Macroscopic Superpositions as Quantum Ground States

Borivoje Dakić[1,2] and Milan Radonjić[2,3]
[1]Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, A-1090 Vienna, Austria
[2]Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria
[3]Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia
(Received 24 June 2016; revised manuscript received 20 June 2017; published 1 September 2017)

We study the question of what kind of a macroscopic superposition can(not) naturally exist as a ground state of some gapped local many-body Hamiltonian. We derive an upper bound on the energy gap of an arbitrary physical Hamiltonian provided that its ground state is a superposition of two well-distinguishable macroscopic "semiclassical" states. For a large class of macroscopic superposition states we show that the gap vanishes in the macroscopic limit. This in turn shows that preparation of such states by simple cooling to the ground state is not experimentally feasible and requires a different strategy. Our approach is very general and can be used to rule out a variety of quantum states, some of which do not even exhibit macroscopic quantum properties. Moreover, our methods and results can be used for addressing quantum marginal related problems.

DOI: 10.1103/PhysRevLett.119.090401

Introduction.—Ever since Schrödinger's cat gedanken experiment [1] the question of whether a macroscopic system can be found in a quantum superposition state remains unanswered. Various attempts were made to address our inability to detect macroscopic quantum superpositions. Decoherence-type arguments are commonly employed in which one advocates that the quantumness of a macroscopic system is lost due to interactions with a noisy environment [2]. Alternatively, it was indicated that classical behavior can emerge because our measurements suffer from limited resolution or limited sensitivity [3–5]. Moreover, various spontaneous collapse models introduce a stochastic nonlinear modification of the Schrödinger equation that causes macroscopic superpositions to quickly appear as classical, while giving the same experimental predictions as quantum theory in the microscopic regime [6].

Naturally, the boundary between the quantum and classical realms should be explored by experiments [7–9]. In recent decades, typical quantum features have been demonstrated in large molecules [10,11], hundreds of photons [12,13], superconducting circuits [14,15], micromechanical oscillators [16,17], and fragmented Bose condensates [18,19]. Nonetheless, quantum superpositions of truly macroscopic objects remain an uncharted territory that will hopefully be revealed by future experiments.

Recently, different measures have been proposed to quantify macroscopicity of quantum states [20–30]. The literature about this topic is diverse and various measures are mutually compared in Refs. [20,21] and summarized in Ref. [22]. Generally speaking, a macroscopic quantum state (MQS) is a state capable of displaying macroscopic quantum effects that can be utilized to validate quantum mechanics (against classical theories) on a macroscopic scale. An important task is the identification of a characteristic parameter that measures the "size" or "macroscopicity" of a certain quantum state [7], such as the characteristic energy, mass, number of elementary constituents, etc. Here we focus on the case of macroscopically large number of particles N that interact via a local Hamiltonian.

An important subclass of MQS are macroscopic superpositions (MS): states of the type $|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$, where $|\psi_{1,2}\rangle$ are macroscopically well-distinguishable states. However, such a definition is not operational as there are infinitely many decompositions of the kind $|\psi_1\rangle + |\psi_2\rangle$ and it might not be clear how to unambiguously identify the "semiclassical" components of the MS. Therefore, we define MS with respect to a measurement of an additive (collective) observable [20,21,23,28,29]. A pure state $|\psi\rangle$ is MS if a measurement of some additive observable $\hat{S}$ can sharply distinguish the semiclassical states that constitute MS; e.g., the distribution of eigenvalues of $\hat{S}$ exhibits two well-resolvable regions (see Fig. 1). Our main focus here is on (i) the possibility of the natural appearance of such states as unique ground states of macroscopic quantum systems and, consequently, (ii) the feasibility of preparing MS by simply cooling down such systems. The latter might be achievable provided that the system has a unique MS ground state; i.e., there is a finite energy gap in the thermodynamic limit. In this respect, it was proven that no MS of "locally distinguishable" states can be the unique ground state of N spins described by a local Hamiltonian whose energy gap is at least $O(1/\mathrm{poly}(N))$ [31]. Conversely, numerical evidence was given in Ref. [32] that the energy gap of a certain N-qubit Hamiltonian decays exponentially fast in the macroscopic limit when its ground state actually is MS. Moreover, relation between the

