Dark-polariton bound pairs in the modified Jaynes-Cummings-Hubbard model

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We investigate a one-dimensional modified Jaynes-Cummings-Hubbard chain of N identical QED cavities with nearest-neighbor photon tunneling and periodic boundary conditions. Each cavity contains an embedded three-level atom which is coupled to a cavity mode and an external classical control field. In the case of two excitations and common large detuning of two Raman-resonant fields, we show the emergence of two different species of dark-polariton bound pairs (DPBPs) that are mutually localized in their relative spatial coordinates. Due to the high degree of controllability, we show the appearance of either one or two DPBPs, having the energies within the energy gaps between three bands of mutually delocalized eigenstates. Interestingly, in a different parameter regime with negatively detuned Raman fields, we find that the ground state of the system is a DPBP which can be utilized for the photon storage, retrieval, and controllable state preparation. Moreover, we propose an experimental realization of our model system.

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I. INTRODUCTION

The interaction between light and matter is one of the most fundamental and basic processes in nature, and it represents a milestone in our understanding of a broad range of physical phenomena. The recent experimental success in engineering strong interactions between photons and atoms in high-quality microcavities opens up the possibility to use light-matter systems as quantum simulators for many-body physics [1]. Key examples as first-principles proposals are quantum phase transitions of light in coupled cavities [2–4], quantum fluids of light (see [5]) and the Mott-insulator-to-superfluid phase transition of polaritons in an array of coupled QED cavities [6–11]. Coupled cavities are realized in a variety of physical systems, among them microcavities and nanocavities in photonic crystals [12]. These have paved the way to study strongly correlated phenomena in a controlled way by using such systems. Richness in these systems emerges from the interplay of two main effects. At one side, light-matter interaction inside the cavity leads to a strong effective Kerr nonlinearity between photons. By controlling the atomic level spacings as well as the cavity-mode frequency, it is possible to achieve a photon-blockade regime [13–16] where photon fluctuations are suppressed in each cavity. On the other side, photon hopping between neighboring cavities supports delocalization and competes with the photon blockade.

At the end of the past century, Fleischhauer and Lukin introduced the theoretical concept of dark-state polaritons (DSPs), form-stable coupled excitations of light and matter associated with the propagation of quantum fields in electromagnetically induced transparency (EIT), and showed their potential usage as quantum memories for photons [17,18]. Since then, DSPs have been in the focus of intense theoretical and experimental investigations [19–33]. The first proposal for realization of strong interactions among DSPs and Mott-insulator-to-superfluid phase transition thereof was given by Hartmann *et al.* [9]. They demonstrated the possibility to generate attractive onsite potentials for polaritons yielding highly entangled states and a phase with particles much more delocalized than in superfluids. Moreover, two-polariton bound states, composite excitations of two polaritons that may be spatially confined together, were predicted by Wong and Law [34]. Very recently, two-polariton bound states have been related to spin-orbit interactions by Li *et al.* [35]. Both are features of the systems described by the one-dimensional Jaynes-Cummings-Hubbard model (JCH) and represent an important connection between condensed matter physics and quantum optics. In such systems, it is possible to realize various many-body effects where the particles of interest are photons rather than electrons.

In this paper, we present a scheme based on a modified Jaynes-Cummings-Hubbard model (MJCH) that enables the formation of two different species of spatially, mutually localized dark-polariton bound pairs (DPBPs). Our scheme is based on N identical coupled QED cavities with periodic boundary conditions. Each cavity embeds a single three-level atom. A cavity mode and an external control field, which are in two-photon Raman resonance, drive the transitions from the two atomic ground states to the excited state. We assume that a common single-photon detuning of the fields is large compared to the coupling strengths. Under such conditions, the description of the three-level atoms is effectively reduced to ground-state two-level systems with tunable coupling strength between the ground levels and controllable level Stark shifts. Hence, our model circumvents the drawbacks of the excited-state spontaneous emission and provides a tunable extension of two-polariton bound states of the classical Jaynes-Cummings-Hubbard model [34]. Furthermore, we find that when the common detuning of the coupling fields is negative, the lowest-energy eigenstate of the system becomes a mutually localized DPBP of a new type that may be used as a quantum memory of light. This may find potential use in quantum information processing and controllable state preparation.

This paper is organized as follows. In Sec. II, we recapitulate the standard Jaynes-Cummings model and focus on its spectrum and eigenstates. In Sec. III, we discuss the modified Jaynes-Cummings model where we derive the modified

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Jaynes-Cummings Hamiltonian from a bare model. Further, we analyze the eigenstates and highlight the differences to the standard Jaynes-Cummings model. In Sec. IV, we present the considered model system and extend the modified Jaynes-Cummings model to a modified Jaynes-Cummings-Hubbard model, highlighting that it features the formation of bound states of two dark-polaritons. In Sec. V, we present a detailed discussion of the two-excitation subspace and explain the formation of dark-polariton bound pairs (DPBPs), accentuating their tunability through the control field Stark shift. In Sec. VI, we demonstrate an application of a ground-state DPBP as a quantum memory on which storage and retrieval of a single photon can be performed, while the second photon remains not influenced by the storage and retrieval process. Even though two photons are bound, exactly one photon can be addressed. The state composition of the ground-state DPBP can be tuned by the relative importance of the intercavity photon hopping, e.g., increasing the common single-photon detuning $|\Delta|$. In Sec. VII, we propose an experimental realization of our model system, where we state not only promising candidates to the creation of one-dimensional chains of N-coupled QED cavities, but also name single Λ atoms which can be considered. In addition, we point out that for Cs the measured strong-coupling constant g_m fits very well with our theoretical prediction, where the formation of DPBPs as well as the storage and retrieval process can be seen. Finally, In Sec. VIII we draw our conclusions.

II. STANDARD JAYNES-CUMMINGS MODEL

Within this section, we recapitulate the standard Jaynes-Cummings model (JC). Especially, we focus on its spectrum and eigenstates. In this model, a two-level atom with ground level $|g\rangle$ and excited level $|e\rangle$ having energies ω_g and ω_e interacts with a single mode of an electromagnetic field of frequency ω_0 that couples the transition $|g\rangle \rightarrow |e\rangle$ with the strength g_0. In the (rotating-wave) approximation (RWA), JC Hamiltonian has the form ($\hbar = 1$) [36,37]

$$\hat{H}^{(\mathrm{JC})} = \omega_0 \hat{n} + \delta \hat{\sigma}^+ \hat{\sigma}^- - g_0(\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger \hat{\sigma}^-), \tag{1}$$

where $\hat{c}^\dagger$ ($\hat{c}$) is the photonic creation (annihilation) operator and $\hat{\sigma}^+ = |e\rangle\langle g|$ ($\hat{\sigma}^- = |g\rangle\langle e|$) is the atomic raising (lowering) operator. $\hat{n} = \hat{c}^\dagger \hat{c} + \hat{\sigma}^+ \hat{\sigma}^-$ is the number operator of the combined photonic and atomic excitations (polaritons) which is a conserved quantity, i.e., $[\hat{H}^{(\mathrm{JC})}, \hat{n}] = 0$. $\delta = \omega_e - \omega_0$ is the detuning. Due to the conservation of $\hat{n}$, $\hat{H}^{(\mathrm{JC})}$ in the subspace $\{|g,n\rangle, |e,n-1\rangle\}$ is represented with the block matrix h_n:

