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2013 Laser Phys. 23 105202

(http://iopscience.iop.org/1555-6611/23/10/105202)

Laser Phys. 23 (2013) 105202 (7pp) doi:10.1088/1054-660X/23/10/105202

Dark-state polaritons in a degenerate two-level system

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Received 26 July 2013
Accepted for publication 23 August 2013
Published 17 September 2013
Online at stacks.iop.org/LP/23/105202

Abstract
We investigate the formation of dark-state polaritons in an ensemble of degenerate two-level atoms admitting electromagnetically induced transparency. Using a generalization of microscopic equation-of-motion technique, multiple collective polariton modes are identified depending on the polarizations of two coupling fields. For each mode, the polariton dispersion relation and composition are obtained in a closed form out of a matrix eigenvalue problem for arbitrary control field strengths. We illustrate the algorithm by considering the $F_g = 2 \to F_e = 1$ transition of the D_1 line in ^{87}Rb atomic vapor. In addition, an application of dark-state polaritons to the frequency and/or polarization conversion, using D_1 and D_2 transitions in cold Rb atoms, is given.

(Some figures may appear in colour only in the online journal)

1. Introduction

At the end of the past century, the novel mechanism of electromagnetically induced transparency (EIT) [1, 2] and its many important applications drew a lot of attention. Nonlinearity of EIT media enables slow, stored and stationary light [3–5]. Mazets and Matisov were the first to introduce the concept of adiabatic Raman polaritons that represent a mixture of photon and collective atomic excitations [6]. Subsequently, Fleischhauer and Lukin further extended the concept to dark-state polaritons (DSPs) in a Λ-type EIT system [7]. They also developed a quantum memory technique [8] in order to transfer quantum states of photon wavepackets onto collective Raman excitations in a loss-free and reversible manner. DSPs in more sophisticated schemes have been studied, e.g. double-Λ [9–11], dual-V [12], inverted-Y [13], four-level [14], tripod [15], M-type [16], cyclic three-level [17] and multi-Λ [18, 19]. Collapses and revivals of the DSP number in an atomic ensemble with ground state degeneracy were found in [20]. Resonance beating of light stored using spinor DSPs in a multilevel-tripod scheme was investigated in [21]. Slow light propagation in a degenerate two-level system was experimentally investigated in [22]. DSPs in these various schemes may find applications in quantum information processing, quantum memory and quantum repeaters. Furthermore, degenerate atomic systems, due to their inherent complexity, could lead to new features of DSPs and building blocks for quantum information and quantum computation.

Most of the works treat DSPs using the perturbative approach to the field operator equations of motion, followed by the adiabatic approximation, which was introduced by Fleischhauer and Lukin. In addition, Zimmer *et al* [12] also used the Morris–Shore transformation [23]. Alternatively, Juzeliunas and Carmichael applied a Bogoliubov-type transformation for exact diagonalization of the model Hamiltonian [24]. Chong and Soljacic [9] elegantly derived the properties of the DSPs in single- and double-Λ systems using the Sawada–Brout technique [25]. In this work, we extend the Sawada–Brout–Chong technique to a degenerate two-level system, having a ground state manifold g and an excited state manifold e, that admits the appearance of EIT, i.e. (multiple) dark states exist within g. We present a general algorithm to identify multiple DSP modes that works for an arbitrary number of degenerate states within manifolds g and e and arbitrary polarizations of two coupling fields. The approach is illustrated by finding DSPs at D_1 line transition $F_g = 2 \to F_e = 1$ in atomic vapor of ^{87}Rb. It is shown

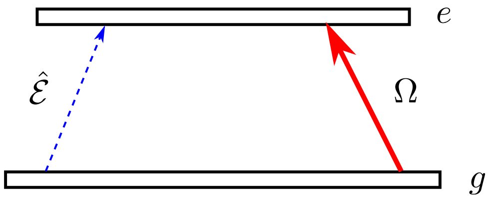

Figure 1. Schematic of a degenerate two-level system, having a ground state manifold g and an excited state manifold e, driven by a strong classical control field (thick line) of Rabi frequency Ω and by a weak quantum probe field $\hat{\mathcal{E}}$ (dashed line) of different polarizations.

that depending on the polarizations of the coupling fields, one or two DSP modes can be determined. In addition, it is shown how DSP modes, originating from different ^{87}Rb transitions, can be utilized for frequency and/or linear polarization conversion.