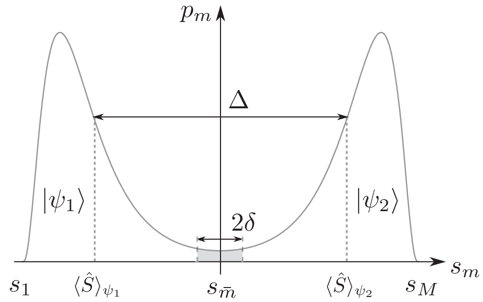

FIG. 1. The distribution p_m of eigenvalues s_m of an additive observable $\hat{S}$ for a MS state $|\psi_1\rangle + |\psi_2\rangle$. A continuous curve is used for aesthetic purposes. The distribution has two well-resolved regions (left and right from the separation point $s_{\bar{m}}$) each corresponding to the superimposed semiclassical states $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. The distance between the regions is $\Delta \coloneqq |\langle\hat{S}\rangle_{\psi_2} - \langle\hat{S}\rangle_{\psi_1}|$. The separation probability related to the finite-sized shaded segment $|s - s_{\bar{m}}| \le \delta = O(N^0)$ should be vanishing in the macroscopic limit $N \to \infty$.

spectral gap and ground state properties of spin lattice systems was studied in Refs. [33,34].

We provide a simple sufficient criterion enforcing the energy gap to vanish in the thermodynamic limit for a very general class of ground states of local many-body Hamiltonians. The most important feature of our approach is an operational method to identify semiclassical states that constitute the macroscopic superposition. We show that in many cases local Hamiltonians are not capable of linking such states, so that the corresponding MS can only represent a degenerate ground state in the macroscopic limit. Our main theorem provides an interesting relation between the energy gap and the order of interaction (i.e., the number K in the case of a K-body interaction). Therefore, one may derive the lowest order of interaction for which a given MS might be a unique ground state. We discuss our results in the context of different physical systems and various proposals for preparation of MS. Furthermore, we show that a certain class of states that are not even considered to be macroscopically quantum (e.g., W states) cannot naturally exist as ground states of gapped local Hamiltonians. Finally, we demonstrate that the methods and results derived here are relevant for quantum marginal related problems.

Preliminaries.—Let us consider a system of N interacting particles described by a K-local Hamiltonian $\hat{H} = \sum_{(i_1,i_2,\ldots,i_K)\in\mathcal{I}_N^{(K)}} \hat{H}_{i_1 i_2 \ldots i_K}$, where $\hat{H}_{i_1 i_2 \ldots i_K}$ is the contribution due to interaction between particles $i_1, i_2, \ldots, i_K$ and $\mathcal{I}_N^{(K)}$ is the set of all K-tuples of N interacting particles. We call K the order of interaction. For instance, usual physical interactions are pairwise with the order $K = 2$.

We begin with the following general lemma:

Lemma.—Let a Hamiltonian $\hat{H}$ have a unique ground state of the form $|\psi\rangle = a_1|\psi_1\rangle + a_2|\psi_2\rangle$, where $|\psi_{1,2}\rangle$ are normalized, $\langle\psi_2|\psi_1\rangle = \lambda$ and $a_1, a_2 > 0$. Then the energy gap ΔE satisfies the inequality

$$\Delta E \le \frac{|\langle\psi_2|\hat{H}|\psi_1\rangle - \lambda E_0|}{a_1 a_2 (1 - |\lambda|^2)}, \tag{1}$$

where E_0 denotes the ground state energy (see Supplemental Material [35] for the proof).

Without loss of generality we set $E_0 = 0$ hereafter. We start our analysis with the simple observation that the energy gap is essentially upper bounded by a magnitude of the matrix element $\langle\psi_2|\hat{H}|\psi_1\rangle = H_{21}$ [assuming that the overlap λ is vanishingly small and $a_{1,2} = O(N^0)$ when $N \to \infty$]. Therefore, the system cannot have a finite gap in the macroscopic limit if H_{21} is vanishing when $N \to \infty$.

An archetypal example of MS is a so-called GHZ state [44], closely related to an original Schrödinger's proposal as it is a superposition of two macroscopically distinct states of N particles, i.e., $|\psi\rangle \propto |\varphi_1\rangle^{\otimes N} + |\varphi_2\rangle^{\otimes N}$. The states $|\varphi_{1,2}\rangle$ are normalized with the fixed nonzero overlap $\omega = |\langle\varphi_1|\varphi_2\rangle| < 1$. Here, one can naturally identify the two constituents $|\psi_{1,2}\rangle = |\varphi_{1,2}\rangle^{\otimes N}$ with exponentially small overlap $|\lambda| = \omega^N$ and $a_{1,2} \overset{N\to\infty}{\to} 1/\sqrt{2}$. Denote by $H_{21}^{[K]}$ the maximal magnitude of all matrix elements $\langle\varphi_2|^{\otimes K} \hat{H}_{i_1,i_2,\ldots,i_K} |\varphi_1\rangle^{\otimes K}$. The value of $H_{21}^{[K]}$ does not scale with N and solely depends on the nature of the interaction. It is not difficult to see that