$$h_n = \begin{pmatrix} \omega_0 n & -g_0\sqrt{n} \\ -g_0\sqrt{n} & \omega_0 n + \delta \end{pmatrix}, \tag{2}$$

with $n = 1,2,3,\ldots$ being the total number of excitations. The matrix in (2) is a 2×2 matrix and can be analytically diagonalized. The eigenenergies are given as

$$E_n = \begin{cases} E_{n\pm} = \omega_0 n + \frac{1}{2}[\delta \pm \chi_n(\delta)], & n \geqslant 1 \\ E_0 = 0, & n = 0 \end{cases} \tag{3}$$

with $\chi_n(\delta) = \sqrt{\delta^2 + 4g_0^2 n}$ being the generalized Rabi frequency and $+$ stands for the higher and $-$ for the lower eigenenergy, while the eigenstates are

$$|n,+\rangle := \sin(\theta_n)|g,n\rangle + \cos(\theta_n)|e,n-1\rangle, \tag{4a}$$

$$|n,-\rangle := \cos(\theta_n)|g,n\rangle - \sin(\theta_n)|e,n-1\rangle. \tag{4b}$$

$n = 0$ corresponds to the state of zero polaritons. It takes on the form

$$|0,\pm\rangle \equiv |0,g\rangle = |0\rangle, \tag{5}$$

whereas the occurring mixing angle θ_n is defined as

$$\theta_n = \frac{1}{2}\arctan\left(\frac{2g_0\sqrt{n}}{\delta}\right). \tag{6}$$

The eigenstates (4) are called polaritons. Polaritons are low-energy quasiparticles which are composed of photonic and atomic excitations in superposition. As we change the mixing angle θ_n by a rotation from 0 to $\frac{\pi}{2}$, which basically corresponds to a change of the detuning δ, we tune the polaritons to either pure photonic or pure atomic excitations in a reversible manner. Due to the contribution of the excited atomic state $|e,n-1\rangle$, these polaritons in a more precise way can be called bright polaritons similar to [17–19,33].

III. MODIFIED JAYNES-CUMMINGS MODEL

For the subsequent discussion, we need to derive the modified Jaynes-Cummings (mJC) Hamiltonian which describes an effective interaction of a Λ system with a highly detuned mode of an electromagnetic and classical field. We show that due to the large, common single-photon detuning Δ, i.e., $|\Delta| \gg |g_m|, |\Omega|$, it is possible to circumvent the drawback of the excited-state spontaneous emission that would plague realizations of the JC model by using atoms and optical cavities [37]. Moreover, we focus on the discussion of the eigenstates and eigenspectrum in two specific cases which naturally arise in our case.

A. Derivation of the modified Jaynes-Cummings model Hamiltonian

We consider a single photon in a single-mode QED cavity in which a Λ three-level atom is embedded. The ground levels are $|g\rangle$ and $|f\rangle$ with their level energies ω_g and ω_f, whereas the excited level $|e\rangle$ with level energy ω_e is detuned by a large, common single-photon detuning Δ with respect to two coupling fields. The cavity field with frequency ω_m couples the transition $|g\rangle \rightarrow |e\rangle$ with strength g_m. Further, a classical control field with frequency ω_c and Rabi frequency Ω couples the transition $|f\rangle \rightarrow |e\rangle$. Our bare model Hamiltonian ($\hbar = 1$) has the form

$$\hat{H}_{\mathrm{bare}}(t) = \hat{H}_c + \hat{H}_a + \hat{H}_{\mathrm{int}}(t), \tag{7a}$$

$$\hat{H}_c = \omega_m \hat{c}^\dagger \hat{c}, \tag{7b}$$

$$\hat{H}_a = \omega_g \hat{\sigma}_{gg} + \omega_f \hat{\sigma}_{ff} + \omega_e \hat{\sigma}_{ee}, \tag{7c}$$

$$\hat{H}_{\mathrm{int}}(t) = -(g_m \hat{c}\hat{\sigma}_{eg} + g_m^* \hat{c}^\dagger \hat{\sigma}_{ge} + \Omega e^{-i\omega_c t}\hat{\sigma}_{ef} + \Omega^* e^{i\omega_c t}\hat{\sigma}_{fe}), \tag{7d}$$

where $\hat{H}_c$ denotes the free-field Hamiltonian of the QED cavity, $\hat{H}_a$ stands for the free-atomic Hamiltonian, and $\hat{H}_{\rm int}(t)$ describes the interaction of the fields with the atom. $\hat{c}^\dagger$ ($\hat{c}$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta} = |\alpha\rangle\langle\beta|$ ($\alpha,\beta \in \{g,f\}$) are the atomic operators. $\hat{H}_{\rm bare}(t)$ in (7) satisfies the time-dependent Schrödinger equation

$$i\partial_t|\Psi(t)\rangle = \hat{H}'(t)|\Psi(t)\rangle. \tag{8}$$

We move to a rotating frame in which (7) is time independent. The corresponding gauge transformation [19,33] has the form ($\hbar = 1$)

$$\hat{H}^T = \hat{U}(t)\hat{H}_{\rm bare}(t)\hat{U}^\dagger(t) + i\partial_t[\hat{U}(t)]\hat{U}^\dagger(t), \tag{9}$$

where $\hat{U}(t)$ is a unitary transformation. Under the gauge (9), $\hat{H}_{\rm bare}(t)$ reads as

$$\hat{H}^T_{\rm bare} = \hat{H}_c + \hat{H}_a + \hat{H}_{\rm int}, \tag{10a}$$

$$\hat{H}_c = \omega_m \hat{c}^\dagger\hat{c}, \tag{10b}$$

$$\hat{H}_a = \omega_g\hat{\sigma}_{gg} + (\omega_f + \omega_c)\hat{\sigma}_{ff} + \omega_e\hat{\sigma}_{ee}, \tag{10c}$$

$$\hat{H}_{\rm int} = -(g_m\hat{c}\hat{\sigma}_{eg} + g_m^*\hat{c}^\dagger\hat{\sigma}_{ge} + \Omega\hat{\sigma}_{ef} + \Omega^*\hat{\sigma}_{fe}). \tag{10d}$$

