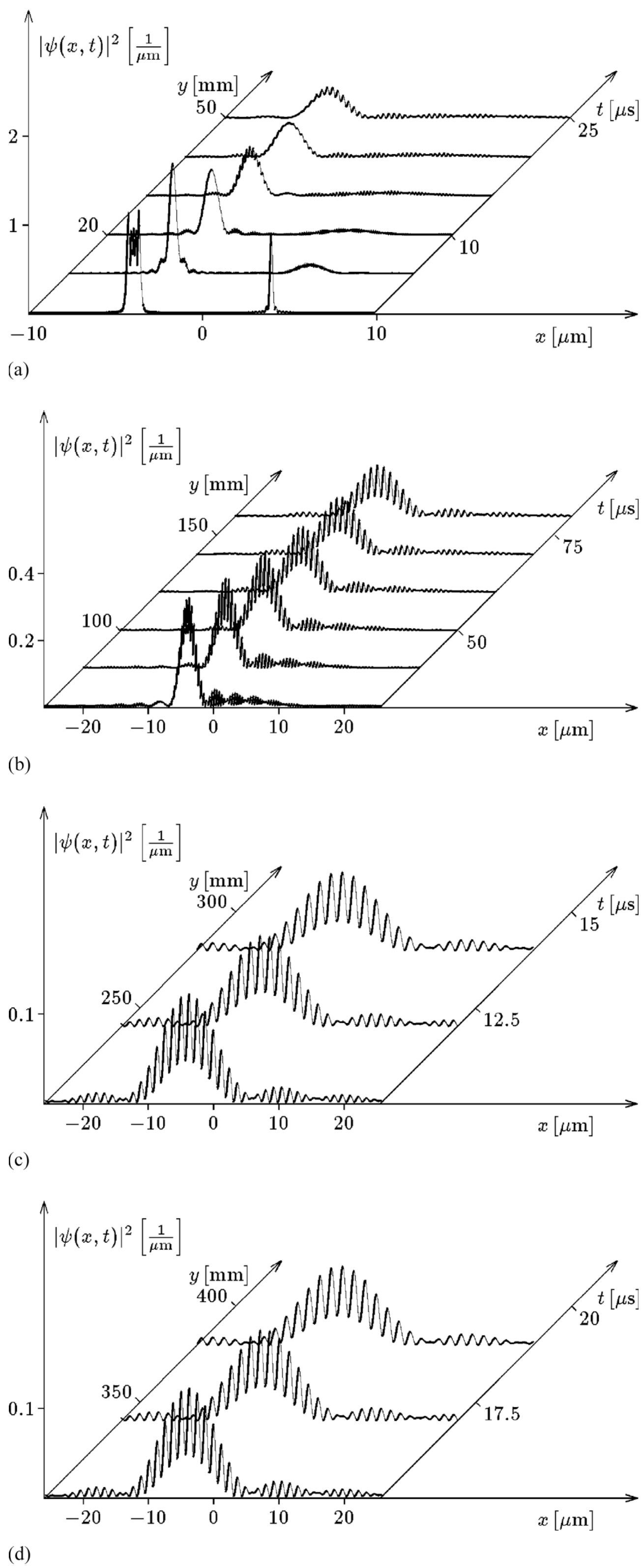
$$\widetilde{P}(x,t) = \int_{-\infty}^{+\infty} dk_x \, |c(k_x)|^2 \left|\psi\left(x - \frac{\hbar k_x t}{m},0\right)\right|^2. \qquad (26)$$

By assuming that the function $\psi(x,0)$ satisfies at the grating the boundary conditions given in Eq. (13), Eq. (26) is transformed into the sum of n terms $[\widetilde{P}_i(x,t)]$. That is

$$\widetilde{P}(x,t) = \frac{1}{\sum_{i=1}^{n}\delta_i}\sum_{i=1}^{n}\int_{(m/\hbar)(x-x_r^i)}^{(m/\hbar)(x-x_l^i)} dk_x |c(k_x)|^2 \equiv \sum_{i=1}^{n}\widetilde{P}_i(x,t). \qquad (27)$$

Here x_l^i and x_r^i are the coordinates of the left and right edges of the ith slit.