$$|H_{21}| \le |\mathcal{I}_N^{(K)}| \omega^{N-K} H_{21}^{[K]} \le \binom{N}{K} \omega^{N-K} H_{21}^{[K]}, \tag{2}$$

since for K fixed the total number of interaction terms grows at most polynomially with N, i.e., $|\mathcal{I}_N^{(K)}| \le \binom{N}{K} = O(N^K)$. Therefore, we conclude that the energy gap vanishes exponentially fast when $N \to \infty$, as long as the order of interaction is fixed. In other words, all the states $|\psi(\alpha)\rangle \propto |\varphi_1\rangle^{\otimes N} + e^{i\alpha}|\varphi_2\rangle^{\otimes N}$ give the same energy in the thermodynamic limit and the ground state becomes at least doubly degenerate. Consequently, cooling down the system towards zero temperature will result in a classical mixture $\frac{1}{2}|\psi(0)\rangle\langle\psi(0)| + \frac{1}{2}|\psi(\pi)\rangle\langle\psi(\pi)|$. In order to make the energy gap finite in the thermodynamic limit, it is necessary that the order of interaction K grows with the number of particles N, which is usually considered nonphysical. This reasoning can be trivially extended to a finite sum $|\varphi_1\rangle^{\otimes N} + \cdots + |\varphi_n\rangle^{\otimes N}$ of macroscopically distinguishable states, i.e., $\langle\varphi_i|\varphi_j\rangle = O(N^0)$ when $i \ne j$. In the Supplemental Material [35] we show that the same result holds for a more general class of states, i.e., the superpositions of locally distinguishable states that have been considered in literature as a natural generalization of the GHZ-like states [20,24].

Whereas the previous examples are fairly easy to grasp, as the superimposed states are identifiable by definition, such a clean prescription is not *a priori* available for arbitrary MQS. Therefore, we continue our analysis by invoking a measurement of some collective observable $\hat{S}$ that should serve as a reference point to identify $|\psi_{1,2}\rangle$.

Consider a system of N particles in a total Hilbert space $\mathcal{H}^N = \otimes_{i=1}^N \mathcal{H}_i$, with $\dim(\mathcal{H}_i) = d$. Let $\hat{S} = \sum_{i=1}^N \hat{S}_i$ be an additive observable. The single-particle operators satisfy $\hat{S}_i|\sigma_i,\mu_i\rangle_i = \sigma_i|\sigma_i,\mu_i\rangle_i$, where $\sigma_i \in \{\varsigma_1 < \varsigma_2 < \ldots < \varsigma_\ell\}$ and $2 \le \ell \le d$, while $\mu_i = 1,\ldots,\mu(\sigma_i)$ enumerate the degeneracies obeying $\sum_{l=1}^\ell \mu(\varsigma_l) = d$. We denote the different eigenvalues of $\hat{S}$ by $s_1 < s_2 < \ldots < s_M$, where $s_m = \sum_{l=1}^\ell n_{m,l}\varsigma_l$, $n_{m,l} \in \mathbb{N}_0$ and $\sum_{l=1}^\ell n_{m,l} = N$. Clearly, $s_1 = N\varsigma_1$ and $s_M = N\varsigma_\ell$. The states $|\boldsymbol{\sigma},\boldsymbol{\mu}\rangle = \otimes_{i=1}^N |\sigma_i,\mu_i\rangle_i$ constitute a complete basis in $\mathcal{H}^N$, i.e., $\sum_{\boldsymbol{\sigma}}\sum_{\boldsymbol{\mu}} |\boldsymbol{\sigma},\boldsymbol{\mu}\rangle\langle\boldsymbol{\sigma},\boldsymbol{\mu}| = \mathbb{1}$, where $\boldsymbol{\sigma} = (\sigma_1,\sigma_2,\ldots,\sigma_N)$ and $\boldsymbol{\mu} = (\mu_1,\mu_2,\ldots,\mu_N)$. This yields a decomposition

$$|\psi\rangle = \sum_{\boldsymbol{\sigma}}\sum_{\boldsymbol{\mu}} |\boldsymbol{\sigma},\boldsymbol{\mu}\rangle\langle\boldsymbol{\sigma},\boldsymbol{\mu}|\psi\rangle = \sum_{m=1}^M \sqrt{p_m}\,|s_m\rangle, \quad (3)$$

where $\hat{S}|s_m\rangle = s_m|s_m\rangle$, and $|s_m\rangle$ contains all the terms from the multisums such that $\sum_{i=1}^N \sigma_i = s_m$. The numbers $p_m \ge 0$ correspond to the probabilities of obtaining the value s_m when measuring the observable $\hat{S}$ in the state $|\psi\rangle$, hence, $\sum_{m=1}^M p_m = 1$.