$\hat{U}(t) = e^{-i\omega_c t\hat{\sigma}_{ff}}$ has been chosen as the unitary transformation in deriving (10). Assume that the Λ three-level atom is initially prepared in the state $|g,n\rangle = |g\rangle\otimes|n\rangle$. n represents the arbitrary but fixed number of excitations with $n = 1,2,3\ldots$ and $|n\rangle$ the corresponding number state. Under the action of $\hat{H}^T_{\rm bare}$ onto the state $|g,n\rangle = |g\rangle\otimes|n\rangle$, we get the relations

$$\hat{H}^T_{\rm bare}|g,n\rangle = (\omega_m n + \omega_g)|g,n\rangle - g_m\sqrt{n}|e,n-1\rangle, \tag{11a}$$

$$\hat{H}^T_{\rm bare}|e,n-1\rangle = [\omega_m(n-1) + \omega_e]|e,n-1\rangle - g_m^*\sqrt{n}|g,n\rangle - \Omega^*|f,n-1\rangle, \tag{11b}$$

$$\hat{H}^T_{\rm bare}|f,n-1\rangle = [\omega_m(n-1) + \omega_f + \omega_c]|f,n-1\rangle - \Omega|e,n-1\rangle. \tag{11c}$$

In the subspace $\{|g,n\rangle,|e,n-1\rangle,|f,n-1\rangle\}$, $\hat{H}^T_{\rm bare}$ has the matrix representation

$$h_{\rm bare} = \begin{pmatrix} (\omega_m n + \omega_g) & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & \omega_m(n-1) + \omega_e & -\Omega^* \\ 0 & -\Omega & (\omega_m(n-1)+\omega_f+\omega_c) \end{pmatrix}. \tag{12}$$

Under Raman resonance condition $\omega_m n + \omega_g = \omega_f + \omega_c = \omega_e - \Delta$, we get

$$h_{\rm bare} = \begin{pmatrix} (\omega_m n + \omega_g) & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & (\omega_m(n-1) + \omega_e) & -\Omega^* \\ 0 & -\Omega & (\omega_m(n-1)+\omega_f+\omega_c) \end{pmatrix}. \tag{13}$$

Under a rotating-wave approximation, (13) is reduced to

$$h^{\rm Raman}_{\rm bare} = \begin{pmatrix} 0 & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & \Delta & -\Omega^* \\ 0 & -\Omega & 0 \end{pmatrix}. \tag{14}$$

In addition, as we have a far detuned excited state $|e,n-1\rangle$, i.e., $|\Delta| \gg |g_m|,|\Omega|$ [36,38] we can adiabatically eliminate the contribution of the excited state $|e,n-1\rangle$ directly on the level of (14). This yields to

$$h^{\rm (mJC)} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -\frac{g_m^*\Omega\sqrt{n}}{\Delta} \\ -\frac{g_m\Omega^*\sqrt{n}}{\Delta} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}. \tag{15}$$

Equation (15) represents the matrix form of the modified Jaynes-Cummings Hamiltonian (mJC) in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$. The operator form of the modified Jaynes-Cummings Hamiltonian (mJC) reads as

$$\hat{H}^{\rm (mJC)} = \hat{H}_S + \hat{H}_{\rm int}, \tag{16a}$$

$$\hat{H}_S = -\left(\frac{|g_m|^2}{\Delta}\hat{c}^\dagger\hat{c}\hat{\sigma}_{gg} + \frac{|\Omega|^2}{\Delta}\hat{\sigma}_{ff}\right), \tag{16b}$$

$$\hat{H}_{\rm int} = -\left(\frac{g_m^*\Omega}{\Delta}\hat{c}^\dagger\hat{\sigma}_{gf} + \frac{g_m\Omega^*}{\Delta}\hat{c}\hat{\sigma}_{\rm fg}\right). \tag{16c}$$

The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{\rm int}$ represents the interaction of the cavity field and the atom, where $G = g_m^*\Omega/\Delta$ is the effective atom-photon coupling constant. Hamiltonians $\hat{H}_S$ and $\hat{H}_{\rm int}$ constitute the modified Jaynes-Cummings Hamiltonian. In the sequel, we are going to discuss the eigenstates of $\hat{H}^{\rm (mJC)}$ and look at the effect of the control field Stark shift.

B. Eigenstates of the modified Jaynes-Cummings model Hamiltonian

In the following, we calculate the eigenenergies and eigenstates of $\hat{H}^{\rm (mJC)}$. We show that dependent on whether one compensates the control field Stark shift by using external fields or not, the eigenenergies, composition of the eigenstates, and the mixing angle θ_n differ significantly. First, we consider the case of noncompensated control field Stark shift. $\hat{H}^{\rm (mJC)}$ of (16) reduces in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$ as

$$h_n^{(m)} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}, \tag{17}$$

with $n = 1,2,3,\ldots$ the total number of excitations and corresponding number state $|n\rangle$. The eigenenergies are given as

$$E^{(m)}_{+,n} = 0, \tag{18}$$

$$E^{(m)}_{-,n} = -\left(\frac{|g_m|^2 n}{\Delta} + \frac{|\Omega|^2}{\Delta}\right). \tag{19}$$

The eigenstates to the eigenenergies $E^{(m)}_{+,n}$ and $E^{(m)}_{-,n}$ read as

$$|n,\mathrm{DP}^{(+)}\rangle := \sin(\theta_n)|f,n-1\rangle - \cos(\theta_n)|g,n\rangle, \tag{20a}$$

$$|n,\mathrm{DP}^{(-)}\rangle := \cos(\theta_n)|f,n-1\rangle + \sin(\theta_n)|g,n\rangle \tag{20b}$$

with the occurring mixing angle θ_n which is defined as

$$\theta_n = \frac{1}{2}\arctan\left(\frac{2|g_m|\sqrt{n}}{|\Omega|}\right). \tag{21}$$

However, $|n,\mathrm{DP}^{(\pm)}\rangle$ are called dark-polaritons. A dark-polariton is a quasiparticle which is a superposition of photonic and atomic excitations, where the atomic excitations have only contributions of ground levels $|g\rangle$ and $|f\rangle$ and not the excited level $|e\rangle$. Such dark-polaritons are very similar to the known dark-state polaritons [17,18], but with one major difference. Dark-state polaritons are defined at Raman resonance of two coupling fields and formed independently of the single-photon detuning. Instead, dark-polaritons, which are also defined at Raman resonance, are formed for a large single, common photon detuning Δ of the two coupling fields, i.e., $|\Delta| \gg |g_m|,|\Omega|$. The dependence on Δ enables to tune the eigenstate $|n,\mathrm{DP}^{(\pm)}\rangle$ from an excited to a ground eigenstate. This follows from the eigenenergy $E_{-,n}^{(m)}$ of the dark-polariton $|n,\mathrm{DP}^{(-)}\rangle$. If $\Delta > 0\,(\Delta < 0)$, $|n,\mathrm{DP}^{(+)}\rangle$ is an excited (a ground) eigenstate and $|n,\mathrm{DP}^{(-)}\rangle$ a ground (an excited) eigenstate. Note that $|n,\mathrm{DP}^{(+)}\rangle$ is a degenerate eigenstate because the corresponding eigenenergy $E_{+,n}^{(m)}$ does not depend on the dark-polariton number n. $|n,\mathrm{DP}^{(-)}\rangle$ is a degenerate eigenstate as well for $n \geqslant 2$. Thus, the spectrum is discrete and degenerate in dependence of the dark-polariton number n.

