Universality and Scaling in an Effective 1-D Model of Planetary Condensation

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Abstract

A recently developed effective 1-D model of planetary condensation [1] is used to study various properties of the produced condensates in terms of a set of uncovered scaling exponents. The dependence of the results upon the choice of the initial mass distribution is investigated, and it is found that the scaling exponents do not depend on the initial conditions, i.e. are universal.

1 Introduction

The interest in the detailed modeling of the planetary accretion process has increased in recent years due to a wealth of new data on extrasolar planets [2]. A large amount of effort has been invested to surmount the principal difficulty in such *ab-initio* simulations, the fact that a huge number of bodies needs to to be considered in order to resolve planets whose masses span four orders of magnitude. To achieve this goal, we have recently developed an effective 1-D model of planetary formation [1]. This approach is conceptually different from the now prevalent brute-force gravitational simulations (e.g. [3, 4]).

Working with a large number of particles N (typically 10^6-10^{10}) has enabled us to clearly distinguish two types of condensates that we identify as light and heavy. The distinction of these two types is based upon their scaling behavior when N is changed.

The problem of not knowing the initial distribution of matter is a serious one. One of the main motivations for developing our model was to investigate how sensitive are the outcomes of the condensation process to variations in the initial conditions. We have identified a set of properties of the final distributions of light and heavy condensates that do not depend on the details of the initial mass distribution. Conversely, it is possible to identify the properties that strongly depend upon the initial conditions. These properties can be fitted to the Solar system data and used to extract the likely initial conditions. Work in this direction is currently in progress.

2 Scaling Exponents

The model we use to simulate the planetary condensation process [1] depends on a single parameter K, a number of initial bodies N, and on the initial mass distribution $\rho(r)$. In order to be able to make a quantitative comparison between the outcomes of accretion simulations for different choices of K, N, and $\rho(r)$, we need to identify a set of quantities that characterize the final distribution of condensates. The first, and principal, such quantity is the ratio of the final and initial number of bodies $\Omega = n/N$. A typical result for Ω is shown in Figure 1. It is seen that Ω monotonically decreases with K from its maximal value of 1 at $K = 0$, to the minimal value of $1/N \sim 0$ at large K. These two extreme cases correspond respectively to no condensation at all, and to a collapse of all the available material into a single large body, i.e. to the formation of a binary system. Thus, the parameter K is seen to regulate the amount of the condensation that takes place.

Since we model the planetary condensation using a stochastic process, the outcomes of various runs do not necessarily coincide. Averaging the results over several runs give their expectation values and error bars. The data shown in Figure 1 are produced by averaging over 100 runs, and error bars can

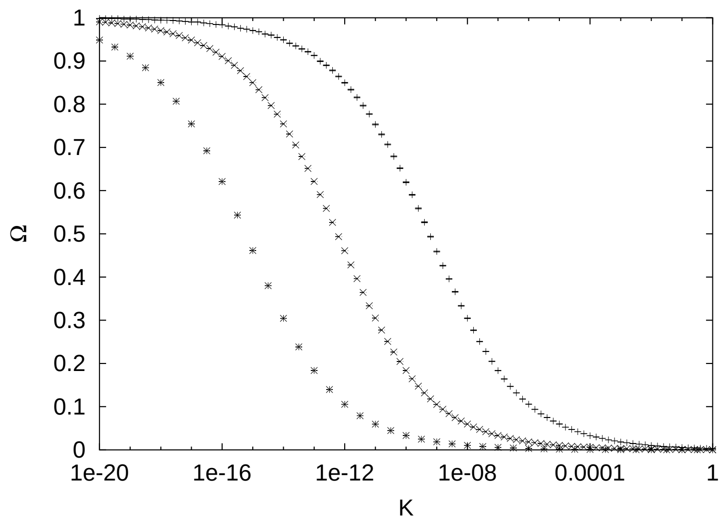

Figure 1: Ω as a function of K for $N = 10^3, 10^4, 10^5$ (from right to left), and triangular $\rho(r)$ peaked at 1.