Now, if the state $|\psi\rangle$ is MS of two states $|\psi_1\rangle$ and $|\psi_2\rangle$, then we expect that the probability distribution $\mathcal{P}_\psi = \{p_m\}_{m=1}^M$ has two distinguishable regions with corresponding probabilities of the order $O(N^0)$ and with vanishingly small probability within the finite-sized bordering segment around some eigenvalue $s_{\bar{m}}$ of $\hat{S}$ (see Fig. 1). Those regions should precisely be related to the semiclassical constituents of the state $|\psi\rangle$. The distance between the regions $\Delta \coloneqq |\langle\hat{S}\rangle_{\psi_2} - \langle\hat{S}\rangle_{\psi_1}|$ is closely related to the fluctuation of the observable $\hat{S}$ in the state $|\psi\rangle$ and it is commonly assumed that MS displays $\Delta = O(N)$ [20,29,30]. However, we will address quantum states from another aspect, which will render our main result independent of Δ. Namely, the prime quantity in our analysis is the separation probability $P_\psi(|s - s_{\bar{m}}| \le \delta)$, i.e., the probability of finding the result s, when measuring $\hat{S}$, within a tiny segment of size $2\delta = O(N^0)$ centered at the separation point $s_{\bar{m}}$. We will provide an upper bound on the energy gap, which essentially depends on the separation probability and the order of interaction. Thus, the interplay between the two will have a crucial role in vanishing of the gap.

Next, we will make use of $s_{\bar{m}}$ to express the ground state in the form of a superposition

$$|\psi\rangle = a_1|\psi_1\rangle + a_2|\psi_2\rangle, \quad (4)$$

with

$$a_1|\psi_1\rangle = \sum_{m=1}^{\bar{m}-1} \sqrt{p_m}\,|s_m\rangle, \quad a_2|\psi_2\rangle = \sum_{m=\bar{m}}^{M} \sqrt{p_m}\,|s_m\rangle, \quad (5)$$

where $a_1 = (p_1 + \cdots + p_{\bar{m}-1})^{1/2}$, $a_2 = (p_{\bar{m}} + \cdots + p_M)^{1/2}$, and, presumably, $a_{1,2} = O(N^0)$. By construction, one has $\langle\psi_2|\psi_1\rangle = 0$. We will employ the introduced separation to derive an upper estimate of the energy gap.

Let us suppose that the Hamiltonian of the physical system is 2-local, i.e., $\hat{H} = \sum_{(i,j)\in\mathcal{I}_N^{(2)}} \hat{H}_{ij}$, where $\hat{H}_{ij}$ represents pairwise interaction between particles i and j and $\mathcal{I}_N^{(2)}$ is the set of pairs of interacting particles. Obviously, the number of interaction terms in the Hamiltonian satisfies $|\mathcal{I}_N^{(2)}| \le N(N-1)/2 = O(N^2)$. The magnitude of the matrix element in the inequality (1) can be estimated in order to obtain the following central result:

Theorem.—Under the assumptions given in the text, the energy gap of the system is bounded as

$$\Delta E \le \frac{|\mathcal{I}_N^{(2)}|}{2a_1^2a_2^2} \max_{(i,j)\in\mathcal{I}_N^{(2)}} \|\hat{H}_{ij}\| \cdot P_\psi(|s - s_{\bar{m}}| \le 2\delta_\varsigma), \quad (6)$$

where $\max_{(i,j)\in\mathcal{I}_N^{(2)}} \|\hat{H}_{ij}\|$ sets the characteristic energy scale (independent of N) and $\delta_\varsigma = \varsigma_\ell - \varsigma_1$. Here, $\|\cdot\|$ denotes the operator spectral norm. The complete proof is given in the Supplemental Material [35].