Now, we switch to the case of compensated control field Stark shift. Compensation is achieved by using an additional field, which couples the ground state $|f\rangle$ with some far-off-resonant excited state [39]. Within (17) we set the control field Stark shift $\frac{|\Omega|^2}{\Delta}$ to zero. Hence, the new block-matrix representation $h_n^{(m,\mathrm{comp})}$ in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$ reads as

$$h_n^{(m,\mathrm{comp})} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & 0 \end{pmatrix}, \tag{22}$$

with $n = 1,2,3,\ldots$ the total number of excitations and corresponding number state $|n\rangle$. The block-matrix (22) is a 2×2 matrix and can be analytically diagonalized. The eigenenergies are given as

$$E_{-,n}^{(\mathrm{comp},m)} = -\frac{|g_m|^2 n + |g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{2\Delta},$$
$$E_{+,n}^{(\mathrm{comp},m)} = \frac{-|g_m|^2 n + |g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{2\Delta}. \tag{23}$$

The respective eigenstates to the eigenenergies $E_{+,n}^{(\mathrm{comp},m)}$ and $E_{-,n}^{(\mathrm{comp},m)}$ are

$$|n,\mathrm{DP}^{(+)}_{\mathrm{comp}}\rangle := \sin(\theta_n)|f,n-1\rangle + \cos(\theta_n)|g,n\rangle, \tag{24a}$$
$$|n,\mathrm{DP}^{(-)}_{\mathrm{comp}}\rangle := \cos(\theta_n)|f,n-1\rangle - \sin(\theta_n)|g,n\rangle, \tag{24b}$$

with the occurring mixing angles θ_n which are defined as

$$\theta_n = \frac{1}{2}\arctan\left[\frac{A(\Omega,n)}{B(g_m,\Omega,n)}\right], \tag{25a}$$
$$A(\Omega,n) = 2\sqrt{2}\times|\Omega|\sqrt{n}, \tag{25b}$$
$$B(g_m,\Omega,n) = \sqrt{C(g_m,\Omega,n)}, \tag{25c}$$
$$C(g_m,\Omega,n) = |g_m|n^2 + 4|\Omega|^2 n + D(g_m,\Omega,n), \tag{25d}$$
$$D(g_m,\Omega,n) = |g_m|n\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}. \tag{25e}$$

$|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$ are dark-polaritons, but of a different type compared to the case of noncompensated control field Stark shift. First of all, the eigenenergies $E_{s,n}^{(\mathrm{comp},m)}$ with $s = +,-$ depend on the generalized Rabi frequency $\xi(n) = \sqrt{|g_m|^2 n + 4|\Omega|^2}$. Second, $|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$ have a common mixing angle θ_n that depends on the generalized Rabi frequency $\xi(n)$ as well. In addition, the two dark-polariton branches, represented through $|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$, are separated by the energy amount

$$E_{-,n}^{(\mathrm{comp},m)} - E_{+,n}^{(\mathrm{comp},m)} = \frac{|g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{\Delta}. \tag{26}$$

The separation energy is directly dependent on the generalized Rabi frequency $\xi(n)$ and the common single-photon detuning Δ as well. This separation is related to the photon-photon repulsion. It is a consequence of the onsite repulsion $U(n)$ which is a measure of the Kerr nonlinearity [40].

C. Comparison to standard Jaynes-Cummings model

On the level of the individual Hamiltonians, major differences are that at first, in $\hat{H}^{(\mathrm{mJC})}$ the number operator depends on the projection operator $\hat{\sigma}_{gg}$ of the ground level $|g\rangle$ which is not the case in $\hat{H}^{(\mathrm{JC})}$. Second, in $\hat{H}^{(\mathrm{mJC})}$ the atom-cavity field coupling strength $G = g_m\Omega/\Delta$ is rescaled by the common single-photon detuning Δ and the Rabi frequency Ω, where G is chosen to be real. Regarding the eigenstates, a key difference between $\hat{H}^{(\mathrm{mJC})}$ and $\hat{H}^{(\mathrm{JC})}$ is that in the modified Jaynes-Cummings model we have eigenstate dependence on the control field Stark shift. In addition, within the modified Jaynes-Cummings model, we only have a dependence on ground levels, whereas in the standard Jaynes-Cummings model there exists a dependence on the excited level. Hence, these dependencies affect the coherences. Namely, the bright polaritons in the standard Jaynes-Cummings model only consist of optical coherences $\hat{\sigma}_{eg}$ and are explored to spontaneous emission, while in the modified Jaynes-Cummings model, dark-polaritons only consist of spin coherences $\hat{\sigma}_{\mathrm{fg}}$ and no exploration to spontaneous emission is present. This enables the usage of dark-polaritons as a quantum memory for photons over their spin coherences likewise the dark-state polaritons [17–33]. Changing the mixing angles in (21) and (25) over rotations from $0 \to \frac{\pi}{2}$, which corresponds to an adiabatical change of the Rabi frequency Ω, photons are transferred to and stored in the spin coherences in a reversible manner. Optical coherences have shorter coherence times compared to the spin coherences which have longer coherence times. Coherence times of spin coherences are in the range of μs to ms in dark-state polaritons [17,18]. Similar is the case for dark-polaritons. In the sequel, we focus on our model system and state the effective model Hamiltonian which is based on our derivation of the modified Jaynes-Cummings model.

IV. MODEL SYSTEM AND EFFECTIVE MODEL HAMILTONIAN

In the previous sections, we have investigated the standard and modified Jaynes-Cummings model on the level of a single QED cavity. In the subsequent step, we extend the modified Jaynes-Cummings model to a one-dimensional array of coupled QED cavities. This will lead us to the modified Jaynes-Cummings Hubbard model as our effective model Hamiltonian. It includes the hopping between adjacent cavities. First, we state the model system and, second, present the effective model Hamiltonian.

A. Model system

The system we consider consists of a one-dimensional array of N-coupled QED cavities. We assume periodic boundary conditions, i.e., the cavity labeled by $n = N + 1$ corresponds to the cavity $n = 1$. Each cavity embeds a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The level energies are ω_g, ω_f, and ω_e, respectively, and the excited level $|e\rangle$ is detuned by the common single-photon detuning Δ. In reality, the levels can be either fine or hyperfine levels of alkali-metal atoms. Their D_1 or D_2 line transitions are nowadays easily accessible via available lasers and optical modes of QED cavities. One mode of a tunable cavity [41,42] of frequency ω_m couples the transition $|g\rangle \to |e\rangle$ with the strength g_m, and the classical control field of frequency ω_c and Rabi-frequency Ω couple the transition $|f\rangle \to |e\rangle$. This configuration is known to feature vacuum induced transparency, as first experimentally demonstrated by the group of Vuletić [43]. Both g_m and Ω are typically in MHz range for alkali-metal atoms, which are strongly coupled to QED cavities, and for moderate laser powers.