barely be seen. It is rather easy to fit the data obtained using a triangular $\rho_1(r)$ peaked at 1 (see [1], eq. (3)) to a simple law. We find

$$\Omega \equiv \frac{n}{N} = \frac{1}{1 + AN^\alpha K^\beta} \ , \tag{1}$$

where $\alpha = 0.737(6)$, $\beta = 0.251(2)$, while $A = 2.10(5)$. When the initial mass distribution $\rho(r)$ is varied within a class of triangular distributions by changing the position of the peak, the exponents α and β remain the same within the error bars. The same happens when we abandon the triangular distribution all together and use $\rho_2 \propto r/(1+r^4)$, the quadratic distribution (ρ_3), or gaussian distribution (ρ_4) instead, as shown in Tables 1 and 2 bellow.

The quantity Ω is a global property of condensates. A more detailed understanding of their structure is achieved by studying their distributions in mass and distance from the star. The typical mass distribution, i.e. the fraction of condensates at mass m, is shown in Figure 2 for $K = 10^{-7}$. It is immediately obvious that the mass distribution of condensates lighter then a certain mass scale m^* can be fitted to a simple K-independent power law

$$n(m_i)/n = BN^{-\tau} m^{-\tau} \ . \tag{2}$$

The condensates that scale according to this law are designated as light, and those that do not as heavy. The mass scale m^* is thus the dividing line distinguishing between light and heavy condensates. For the triangular $\rho(r)$ peaked at 1, the scaling exponent takes the value $\tau = 1.20(5)$. Again, the same value (within error bars) is obtained for all other density distributions considered, i.e. it is universal.

The best fit of our model (in particular the heavy condensates) to Solar system data is obtained for $K = 0.1$ [1]. The mass distribution of condensates for that value of K is shown in Figure 3. The distinction between light and heavy condensates is apparent again, the only difference being that m^* has developed a stronger N dependence. In order for heavy condensates to span four orders of magnitude in mass, as in the Solar system, we must have $m^* < 10^{-4}$. This is achieved for $N > 10^6$. If we further impose that light and heavy condensates be well separated, then at least $N = 10^8$ initial bodies are needed.

The radial distribution of light condensates, i.e. the fraction of condensates found at distance r from the star, also displays a power law behaviour $\Lambda(r) \propto r^{-\zeta}$. For the triangular $\rho(r)$ peaked at 1, and $K = 0.1$ the scaling exponent is $\zeta = 1.85(5)$. However, unlike the previously considered exponents, this exponent is *not* universal.

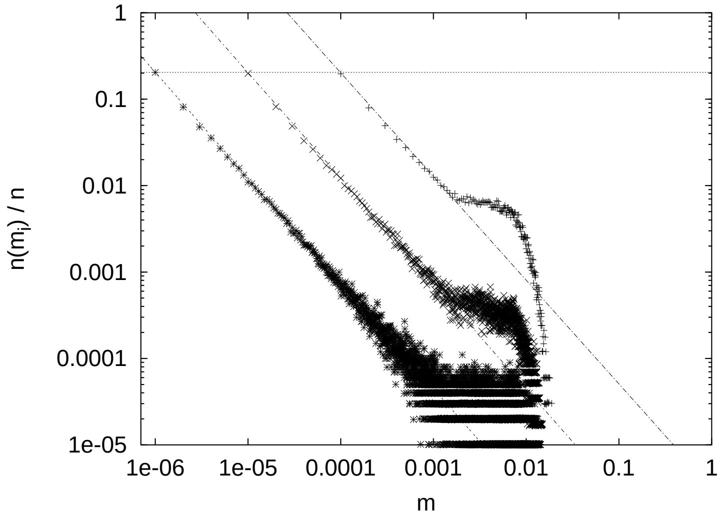

Figure 2: Plot of the relative number of condensates of mass $m_i = i/N$ (averaged over 100 runs), for $K = 10^{-7}$ and $N = 10^4$, 10^5 and 10^6 initial particles (from right to left).