The bound (6) is valid for any $s_{\bar{m}}$, which has been arbitrary up to now. Clearly, one should select $\hat{S}$ and the corresponding $s_{\bar{m}}$ so that $P_\psi(|s - s_{\bar{m}}| \le 2\delta_\varsigma)$ vanishes as fast as possible for $N \to \infty$. In the previously discussed GHZ-like case, the separation probability scales as $\exp[-O(N)]$ and the energy gap vanishes exponentially fast with N. Furthermore, it is clear that for any state exhibiting $P_\psi = o(1/N^2)$ the gap will vanish in the thermodynamic limit and the state can only represent a degenerate ground state. In general, such a state does not necessarily display anomalous fluctuation of $\hat{S}$. One can even find examples where $\Delta = O(N^0)$ [such as $|\psi\rangle = (|s_{m_1}\rangle + |s_{m_2}\rangle)/\sqrt{2}$, where $s_{m_1} = s_{\bar{m}} - 2\delta$ and $s_{m_2} = s_{\bar{m}} + 2\delta$, with $\delta > \delta_\varsigma$]. Conversely, when the system features a finite energy gap, the relation (6) puts a lower bound $P_\psi(|s - s_{\bar{m}}| \le 2\delta_\varsigma) \ge O(1/N^2)$ for any gapped 2-local Hamiltonian and arbitrary observable $\hat{S}$.

The appearance of probabilities corresponding to the interval of size $4\delta_\varsigma$ centered at $s_{\bar{m}}$ is a direct consequence of the 2-local nature of the Hamiltonian. We note that the Theorem could easily be generalized for arbitrary K-local Hamiltonians. In that case, one would consider the set $\mathcal{I}_N^{(K)}$ of K-tuples of interacting particles, for which $|\mathcal{I}_N^{(K)}| \le \binom{N}{K} = O(N^K)$, and the corresponding estimate of the gap would involve the probability $P_\psi(|s - s_{\bar{m}}| \le 2K\delta_\varsigma)$. Thus, for a gapped K-local Hamiltonian we conclude that the best possible separation probability one can achieve for a ground state is asymptotically lower bounded by $O(1/N^K)$. Consequently, all the states exhibiting the scaling $P_\psi = o(1/N^K)$ are excluded as possible unique ground states.

Various examples.—Our general result nicely complies with the investigation of ground states of various physical systems. For example, a twofold fragmented condensate of interacting bosons trapped in a single well [18] features a doubly degenerate ground state, in the thermodynamic limit. It was shown in Ref. [19] that in the appropriate Fock space basis the corresponding ground states are identical to the photon cat states. In accordance with our findings, the proposed preparation of these states requires other means than simple cooling, i.e., the rapid sweep of interaction couplings [45]. Another example is a one-dimensional array of circuit quantum electrodynamic (cQED) systems in the ultrastrong cavity-qubit coupling regime [46]. The authors showed that the photon hopping between cavities can be mapped to the Ising interaction between the lowest two levels of individual cQED of the chain. Based on the mapping, they found two nearly degenerate GHZ-type ground states with energy splitting exponentially small in the system size. Again, this is in perfect agreement with our results. Moreover, we mention the study of a bosonic Josephson junction made of N ultracold and dilute atoms confined by a quasi-one-dimensional double-well potential within the two-site Bose-Hubbard model framework [47]. Detailed treatment showed that the ground state of the system evolves towards NOON state when increasing attractive interatomic interaction. The estimated gap between two lowest energy states vanishes exponentially with N, in full compliance with our considerations. Our work also nicely agrees with Ref. [48] where the possibility of creating many-particle catlike states was examined for a Bose-Einstein condensate trapped in a double-well potential. It was discussed in detail that creating cat states via adiabatic manipulation of the many-body ground state is experimentally unfeasible due to the fact that the end state is nearly degenerate with the first-excited state; hence, such a process would require an exponentially long time. This difficulty was surpassed by proposing to exploit dynamic evolution following a sudden flipping of the sign of the atomic interaction, accomplished via Feshbach resonance technique [49]. Finally, we mention that our treatment assumes a close correspondence between the macroscopicity of the system and the number of its constituents. However, the macroscopicity might be related to other quantities and only weakly depend on the system size. SQUID systems, which were proposed as good candidates to host the "genuine" MS [7], are a paramount example of that. Although our results are not directly applicable to such a case, in the Supplemental Material [35] we provide a discussion of SQUIDs showing some similarities with our findings.

Our generic analysis demonstrates that more sophisticated experimental techniques are needed for the preparation of a variety of macroscopic superpositions in the thermodynamic limit. This may require some form of dynamical driving of a system, as in the mentioned examples, advanced matter-wave interferometric approaches [50] or use of demanding postselection techniques [51].