B. Effective model Hamiltonian

As we consider a one-dimensional chain of N identical coupled QED cavities, the derived modified Jaynes-Cummings model for a single QED cavity is valid for all QED cavities in the one-dimensional chain. Therefore, our effective model Hamiltonian (modified Jaynes-Cummings Hubbard model) ($\hbar = 1$) has the form

$$\hat{H}^{(\mathrm{mJCH})} = \hat{H}^{(\mathrm{mJC})} + \hat{H}_{\mathrm{hop}}, \tag{27a}$$

$$\hat{H}^{(\mathrm{mJC})} = \hat{H}_S + \hat{H}_{\mathrm{int}}, \tag{27b}$$

$$\hat{H}_S = -\sum_{\mu=1}^{N}\left(\frac{g_m^2}{\Delta}\hat{c}_\mu^\dagger\hat{c}_\mu\hat{\sigma}_{gg}^{(\mu)} + \frac{\Omega^2}{\Delta}\hat{\sigma}_{ff}^{(\mu)}\right), \tag{27c}$$

$$\hat{H}_{\mathrm{int}} = -G\sum_{\mu=1}^{N}\left(\hat{c}_\mu^\dagger\hat{\sigma}_{gf}^{(\mu)} + \hat{c}_\mu\hat{\sigma}_{\mathrm{fg}}^{(\mu)}\right), \tag{27d}$$

$$\hat{H}_{\mathrm{hop}} = -J\sum_{\mu=1}^{N}(\hat{c}_{\mu+1}^\dagger\hat{c}_\mu + \hat{c}_\mu^\dagger\hat{c}_{\mu+1}), \tag{27e}$$

where $\hat{c}_\mu^\dagger$ ($\hat{c}_\mu$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta}^{(\mu)} = |\alpha\rangle_\mu\langle\beta|$ ($\alpha,\beta \in \{g,f\}$) are the atomic operators for the site number μ. The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{\mathrm{int}}$ represents the interaction of the cavity field and the atom, where $G = g_m\Omega/\Delta$ is the effective atom-photon coupling constant which is set to be real. Hamiltonians $\hat{H}_S$ and $\hat{H}_{\mathrm{int}}$ constitute the modified Jaynes-Cummings Hamiltonian. As will be shown in the sequel, the Stark shifts have profound influence on the energy eigenspectrum. $\hat{H}_{\mathrm{hop}}$ describes the photon hopping between adjacent cavities, based on evanescent field coupling, with J as the intercavity photon hopping strength. Similar effective Hamiltonian has been previously used to describe a network of fiber coupled cavities, embedded with three-level atoms [39]. However, while that scheme requires the compensation of the level Stark shifts, here we utilize the individual Stark shifts to achieve tunability. Our effective model Hamiltonian (27) supports the formation of dark-polariton bound pairs. We will see that the different dark-polaritons, which have been discussed in Sec. III, are actually involved in the formation of the energy bands and the bound states. Moreover, we show and discuss that the bound states are formed due to the presence of a force called Kerr nonlinearity which is determined by the onsite repulsion.

V. FORMATION OF DARK-POLARITON BOUND PAIRS

In the following, we discuss the formation of dark-polariton bound pairs in our system. In order to exploit the invariance of the system under cyclic permutations of the sites, we introduce the following operators via discrete Fourier transforms:

$$\hat{b}_k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N}e^{-\frac{2\pi i}{N}\mu k}\,\hat{c}_\mu, \tag{28a}$$

$$\hat{s}_{gf}^{(k)} = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N}e^{-\frac{2\pi i}{N}\mu k}\,\hat{\sigma}_{gf}^{(\mu)}, \tag{28b}$$

where $k = 0,1,\ldots,N-1$ is related to the (discrete) quasimomentum of the excitation. Similarly to [34], we work in the two-excitation subspace that is spanned by the states $|kj\rangle_F \equiv \hat{b}_k^\dagger\hat{b}_j^\dagger|\Phi_0\rangle$, $|k\rangle_F|j\rangle_A \equiv \hat{b}_k^\dagger\hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$, and $|kj\rangle_A \equiv \hat{s}_{gf}^{(k)\dagger}\hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$. The subscripts F and A stand for the photonic and atomic excitations, respectively. The state $|\Phi_0\rangle = \otimes_{\mu=1}^{N}|g\rangle_\mu|0\rangle_\mu$ is the ground state of the system, where $|0\rangle_\mu$ denotes the vacuum state of the cavity number μ. We note that the excitations (polaritons) are in our case dark in a sense that they do not have the contribution of the excited levels $|e\rangle$ and are not subjected to spontaneous emission. The atomic excitations $|kj\rangle_A$ are in general not orthogonal to each other because of ${}_A\langle k'j'|kj\rangle_A = \delta_{k,k'}\delta_{j,j'} + \delta_{k,j'}\delta_{j,k'} - \frac{2}{N}\delta_{k+j,k'+j'}$. $\hat{b}_k$ and $\hat{b}_j^\dagger$ fulfill the bosonic commutation relation $[\hat{b}_k,\hat{b}_j^\dagger] = \delta_{kj}$, while the atomic operators fulfill the commutation relation $[\hat{s}_{gf}^{(k)},\hat{s}_{gf}^{(j)\dagger}] = -\frac{1}{N}\sum_{\mu=1}^{N}e^{\frac{2\pi i}{N}\mu(j-k)}\hat{\sigma}_z^{(\mu)}$ with $\hat{\sigma}_z^{(\mu)}$ as the Pauli z matrix for the atom in the μth cavity. Under the action of $\hat{H}$ on the states which form the two-excitation subspace, we get the relations

$$\hat{H}|kj\rangle_F = (\omega_k + \omega_j - 2a)|kj\rangle_F - G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F), \tag{29a}$$

$$\hat{H}|k\rangle_A|j\rangle_F = (\omega_j - a - b)|k\rangle_A|j\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A, \tag{29b}$$

$$\hat{H}|j\rangle_A|k\rangle_F = (\omega_k - a - b)|j\rangle_A|k\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A, \tag{29c}$$

$$\hat{H}|kj\rangle_A = -G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F) - 2b|kj\rangle_A, \tag{29d}$$

where $\omega_l = -2J\cos(\frac{2\pi l}{N})$ for $l \in \{k,j\}$, $a = g_m^2/\Delta$, and $b = \Omega^2/\Delta$. Within Eqs. (29b) and (29c), we have a sum over the set $S_P = \{(k,j) \mid 0 \leqslant k < j \leqslant N-1, k+j \equiv P \,(\mathrm{mod}\, N)\}$ that is determined by the quasimomentum P. From Eqs. (29a)–(29d) we can deduce that the quasimomentum P is a conserved quantity and hence a good quantum number. Apart from the quasimomentum, the total number of excitations (dark-polaritons) $\hat{N} = \sum_{\mu=1}^{N}(\hat{c}_\mu^\dagger\hat{c}_\mu + \hat{\sigma}_{ff}^{(\mu)})$ is a conserved quantity.