Numerical calculations for various values of n graphically presented in Ref. [23] and in Fig. 4 show that far from the slits (Fraunhofer region) the function $\widetilde{P}(x,t)$ is nearly equal



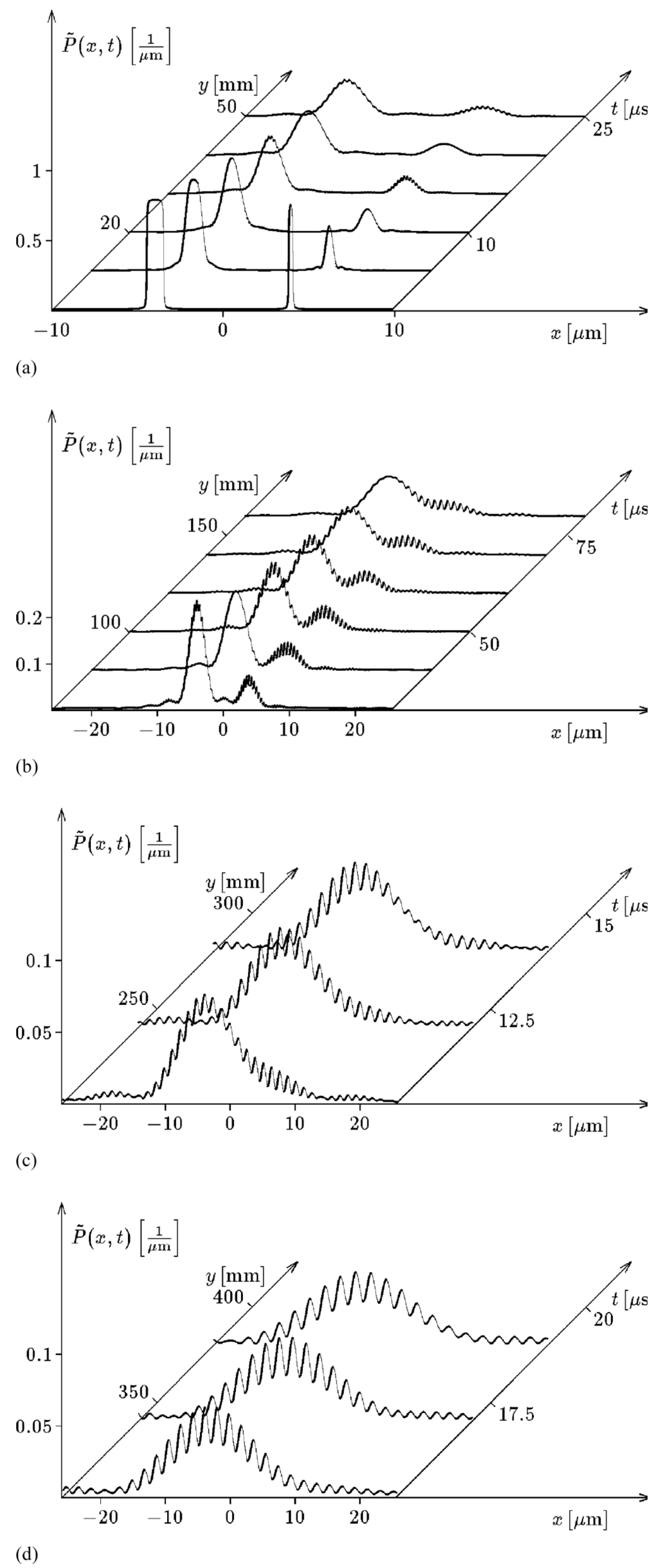


FIG. 3. The particle distribution function $|\psi(x,t)|^2$ behind the asymmetric double-slit grating ($\delta_1=1$ μm, $\delta_2=0.25$ μm, $d=8$ μm) close to the slits (a,b) and far from the slits (c,d). The initial longitudinal wave vector is $k=4\pi\times10^{10}$ m^{-1}, the particle mass is $m=3.8189\times10^{-26}$ kg.