Furthermore, we present an example to demonstrate that our results can be used to address the states that are more general than MQS (see Supplemental Material [35]). Consider a lattice model of N spin-$1/2$ particles interacting with the fixed number of neighbors. Thus, one has $d=2$, $\ell=2$, $\delta_\varsigma=1$, and $|\mathcal{I}_N^{(2)}|=O(N)$. In order to prove that the model becomes gapless in the limit $N\to\infty$, one has to find an appropriate additive observable $\hat{S}$ for which the ground-state-related separation probability vanishes as $o(1/N)$. Collective states that naturally appear in spin systems are the Dicke states [52] $|j,m\rangle$ $(m=-j,\ldots,j)$, where $j=N/2$. They are permutation invariant and satisfy $\hat{J}^2|j,m\rangle=j(j+1)|j,m\rangle$ and $\hat{J}_z|j,m\rangle=m|j,m\rangle$. All Dicke states are unique ground states of some fully 2-local, gapped Hamiltonian for which $|\mathcal{I}_N^{(2)}|=N(N-1)/2$ (all the particles mutually interact pair wisely, such as indistinguishable particles) [31]. However, such Hamiltonians do not correspond to the present case. Therefore, we will show that, for example, an N-qubit W state $|j,j-1\rangle$, which represents the case of symmetrically distributed one-spin excitations, cannot be a unique ground state of any considered spin-lattice model. First, we find the appropriate collective observable to be $\hat{J}_x$. Let $|j,m\rangle_x$ $(m=-j,\ldots,j)$ be the common eigenbasis of $\hat{J}^2$ and $\hat{J}_x$. The related probability distribution is $p_m=|\langle j,j-1|j,m\rangle_x|^2$ (see Fig. 1 in the Supplemental Material [35]), $s_m=m$, and we choose $s_{\bar{m}}=0$ for j integer or $s_{\bar{m}}=1/2$ for j half-integer. As presented in the Supplemental Material [35], we find

$$p_m=\frac{2m^2}{2^{2j}j}\binom{2j}{j+m}\sim\frac{2m^2}{\sqrt{\pi}j^{3/2}}, \tag{7}$$

where the last asymptotic behavior holds for fixed m and $j\to\infty$. We conclude that the separation probability $P_\psi(|s-s_{\bar{m}}|\leq 2)$ scales as $O(1/j^{3/2})$, i.e., $O(1/N^{3/2})$. Thus, the W state can only be a degenerate ground state of the arbitrary spin-lattice model considered here. Moreover, the distance between the two peaks has sublinear asymptotic scaling $\sim\sqrt{2N}$. Hence, the W state is an example of a state that is not even a MQS according to the anomalous fluctuation criterion, but is nevertheless amenable to our present analysis.

Finally, our results can be naturally related to quantum marginal problem [53,54]. There, the main task is to check whether or not a given set of marginal states $\hat{\boldsymbol{\rho}}=(\hat{\rho}_{s_1},\hat{\rho}_{s_2},\ldots)$ can be extended to some N-particle quantum state $\hat{\varrho}^{[N]}$, i.e., $\hat{\rho}_{s_k}=\mathrm{Tr}_{\bar{s}_k}\hat{\varrho}^{[N]}$, where s_k denotes a subset of N particles. The set of all representable marginals $\hat{\boldsymbol{\rho}}$ is convex and completely characterized by its extremal points (for finite-dimensional systems); therefore, their identification is of great importance. On the other hand, the set of extremal points is in unique correspondence to the set of N-particle nondegenerate ground states of the local Hamiltonians [54]. Namely, for a given Hamiltonian $\hat{H}=\sum_k\hat{H}_{s_k}$, where $\hat{H}_{s_k}$ denotes local Hamiltonian acting on the subset of particles s_k, we have $E=\mathrm{Tr}(\hat{\varrho}^{[N]}\hat{H})=\sum_k\mathrm{Tr}_{s_k}(\hat{\rho}_{s_k}\hat{H}_{s_k})=\mathrm{Tr}(\hat{\boldsymbol{\rho}}\hat{\boldsymbol{H}})$, where $\hat{\boldsymbol{H}}=(\hat{H}_{s_1},\hat{H}_{s_2},\ldots)$.

Thus, the energy E is a linear functional on the set of all representable marginals $\hat{\rho}$ and it reaches its extreme values on the set of nondegenerate ground states. Our criterion (6) implies that a large class of degenerate ground states (in the thermodynamic limit) has the set of marginals that cannot be extremal.

Summary and outlook.—In this Letter we provided a powerful generic method to analyze the possibility for ground states of gapped many-body quantum systems to be superpositions of macroscopically distinct quantum states. We have ruled out a large class of quantum states that cannot be prepared by simply cooling macroscopic quantum systems that exhibit interactions involving some finite number of their constituents. For such a state, we require that the separation probability, related to the small segment around the separation point between its two semiclassical components, vanishes sufficiently fast in the thermodynamic limit. We expect our results to be valuable for future experiments aiming at preparing quantum states that exhibit macroscopic quantum properties. Furthermore, we have shown that our study is relevant for quantum marginal problem.