We can construct the complete set of eigenvectors by solving the eigenproblem within each of the subspaces $P = 0,1,\ldots,N-1$. Following [34], we restrict the discussion to the case of even N and odd P. A general dark two-polariton eigenvector $|\Psi_P^{(D)}\rangle$ has the form

$$\big|\Psi_P^{(D)}\big\rangle = \sum_{(k,j)\in S_P}(\alpha_{kj}|kj\rangle_F + \beta_{kj}|k\rangle_A|j\rangle_F + \beta'_{kj}|j\rangle_A|k\rangle_F + \gamma_{kj}|kj\rangle_A). \tag{30}$$

$|\Psi_P^{(D)}\rangle$ satisfies the time-independent Schrödinger equation $\hat{H}|\Psi_P^{(D)}\rangle = \lambda|\Psi_P^{(D)}\rangle$ which yields within each of the subspaces $P = 1,3,\ldots,N-1$ an eigenproblem that is given by the subsequent set of linear equations

$$\lambda\alpha_{kj} = (\omega_k + \omega_j - 2a)\alpha_{kj} - G(\beta_{kj} + \beta'_{kj}), \tag{31a}$$

$$\lambda\beta_{kj} = -G\alpha_{kj} + (\omega_j - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'} \tag{31b}$$

$$\lambda\beta'_{kj} = -G\alpha_{kj} + (\omega_k - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'}, \tag{31c}$$

$$\lambda\gamma_{kj} = -G(\beta_{kj} + \beta'_{kj}) - 2b\gamma_{kj}, \tag{31d}$$

where λ is the corresponding eigenvalue. As it was demonstrated in [34], for various values of the quasimomentum P the majority of eigenvalues are at most distributed among three bands. When all three bands are well resolved, it was shown that each of the two band gaps contains an eigenenergy of the single two-polariton bound state. For sufficiently large intercavity photon hopping strength J comparing to the strength of the atom-photon interaction, the bands start to overlap.

However, since we are not dealing with the standard JCH model, but rather with a modified one, we find some important differences and new features. Namely, as opposed to [34] there is only one mutually localized DPBP within one of the existing band gaps, while the other one joins the adjacent outer band. The other DPBP can reappear provided that the Stark shift of the control field is compensated. In both cases, when $\Delta < 0$, $g_m \gg \Omega$ and $g_m^2/|\Delta| \gtrsim 1.5\,J$, the ground state of the system is DPBP of a different type than the aforementioned ones. In the sequel, we report on the state composition of the different DPBP types.

The Kerr nonlinearity is a known force in light-atom interactions which depends on the atomic level structure as well as on the coupling strength of light-atom interactions. In our case, the strength of light-atom interaction is described by the effective coupling strength $G = g_m\Omega/\Delta$. Tuning g_m and/or Ω directly affects the Kerr nonlinearity. Compared to [34], we can not only tune and control the Kerr nonlinearity by the cavity-mode coupling strength g_m, but also by the Rabi frequency Ω. This force can be attractive or repulsive [1,13–16]. This force generates the bound state of two dark-polaritons in our case. A measure of the Kerr nonlinearity is the onsite repulsion $U(n)$ which is in general defined as

$$U(n) := (E_+ - E_-)(n+1) - (E_+ - E_-)(n) \tag{32}$$

with $E_\pm$ the eigenenergies of the considered eigenstates. In case of the standard Jaynes-Cummings model, the onsite repulsion $U(n) = \chi(n+1) - \chi(n)$ is determined by the generalized Rabi frequency $\chi(n)$ [3]. This will be different in our case as we will see in the following. In our DPBPs we have bound photons and bound atoms. In [44], they have experimentally shown bound states of atoms in coupled QED cavities, when atoms occupy the same site.

A. Dark-polariton bound pairs in the regime of noncompensated control field Stark shift

We focus on the single DPBP solution of Eqs. (31) which is given in red color within Fig. 1(a) representing the energy eigenspectrum of the model Hamiltonian $\hat{H}$ in dependence of odd values of quasimomentum P. Three energy bands are visible for the used parameter values. We define the gap between the two upper energy bands as the high-energy band gap and in accordance the gap between the two lower-energy bands as the low-energy band gap. The dark-polaritons, which are involved in the formation of energy bands and the single DPBP in Fig. 1(a), are given in (20). This can be seen by solving Eqs. (31) for intercavity hopping $J = 0$. Note that the bands are a consequence of repulsively interacting dark-polaritons of different types with respect to the eigenenergies $E_{\pm,n}^{(m)}$. By different types here, we mean that the dark-polariton with eigenenergy $E_{+,n}^{(m)}$ interacts with the dark-polariton of eigenenergy $E_{-,n}^{(m)}$ in a repulsive way at the same site μ. This is a consequence of the onsite repulsion $U(n)$. On different sites, dark-polaritons with eigenenergies $E_{+,n}^{(m)}$ and $E_{-,n}^{(m)}$ are noninteracting. Instead, the mentioned Kerr nonlinearity, expressed through the onsite repulsion $U(n) = \frac{g_m^2}{\Delta}$, enables the single DPBP state formation by the two dark-polaritons with eigenenergies $E_{-,n}^{(m)}$ which is placed at the same site μ in case of $\Delta > 0$. There is an additional DPBP, formed by the two dark-polaritons with eigenenergies $E_{+,n}^{(m)}$ in case of $\Delta > 0$, but is not visible in the spectrum as it is attached to the central

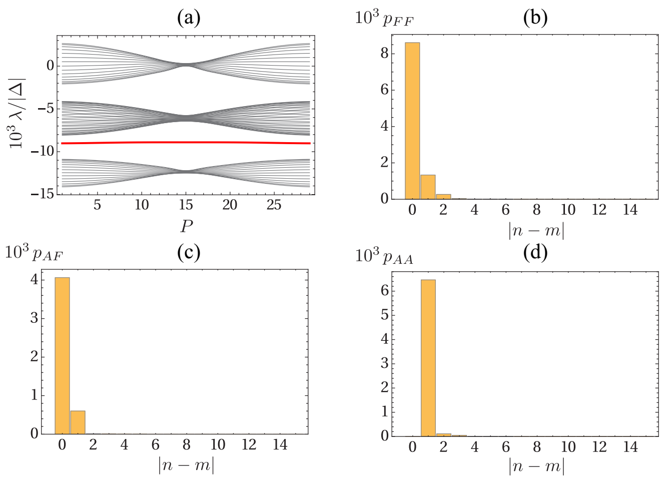

FIG. 1. (a) Normalized eigenvalues dependence on the quasimomentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears in the low-energy band gap. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$, and $J = 0.001\,|\Delta|$.