FIG. 4. The probability density $\widetilde{P}(x,t)$ of particle's arrival to the point x at time t ($y=vt$) behind the asymmetric double-slit grating ($\delta_1=1$ μm, $\delta_2=0.25$ μm, $d=8$ μm) close to the slits (a,b) and far from the slits (c,d). It is evaluated from Eq. (27). Particle's diameter D is negligible with respect to the widths of the slits. The initial longitudinal wave vector is $k=4\pi\times10^{10}$ m^{-1} and the particle mass is $m=3.8189\times10^{-26}$ kg.

to the exact probability density $|\Psi(x,y,t)|^2=|\psi(x,t)|^2$. Near the slits (Frenel's and Talbot's regions) $\tilde{P}(x,t)$ and $|\psi(x,t)|^2$ qualitatively look similar but they differ numerically.

By taking into account that $\tilde{P}(x,t)$ is obtained by summing the probabilities of the particle's arrival at the point (x,y) at time t along various possible trajectories, we conclude [22] that far from the slits the function $\tilde{P}_i(x,t)$ is equal to the probability that a particle reaches $(x,y=vt)$ at time t after passing at $t=0$ through the ith slit of the n-slits grating. Near the slits $\tilde{P}_i(x,t)$ is not equal to that probability because the particles' trajectories near the slits are more complicated than further from the slits. Only far from the slits the straight lines could approximate the particles' trajectories.

V. ON THE POSSIBLE INFLUENCE OF THE PARTICLE'S DIAMETER ON THE INTERFERENCE PATTERN

Up to now the size of the particles has not been taken into account in the quantum theory of interference [1–6,25]. This is because in the standard quantum theory the particle's size is absent from the probability interpretation of the modulus square of a wave function. However, in light of molecular interference experiments, it is necessary to reconsider the quantum theory of interference by taking into account the size of the particle.

An asymmetric double-slit grating seems to be very suitable for the study of the influence of a particle's size on the interference pattern [24]. Its usefulness comes from the fact that one can identify three characteristic cases for the ratio of slit widths δ_1 and δ_2 and the diameter of the particle D: (a) the diameter D is negligible with respect to the widths δ_1 and δ_2; (b) the diameter D satisfies $\delta_2<D<\delta_1$; (c) the diameter D is greater than both widths δ_1 and δ_2.

A full quantum description of interference, assuming that the characteristic sizes of the diffraction structure are of the order of the diameter of a particle, is rather complex. It requires investigating the influence of particle-wall interaction on interference. This interaction is neglected in the standard quantum theory of interference of particles of negligible size.

In this paper we shall treat the particle-wall interaction in the simple fashion, such that if the particle size is greater than the slit opening, there is no transmission. But, we shall take properly into account the influence of the slits on the wave function, as it has been necessary in the standard quantum theory of interference of particles having size negligible with respect to the widths of the slits. We shall especially take into account the well-established fact that the behavior of the photon, electron, atom, and small molecule behind the grating depends on the number of open slits, widths, and distances. This implies that behavior of each particle is determined by all slits illuminated by the initial particle wave, not only by the one through which the particle had passed [22].

A. The particle's diameter is negligible

If a particle diameter D is negligible with respect to the widths δ_1 and δ_2, the particle can go through either of the slits. The transverse wave function $\psi(x,t)$ and probability amplitude of transverse momenta $\overline{c(p_x)}$ are given by Eqs. (11) and (17). The momentum distribution $|\overline{c(p_x)}|^2$ and time dependence of space distribution $|\psi(x,t)|^2$ are graphically represented for chosen set of parameters in Figs. 2(c) and 3, respectively.

The evident dependence of transverse momentum distribution on the grating parameters and the close relation between momentum distribution and the space distribution clearly show that the behavior of each particle is determined by the superposition of all waves emerging from every slit illuminated by the initial particle wave.

B. The particle diameter D satisfies $\delta_2<D<\delta_1$

In this case we are faced with the question of how and where to take the diameter of the particle into account. We know that the diameter of the particle is not incorporated anywhere in the Schrödinger equation. But, we expect that a particle having diameter D which satisfies $\delta_1>D>\delta_2$ could not pass through the second slit.