We thank Časlav Brukner, Nikola Paunković, and Jacques Pienaar for helpful comments and gratefully acknowledge financial support from the European Commission through the projects RAQUEL (No. 323970) and QUCHIP (No. 641039).

[1] E. Schrödinger, Die gegenwärtige Situation in der Quantenmechanik, Naturwissenschaften **23**, 807 (1935).

[2] W. H. Zurek, Decoherence, einselection, and the quantum origins of the classical, Rev. Mod. Phys. **75**, 715 (2003).

[3] J. Kofler and Č. Brukner, Classical World Arising out of Quantum Physics under the Restriction of Coarse-Grained Measurements, Phys. Rev. Lett. **99**, 180403 (2007).

[4] S. Raeisi, P. Sekatski, and C. Simon, Coarse Graining Makes It Hard to See Micro-Macro Entanglement, Phys. Rev. Lett. **107**, 250401 (2011).

[5] P. Sekatski, N. Gisin, and N. Sangouard, How Difficult Is It to Prove the Quantumness of Macroscropic States?, Phys. Rev. Lett. **113**, 090403 (2014).

[6] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Models of wave-function collapse, underlying theories, and experimental tests, Rev. Mod. Phys. **85**, 471 (2013).

[7] A. J. Leggett, Testing the limits of quantum mechanics: motivation, state of play, prospects, J. Phys. Condens. Matter **14**, R415 (2002).

[8] M. Arndt and K. Hornberger, Testing the limits of quantum mechanical superpositions, Nat. Phys. **10**, 271 (2014).

[9] T. Farrow and V. Vedral, Classification of macroscopic quantum effects, Opt. Commun. **337**, 22 (2015).

[10] O. Nairz, M. Arndt, and A. Zeilinger, Quantum interference experiments with large molecules, Am. J. Phys. **71**, 319 (2003).

[11] S. Eibenberger, S. Gerlich, M. Arndt, M. Mayor, and J. Tüxen, Matter-wave interference of particles selected from a molecular library with masses exceeding 10000 amu, Phys. Chem. Chem. Phys. **15**, 14696 (2013).

[12] N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R. T. Thew, and N. Gisin, Displacement of entanglement back and forth between the micro and macro domains, Nat. Phys. **9**, 545 (2013).

[13] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, and C. Simon, Observation of micro-macro entanglement of light, Nat. Phys. **9**, 541 (2013).

[14] C. H. van der Wal *et al.*, Quantum Superposition of Macroscopic Persistent-Current States, Science **290**, 773 (2000).

[15] J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Quantum superposition of distinct macroscopic states, Nature (London) **406**, 43 (2000).

[16] A. D. O'Connell *et al.*, Quantum ground state and single-phonon control of a mechanical resonator, Nature (London) **464**, 697 (2010).

[17] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, Cavity cooling of an optically levitated submicron particle, Proc. Natl. Acad. Sci. U.S.A. **110**, 14180 (2013).

[18] P. Bader and U. R. Fischer, Fragmented Many-Body Ground States for Scalar Bosons in a Single Trap, Phys. Rev. Lett. **103**, 060402 (2009).

[19] U. R. Fischer and M.-K. Kang, "Photonic" Cat States from Strongly Interacting Matter Waves, Phys. Rev. Lett. **115**, 260404 (2015).

[20] F. Fröwis and W. Dür, Measures of macroscopicity for quantum spin systems, New J. Phys. **14**, 093039 (2012).

[21] F. Fröwis, N. Sangouard, and N. Gisin, Linking measures for macroscopic quantum states via photon-spin mapping, Opt. Commun. **337**, 2 (2015).

[22] H. Jeong, M. Kang, and H. Kwon, Characterizations and quantifications of macroscopic quantumness and its implementations using optical fields, Opt. Commun. **337**, 12 (2015).

[23] G. Björk and P. G. L. Mana, A size criterion for macroscopic superposition states, J. Opt. B **6**, 429 (2004).

[24] J. I. Korsbakken, K. B. Whaley, J. Dubois, and J. I. Cirac, Measurement-based measure of the size of macroscopic quantum superpositions, Phys. Rev. A **75**, 042106 (2007).

[25] F. Marquardt, B. Abel, and J. von Delft, Measuring the size of a quantum superposition of many-body states, Phys. Rev. A **78**, 012109 (2008).

[26] S. Nimmrichter and K. Hornberger, Macroscopicity of Mechanical Quantum Superposition States, Phys. Rev. Lett. **110**, 160403 (2013).