band. On the contrary, formation of single DPBP interchanges for $\Delta < 0$. Our determined $U(n)$ from [3] is mainly affected by the cavity field coupling strength g_m. By increasing g_m we increase the onsite repulsion $U(n)$ which directly enhances the interaction between the two dark-polaritons with eigenenergies $E^{(m)}_{-,n}$ at the same site μ with $\Delta > 0$. Thus, single DPBP is strengthened. Due to the interaction, the single DPBP lies inside the energy band gaps. Depending on the sign of the common single-photon detuning Δ, DPBP lies either in the high- or low-energy band gap. In the case $\Delta > 0$, DPBP lies in the low-energy band gap, whereas in the opposite case it resides within the high-energy band gap. In order to get some information on the inherent state composition of the single DPBP, we calculate, in line with [34], the joint probabilities

$$p_{FF} = \left|\langle\Psi^{(D)}_P|\frac{\hat{c}^\dagger_n\hat{c}^\dagger_m}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle\right|^2, \tag{33a}$$

$$p_{AF} = \left|\langle\Psi^{(D)}_P|\hat{c}^\dagger_n\hat{\sigma}^{(m)\dagger}_{gf}|\Phi_0\rangle\right|^2, \tag{33b}$$

$$p_{AA} = \left|\langle\Psi^{(D)}_P|\hat{\sigma}^{(n)\dagger}_{gf}\hat{\sigma}^{(m)\dagger}_{gf}|\Phi_0\rangle\right|^2 \tag{33c}$$

of finding pure photonic, photon-atom, and pure atomic excitations, respectively, in cavities at positions n and m. These excitations (pure photonic, pure atomic, and photon-atom) reflect the unique property of dark-polaritons in which the superposition of photonic and collective atomic excitations can be tuned by changing Ω in first place. In our case, we can not only change Ω, but also g_m as we use tunable cavities [41,42]. For a given value of quasimomentum P, all three joint probabilities only depend on the relative distance $|n - m|$ within the cavities.

In Figs. 1(b)–1(d) we present the joint probabilities for the single DPBP state of Fig. 1(a). We have chosen the number of coupled QED cavities to be $N = 30$, single-photon detuning $\Delta > 0$, cavity-mode coupling strength $g_m = 0.05\,\Delta$, the control field Rabi frequency $\Omega = 0.06\,\Delta$, intercavity photon hopping strength $J = 0.001\,\Delta$, and subspace $P = 1$. One can see that the DPBP excitations are well confined together, and all three possible excitation types coexist with roughly equal contributions. The state composition gradually changes by decreasing the contribution of double atomic excitations when P approaches the midrange values. This regime is roughly characterized by $g_m \approx \Omega$ and $(g_m^2 + \Omega^2)/|\Delta| > 5J$. The energy band gaps close when decreasing the ratio of $(g_m^2 + \Omega^2)/|\Delta|$ and J. At the same time, DPBP becomes relatively delocalized, similarly as in [34].

B. Dark-polariton bound pairs in the regime of compensated control field Stark shift

The tunability of our model enables not only the control of the shape of the energy bands, but also the emergence of an additional DPBP state. Namely, if the control field Stark shift is compensated by using an additional field, which couples the ground state $|f\rangle$ with some far-off-resonant excited state [39], another DPBP state appears in the formerly empty energy band gap. Such an add reflects in the removal of the parameter b from Eqs. (31). The energy bands in Fig. 2(a), shown for discrete and distinct quasimomenta P, are formed by the dark-polaritons in (24). This can be seen by solving Eqs. (31) for the intercavity hopping strength $J = 0$ and set the parameter b equal to zero. The onsite repulsion $U(n)$, which ensures the formation of the two DPBPs, is given as $U(n) = \frac{g_m\sqrt{n+1}\sqrt{g_m^2(n+1)+4\Omega^2} - g_m\sqrt{n}\sqrt{g_m^2 n+4\Omega^2}}{\Delta}$ for positive and negative common single-photon detuning Δ. Thus, the onsite repulsion $U(n)$ is invariant under the sign change of Δ. Distinctly to the DPBP formation under noncompensated control field Stark shift, the onsite repulsion $U(n)$ apart from the cavity field coupling strength g_m directly depends on the Rabi frequency Ω. This gives the opportunity to effectively control and enhance the interaction through g_m and Ω. Further, in Fig. 2(a) one can observe that each of the two energy band gaps now contains a single DPBP state (blue and red curves). We used the same parameter values as in Fig. 1, but with compensated control field Stark shift. In Figs. 2(b)–2(d) and Figs. 2(e)–2(g) we characterize the state composition of lower- and higher-energy DPBP states, respectively, by considering the joint probabilities as in the previous subsection. The DPBP in the lower-energy band gap is dominantly composed of two-photon excitation, while in the other DPBP state atom-photon excitation prevails. Moreover, higher-energy DPBP state is further apart from the outer energy band and it is relatively more localized than the lower-energy DPBP state. We checked that the same behavior persists for other values of quasimomentum P. Note that the described situation is for $\Delta > 0$, while it interchanges for $\Delta < 0$.

VI. QUANTUM MEMORY OF LIGHT IN A DARK-POLARITON BOUND PAIR

In the parameter regime where the common single-photon detuning Δ is negative and the cavity-atom coupling strength g_m is significantly larger than the control field Rabi frequency Ω, we have a single DPBP state which is the ground state

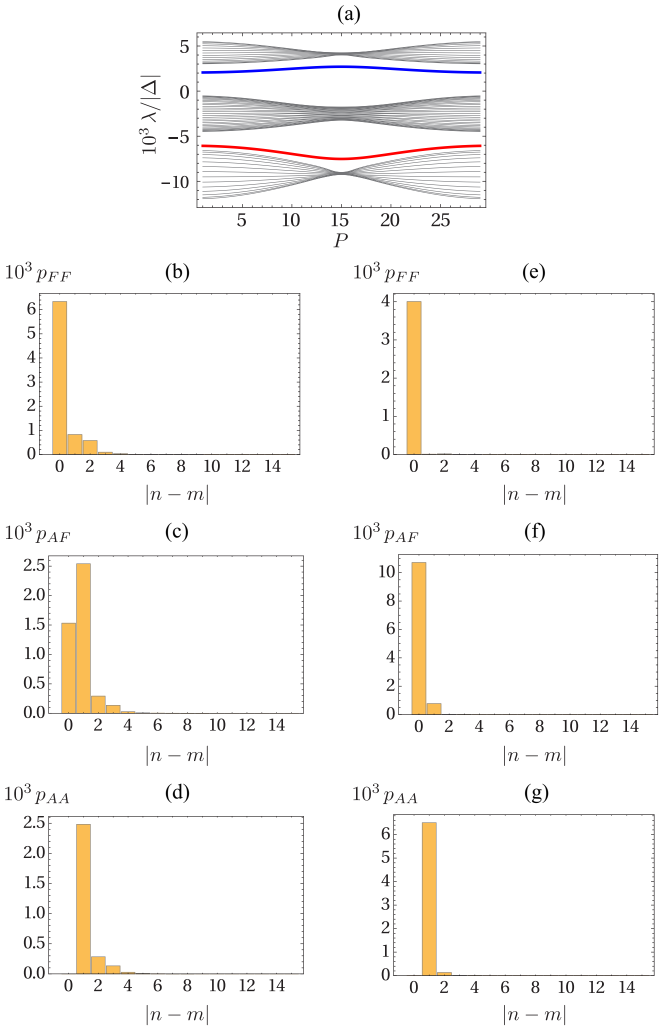

FIG. 2. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Two dark-polariton bound pair states (blue and red curves) appear in both energy band gaps. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to lower-energy DPBP state. (e)–(g) Joint probabilities for different types of double excitations associated to higher-energy DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$, and $J = 0.001\,|\Delta|$.

of the system. It is well separated from the rest of the energy spectrum when $g_m^2/|\Delta| \gtrsim 1.5\,J$. This is presented in Fig. 3(a). DPBP state composition, given in Figs. 3(b)–3(d) by the corresponding joint probabilities, reveals that the state is dominantly composed of combined atomic and photonic excitations which are localized in their relative spatial coordinates. Note that this DPBP state is of a completely different type than the ones found in the previous section.