We foresee two possible reasonings. First, one can regard $\psi(x,t)$ as a probability amplitude for a particle being at certain place at time t, without any other physical reality. Then, one has to conclude that particle distribution in this case should be identical to the one-slit distribution (no interference). Second, one can consider $\psi(x,t)$ as a real wave associated with a particle. Then, one should conclude that the solution of the Schrödinger equation behind a grating is the same in case (b) as in case (a) [given by Eqs. (11) and (17)]. This is because the boundary conditions for the wave function are the same in both cases. Consequently, the momentum distribution $|c(k_x)|^2$ of particles is given by Eq. (21) because it is determined by the values of the wave function at the boundary.

But the space distribution of particles in case (b) should be different from the space distribution in case (a) because the particles arriving at the smaller slit cannot go through. Only particles arriving at the larger slit can go through.

Will those particles behave classically or quantum mechanically? Will those particles be influenced by waves spreading from both slits? We shall assume that those particles will behave as quantum particles and that their motion will be influenced by waves spreading from both slits.

We found that Eq. (27) is satisfied far from the slits if the particle size is negligible with respect to the widths of the slits. $\tilde{P}_i(x,t)$ is equal to the probability that a particle reaches $(x,y=vt)$ at time t after passing through the ith slit at $t=0$ of the n-slits grating. If we go to Eq. (25) from which Eq. (27) was derived, we find that we assumed implicitly that each particle passed through. However, the assumption is not appropriate for slits widths smaller then the particle size. This means that the integration in Eq. (25) should be only over x'' belonging to the slit through which particles can pass. This is to say that probability of a large particle reaching (x,y) at time t, $\tilde{P}^l(x,t)$, is equal to the probability $\tilde{P}_1(x,t)$ of a particle reaching (x,y) at time t after passing through the larger slit:

$$\widetilde{P}^{l}(x,t) = \widetilde{P}_1(x,t) = \frac{1}{\delta_1 + \delta_2}\int_{(m/\hbar)(x-x_r^1)}^{(m/\hbar)(x-x_l^1)} |c(k_x)|^2 dk_x. \quad (28)$$

The probability $\widetilde{P}_1(x,t)$ is graphically represented in Fig. 5 for a chosen set of parameters. $\widetilde{P}_1(x,t)$ shows presence of interference, despite the fact that particles are not allowed to pass through the smaller slit. The interference is due to the influence of wave spreading through the smaller and through the larger slit on the momentum distribution and motion of particles going through the larger slit. The visibility in case (b) is slightly less than in case (a), as seen by comparing Fig. 5 with Figs. 3 and 4.

C. The diameter D is greater than both widths δ_1 and δ_2

In this case, transmission through both slits will be zero, and consequently there will be no interference.

VI. CONCLUSION

Inspired by the current efforts to perform diffraction and interference experiments with objects of size equal to or even larger than the diffraction structure, we outline an approach to investigate how the particle diameter influences the interference pattern in an asymmetric double-slit interferometer. The approach is based on the use of the time-dependent wave function of a particle's transverse motion and the probability amplitude of transverse momentum. We evaluated these functions using the stationary solution of the two-dimensional Schrödinger equation and assuming classical motion along the longitudinal axis.

Similar solutions were determined and used by Dubetsky and Berman [28] for infinite periodic gratings. The distributions for transverse momenta behind one slit, symmetric and asymmetric double slits, and n-slits grating that we determined in Refs. [21–23] and in this paper are continuous since these gratings are not periodic.

For the asymmetric double-slit grating we identify three characteristic cases for the ratio of slit widths δ_1 and δ_2 and the diameter D of the particle: (a) $D \ll \delta_1$ and $D \ll \delta_2$, (b) $\delta_1 > D > \delta_2$, and (c) $D > \delta_1 > \delta_2$.

The wave function behind the grating has the same form in cases (a) and (b) because it is the solution of the Schrödinger equation which is not sensitive to the diameter of the particle.