[27] P. Sekatski, N. Sangouard, and N. Gisin, Size of quantum superpositions as measured with classical detectors, Phys. Rev. A **89**, 012116 (2014).

[28] A. Shimizu and T. Miyadera, Stability of Quantum States of Finite Macroscopic Systems against Classical Noises, Perturbations from Environments, and Local Measurements, Phys. Rev. Lett. **89**, 270403 (2002).

[29] A. Shimizu and T. Morimae, Detection of Macroscopic Entanglement by Correlation of Local Observables, Phys. Rev. Lett. **95**, 090401 (2005).

[30] C.-W. Lee and H. Jeong, Quantification of Macroscopic Quantum Superpositions within Phase Space, Phys. Rev. Lett. **106**, 220401 (2011).

[31] F. Fröwis, M. van den Nest, and W. Dür, Certifiability criterion for large-scale quantum systems, New J. Phys. **15**, 113011 (2013).
[32] T. Morimae, Low-temperature coherence properties of Z_2 quantum memory, Phys. Rev. A **81**, 022304 (2010).
[33] T. Kuwahara, I. Arad, L. Amico, and V. Vedral, Local reversibility and entanglement structure of many-body ground states, Quantum Sci. Technol. **2**, 015005 (2017).
[34] T. Kuwahara, Asymptotic behavior of macroscopic observables in generic spin systems, J. Stat. Mech. (2016) 053103.
[35] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.090401 for the proofs, further examples and discussion. Additional Refs. [36–43] are also included therein.
[36] J. Watrous, *Theory of Quantum Information* (University of Waterloo, Waterloo, 2016).
[37] L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics: Theory and Application* (Addison-Wesley, Reading, MA, 1981).
[38] C. L. Frenzen and R. Wong, A uniform asymptotic expansion of the Jacobi polynomials with error bounds, Can. J. Math. **37**, 979 (1985).
[39] R. Wong and Y.-Q. Zhao, Uniform asymptotic expansion of the Jacobi polynomials in a complex domain, Proc. R. Soc. A **460**, 2569 (2004).
[40] X.-X. Bai and Y.-Q. Zhao, A uniform asymptotic expansion for Jacobi polynomials via uniform treatment of Darboux's method, J. Approx. Theory **148**, 1 (2007).
[41] U. Eckern, G. Schön, and V. Ambegaokar, Quantum dynamics of a superconducting tunnel junction, Phys. Rev. B **30**, 6419 (1984).
[42] M. Robnik, L. Salasnich, and M. Vranicar, WKB Corrections to the Energy Splitting in Double Well Potentials, Nonlin. Phenom. Complex Syst. (Minsk) **2**, 49 (1999).
[43] A. L. DiRienzo, Ph.D. thesis, The University of Arizona, 1982.
[44] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), p. 69.
[45] U. R. Fischer, K.-S. Lee, and B. Xiong, Emergence of a new pair-coherent phase in many-body quenches of repulsive bosons, Phys. Rev. A **84**, 011604(R) (2011).
[46] M.-J. Hwang and M.-S. Choi, Large-scale maximal entanglement and Majorana bound states in coupled circuit quantum electrodynamic systems, Phys. Rev. B **87**, 125404 (2013).
[47] G. Mazzarella, L. Salasnich, A. Parola, and F. Toigo, Coherence and entanglement in the ground state of a bosonic Josephson junction: From macroscopic Schrödinger cat states to separable Fock states, Phys. Rev. A **83**, 053607 (2011).
[48] Y. P. Huang and M. G. Moore, Creation, detection, and decoherence of macroscopic quantum superposition states in double-well Bose-Einstein condensates, Phys. Rev. A **73**, 023606 (2006).
[49] H. Feshbach, *Theoretical Nuclear Physics* (Wiley, New York, 1992).
[50] N. Dörre, J. Rodewald, P. Geyer, B. von Issendorff, P. Haslinger, and M. Arndt, Photofragmentation Beam Splitters for Matter-Wave Interferometry, Phys. Rev. Lett. **113**, 233001 (2014).
[51] X.-L. Wang *et al.*, Experimental Ten-Photon Entanglement, Phys. Rev. Lett. **117**, 210502 (2016).
[52] R. Dicke, Coherence in spontaneous radiation processes, Phys. Rev. **93**, 99 (1954).
[53] A. A. Klyachko, Quantum marginal problem and N-representability, J. Phys. Conf. Ser. **36**, 72 (2006).
[54] A. J. Coleman, Structure of fermion density matrices, Rev. Mod. Phys. **35**, 668 (1963).