It is important that this state also enables the storage of a single photon in the form of a collective atomic spin coherence excitation to which the other photon is closely bound. Namely, when $\Omega \to 0$ adiabatically, a DPBP becomes a pure combination of an atomic and photonic excitation. From this we can deduce that one photon remains attached

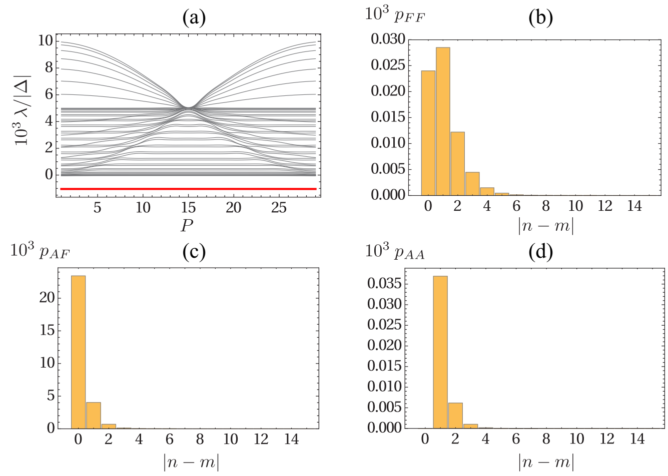

FIG. 3. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$, and $J = 0.00125\,|\Delta|$.

to the atomic spin coherence wave. This is reminiscent of the atom-photon molecule [36].

The state composition can be tuned by increasing the relative importance of the intercavity photon hopping, e.g., by increasing $|\Delta|$. This is achieved gradually for distinct values of quasimomentum, starting from the values $P = 1$, $N-1$ and proceeding towards the midrange values of P. Figure 4(a) shows the energy spectrum in such a case. For $P \in \{1, 3, N-3, N-1\}$ the DPBP state is predominantly composed of two-photon excitations which become delocalized in their relative

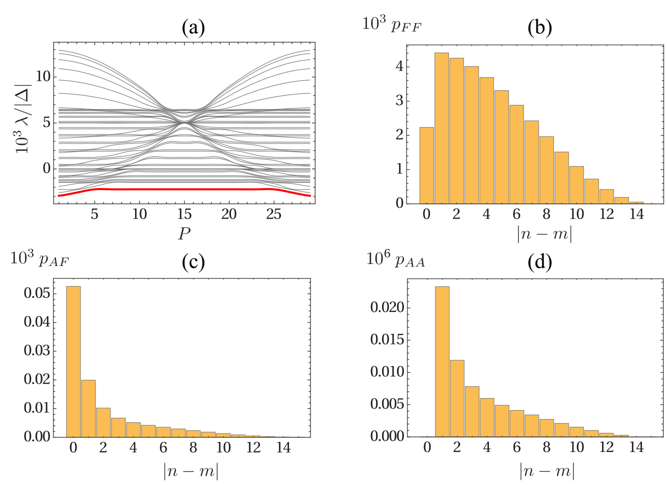

FIG. 4. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$, and $J = 0.002\,|\Delta|$.

spatial positions, as can be seen in Figs. 4(b)–4(d). The reason for such behavior can be traced back to the emergence of the avoided crossings of the ground state and the first excited state near the edges of the quasimomentum zone. The crossings shift towards the P-zone center as the influence of the photon hopping is being increased. For the quasimomentum values between the crossings, the DPBP state remains dominantly of the atom-photon type. In the case when the control field strength adiabatically reduces to zero, the DPBP state becomes of a pure two-photon type. Therefore, this corresponds to the retrieval procedure of the previously stored photon excitation.

VII. EXPERIMENTAL REALIZATION

Our model system is a large, one-dimensional mJCH chain of N-coupled QED cavities. In order to realize it, we need a structure, in which large arrays of coupled QED cavities can be realized. Promising candidates are photonic band-gap cavities [12,45]. It is manageable to produce and position them with high precision and in large numbers. A tempting alternative are photonic crystals as they offer the possibility of fabricating large arrays of QED cavities in one- or two-dimensional lattices as well as networks [46–48]. A third possibility would be the use of toroidal micro-QED cavities that are coupled via tapered optical fibers [49]. Single atoms, embedded in each QED cavity, are three-level atoms where the excited level is far detuned by the common single-photon detuning with respect to the two coupling fields. In real experiments, Cs and ultracold ^{87}Rb atoms have shown to be very suitable [44,50,51]. For Cs in a toroidal micro-QED cavity it has be shown that g_m in the strong-coupling regime reaches the value of ~ 50 MHz [50]. This fits pretty well with our theoretically chosen value for the formation of individual DPBP inside the energy band gaps, but also for the ground DPBP at $\Delta < 0$ with its potential use as a quantum memory for a single photon.

VIII. CONCLUSION

To summarize, we have derived a modified Jaynes-Cummings model from the bare model under two conditions: (i) two-photon Raman resonance of the cavity mode and classical control field, (ii) common single-photon detuning $|\Delta| \gg g_m,\Omega$. We have shown that the eigenstates on one hand depend on the common single-photon detuning and, on the other hand, their composition differs with respect to the control field Stark shift. Moreover, we have extended the modified Jaynes-Cummings model to a modified Jaynes-Cummings-Hubbard model where an array of N-coupled QED cavities, each having an embedded single three-level atom, is considered. The modified Jaynes-Cummings-Hubbard model supports DPBPs. The formation of two different species of spatially localized dark-polariton bound pairs (DPBPs) has been elaborated when there are exactly two excitations in the system. It was shown that the onsite repulsion $U(n)$ as a consequence of the Kerr nonlinearity represents the attractive force between interacting dark-polaritons and enables the existence of DPBP states. Furthermore, it is demonstrated that our model system offers a high degree of tunability that can affect both quantitative and qualitative behavior. In particular, the number of DPBP states can be controlled by (not) compensating the Stark shift due to the control field. Further, in the regime when cavity-atom coupling overwhelms the influence of the control field, and the common single-photon detuning of the fields is negative, we obtained a ground DPBP eigenstate on which the storage and readout of a single photon can be effectively performed. An experimental realization is proposed for our model system. Cs atom has been mentioned as a promising candidate as its value of the cavity-mode coupling strength g_m fits very well with our theoretically chosen and determined one. We expect that future investigations of this kind of system under different settings, i.e., with distinct and alternating hopping strengths between the cavities, in the presence of disorder, or in two-dimensional lattice configurations, may lead to various effects and rich physics.

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