The space distribution of particles in case (a) is given as usual by the modulus square of this function. Using the same wave function and assuming that a particle with diameter D, such that $\delta_1 > D > \delta_2$, could not pass through the second slit, we determine the space distribution in case (b). We conclude that the momentum distribution of particles behind the grat-

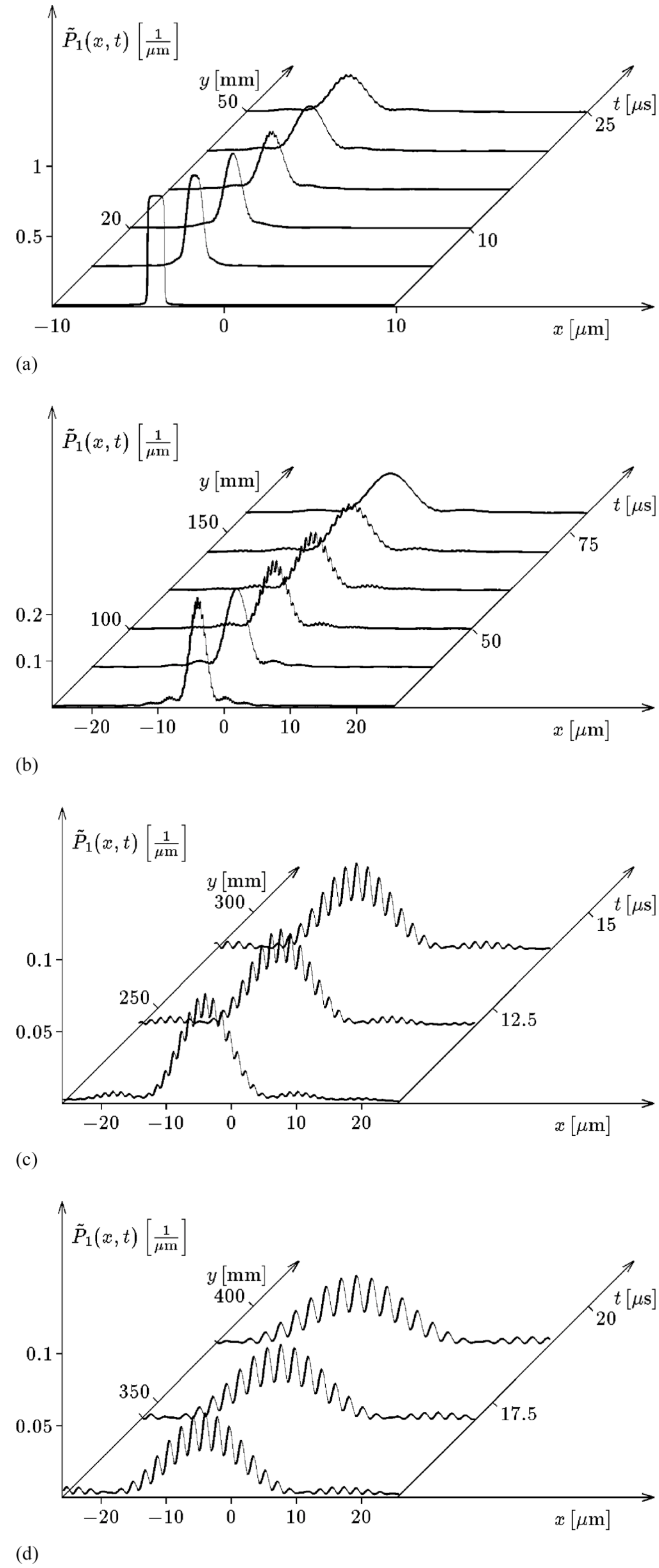


FIG. 5. The probability density $\widetilde{P}^{l}(x,t) = \widetilde{P}_1(x,t)$ of large particles reaching (x,y) at time t after passing through the larger slit, near the slits (a,b) and far from the slits (c,d). It is evaluated from Eq. (28). D is assumed to be larger than δ_2 and smaller than δ_1. The values of parameters are the same as in captions of Figs. 2–4.

ing is the same in cases (a) and (b). As a consequence we conclude that there should be interference in both cases (a) and (b). The interference in case (b) is due to the influence of wave spreading through the smaller slit on the momentum distribution and motion of particles going through the larger slit.

The interference is absent only in case (c) because transmission should be zero in this case.

[1] M. H. MacGregor, *The Enigmatic Electron* (Kluwer, Dordrecht, 1992).