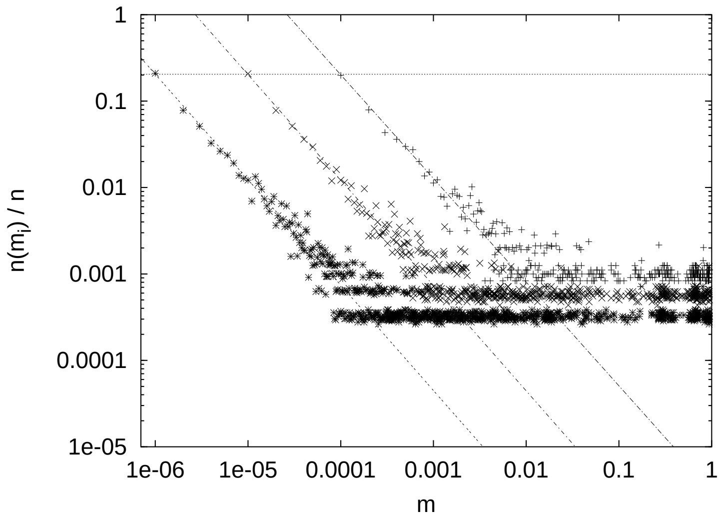

Figure 3: The same as in Figure 2, for $K = 0.1$, the value that best fits our Solar system.

A comparison of the various scaling exponents for different initial densities is given in Table 1. The triangular mass density listed here is the one peaked at 1. Note that the universal scaling exponents ω and ϵ correspond to the power law linking the spin and mass of the condensates, i.e. $s \propto m^\omega K^\epsilon$ ([1], eq. 6). The results have been obtained from averaging over 100 runs for each value of K and N. We have looked at 100 values of K on the interval $[10^{-8}, 100]$. The N dependence was investigated by doing runs with $N = 10^3, 10^{3.5}, 10^4, 10^{4.5}, \ldots 10^6$ initial particles. The coefficient β for $\rho_4(r)$ has not obtained its asymptotic value for $N = 10^6$. Up to this value of N all that can be said is that $\beta > 0.24$.

We have also looked at how the scaling exponents depend on the location of the peak of the general triangular initial distribution peaked at $r = b$. These results are given in Table 2. The simulations for $b \neq 1$ were only done for $N = 10^6$, so the coefficients α and ϵ could not be calculated. From the tables we see that, as we change $\rho(r)$, the scaling exponents α, β, τ, ϵ and ω remain the same, within the error bars.

	α	β	τ	ϵ	ω
$\rho_1(r)$	0.737 ± 0.006	0.251 ± 0.002	1.20 ± 0.05	0.40 ± 0.02	1.75 ± 0.03
$\rho_2(r)$	0.735 ± 0.008	0.247 ± 0.002	1.21 ± 0.01	0.44 ± 0.02	1.73 ± 0.03
$\rho_3(r)$	0.746 ± 0.004	0.249 ± 0.001	1.20 ± 0.02	0.43 ± 0.03	1.75 ± 0.03
$\rho_4(r)$	0.72 ± 0.01	> 0.24	1.23 ± 0.02	0.42 ± 0.01	1.78 ± 0.03

Table 1: Scaling exponents for different initial densities.

	β	τ	ω
$b = 1$	0.251 ± 0.002	1.20 ± 0.05	1.75 ± 0.03
$b = 2$	0.250 ± 0.004	1.2 ± 0.1	1.71 ± 0.03
$b = 4$	0.249 ± 0.004	1.2 ± 0.1	1.73 ± 0.03
$b = 6$	0.248 ± 0.004	1.2 ± 0.1	1.70 ± 0.03
$b = 8$	0.246 ± 0.004	1.2 ± 0.1	1.70 ± 0.03
$b = 10$	0.244 ± 0.004	1.2 ± 0.1	1.70 ± 0.03

Table 2: Scaling exponents for triangular initial distributions peaked at $r = b$.

In conclusion, using the effective 1-D model presented in [1], we have obtained a set of scaling exponents and have shown that they are universal within a one hump universality class of initial mass distributions $\rho(r)$.

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