[2] C. Davisson and L. H. Germer, Phys. Rev. **30**, 705 (1927).

[3] C. Jonsson, Z. Phys. **161**, 454 (1961).

[4] G. Mollenstedt, Physica B & C **151**, 201 (1988).

[5] A. Tonomura, J. Endo, J. Matsuda, T. Kawasaki, and H. Ezawa, Am. J. Phys. **57**, 117 (1989).

[6] H. Rauch and S. A. Werner, *Neutron Interferometry, Lessons in Experimental Quantum Mechanics* (Clarendon Press, Oxford, 2000).

[7] A. Zeilinger, R. Gahler, C. G. Schull, W. Treimer, and W. Mampe, Rev. Mod. Phys. **60**, 1067 (1988).

[8] G. Hunter and R. L. P. Wadlinger, Phys. Essays **2**, 158 (1989).

[9] P. R. Berman, *Atom Interferometry* (Academic Press, New York, 1997).

[10] S. T. Thornton and A. Rex, *Modern Physics for Scientists and Engineers* (Saunders, New York, 1993).

[11] C. Fabre, M. Gross, J. M. Raimond, and S. Haroche, J. Phys. B **16**, L671 (1983).

[12] J. Schmiedmayer, C. R. Ekstrom, M. S. Chapman, T. D. Hammond, and D. E. Pritchard, in *Fundamentals of Quantum Optics III*, edited by F. Ehlotzky (Springer-Verlag, Berlin, 1993), p. 21.

[13] M. S. Chapman, C. R. Ekstrom, T. D. Hammond, R. A. Rubenstein, J. Schmiedmayer, S. Wehinger, and D. E. Pritchard, Phys. Rev. Lett. **74**, 4783 (1995).

[14] J. Schmiedmayer, M. S. Chapman, C. R. Ekstrom, T. D. Hammond, D. A. Kokorowski, A. Lenef, R. A. Rubenstein, E. T. Smith, and D. E. Pritchard, in *Atom Interferometry*, edited by P. R. Berman (Academic Press, New York, 1997), p. 1.

[15] J. F. Clauser, in *Experimental Metaphysics*, edited by R. S. Cohen *et al.* (Kluwer, Dordrecht, 1997).

[16] M. Arndt, O. Nairz, G. van der Zouw, and A. Zeilinger, in *Epistemological and Experimental Perspectives on Quantum Physics*, edited by D. Greenberger *et al.* (Kluwer, Dordrecht, 1999), p. 221.

[17] M. Arndt, O. Nairz, J. V. Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, Nature (London) **401**, 680 (1999).

[18] O. Nairz, M. Arndt, and A. Zeilinger, J. Mod. Opt. **47**, 2811 (2000).

[19] B. Brezger, L. Hackermuller, S. Uttenthaler, J. Petschinka, M. Arndt, and A. Zeilinger, Phys. Rev. Lett. **88**, 100404 (2002).

[20] L. Hackermuller K. Hornberger, B. Brezger, A. Zeilinger, and N. Armdt, Appl. Phys. B: Lasers Opt. **77**, 781 (2003).

[21] M. Božić, D. Arsenović, and L. Vušković, Z. Naturforsch., A: Phys. Sci. **56**, 173 (2001).

[22] D. Arsenović, M. Božić, and L. Vušković, J. Opt. B: Quantum Semiclassical Opt. **4**, S358 (2002).

[23] L. Vušković, D. Arsenović, and M. Božić, Found. Phys. **32**, 1329 (2002).

[24] M. Božić, D. Arsenović, and L. Vušković, Bull. Am. Phys. Soc. **47**, 74 (2002).

[25] Ch. Kurtsiefer, T. Pfau, and J. Mlynek, Nature (London) **386**, 150 (1997).

[26] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Oxford, 1965).

[27] V. Manko, O. Manko, M. Božić, and D. Arsenović (unpublished).

[28] B. Dubetsky and P. A. Berman, in *Atom Interferometry* (Ref. [14]), p. 407.

[29] K. Gottfried, *Quantum Mechanics Volume I: Fundamentals* (W.A. Benjamin, New York, 1966), p. 29.