2. Degenerate two-level system

In this section, we present a general formalism of dark-state polaritons in a degenerate two-level system. It is a generalization of the neat approach of [9]. We consider a gas sample of N atoms, where N is large. Let us denote by $\mathcal{H}_{\rm g}$ the Hilbert space of the atomic states in the ground state manifold g and let $\mathcal{H}_{\rm e}$ be the Hilbert space of atomic excited states in the manifold e. The corresponding ground- and excited-state energies are denoted by $\hbar\omega_{\rm g}$ and $\hbar\omega_{\rm e}$, respectively. A strong classical control field of Rabi frequency Ω and a weak quantum probe field $\hat{\mathcal{E}}$, which differ in polarizations and both propagate along the z axis, couple the transition g → e (see figure 1). The corresponding raising and lowering operators of the control (probe) field, $\hat{V}_{\rm c}^\dagger$ and $\hat{V}_{\rm c}$ ($\hat{V}_{\rm p}^\dagger$ and $\hat{V}_{\rm p}$), connect the states in manifold g to the states in manifold e and vice versa. We assume that $\dim\mathcal{H}_{\rm g} \geq \dim\mathcal{H}_{\rm e}$ holds, so that the system admits EIT [26]. This assures the existence of the Hilbert space $\mathcal{H}_{\rm g}^{\rm d}$ of the states in manifold g that are dark to the g → e transition for the control field [27, 28]. Formally, we can view the raising operator $\hat{V}_{\rm c}^\dagger$ as a linear mapping $\hat{V}_{\rm c}^\dagger\colon \mathcal{H}_{\rm g} \to \mathcal{H}_{\rm e}$. The space $\mathcal{H}_{\rm g}^{\rm d}$ is then the null space of the mapping $\hat{V}_{\rm c}^\dagger$

$$\mathcal{H}_{\rm g}^{\rm d} = \{|g\rangle \in \mathcal{H}_{\rm g} \mid \hat{V}_{\rm c}^\dagger|g\rangle = 0\}. \tag{1}$$

2.1. Model Hamiltonian

We will now present the model Hamiltonian and the dynamics of the lowest energy excitations of the ensemble of degenerate two-level atoms. The free atomic Hamiltonian has the form

$$\hat{H}_{\rm at} = \sum_r (\hbar\omega_{\rm g}\hat{\mathbb{I}}_{\rm g}(r) + \hbar\omega_{\rm e}\hat{\mathbb{I}}_{\rm e}(r)), \tag{2}$$

where the summation index r counts the atomic positions, while $\hat{\mathbb{I}}_{\rm g}$ and $\hat{\mathbb{I}}_{\rm e}$ are the projection operators onto the states in the manifolds g and e, respectively. The free photon Hamiltonian, including multiple quantum probe field modes, is

$$\hat{H}_{\rm ph} = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k, \tag{3}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators of the probe photons with the wavevector k and frequency $\omega_k = c|k| \sim \omega_{\rm eg} \equiv \omega_{\rm e} - \omega_{\rm g}$. The atom interaction with the probe field is given through the minimal coupling Hamiltonian

$$\hat{H}_{\rm p} = -\sum_k \sum_r \hbar g_k \hat{a}_k \exp({\rm i}kr)\hat{V}_{\rm p}^\dagger(r) + {\rm H.c.} \tag{4}$$

with coupling constant $\hbar g_k = \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\, d_{ge}$, where d_{ge} is the effective electric dipole moment of the g → e transition, ϵ_0 is the vacuum permittivity and V is the quantization volume. The interaction of the atomic ensemble with the classical control field of the carrier frequency $\omega_{\rm c} \sim \omega_{\rm eg}$ and the wavevector $k_{\rm c}$ is of the form

$$\hat{H}_{\rm c}(t) = -\sum_r \hbar\Omega \exp[-{\rm i}(\omega_{\rm c}t - k_{\rm c}r)]\hat{V}_{\rm c}^\dagger(r) + {\rm H.c.} \tag{5}$$

For simplicity, we have used the rotating-wave approximation. In addition, for an atomic operator $\hat{A}(r)$ we define a Fourier-transformed operator $\hat{A}(k) = \sum_r \hat{A}(r)\exp({\rm i}kr)/\sqrt{N}$. Note that $\left(\hat{A}(k)\right)^\dagger = \hat{A}^\dagger(-k)$. Especially, one has $\sum_r \hat{A}(r) = \sqrt{N}\hat{A}(k=0)$. In terms of the Fourier-transformed operators, various Hamiltonian parts are

$$\hat{H}_{\rm at} = \hbar\omega_{\rm g}\sqrt{N}\,\hat{\mathbb{I}}_{\rm g}(k=0) + \hbar\omega_{\rm e}\sqrt{N}\,\hat{\mathbb{I}}_{\rm e}(k=0), \tag{6a}$$

$$\hat{H}_{\rm p} = -\sum_k \hbar g_k \sqrt{N}\,\hat{a}_k \hat{V}_{\rm p}^\dagger(k) + {\rm H.c.}, \tag{6b}$$

$$\hat{H}_{\rm c}(t) = -\hbar\Omega\sqrt{N}{\rm e}^{-{\rm i}\omega_{\rm c}t}\hat{V}_{\rm c}^\dagger(k_{\rm c}) + {\rm H.c.} \tag{6c}$$

The entire Hamiltonian of the ensemble of degenerate two-level atoms interacting with the probe and the control field is $\hat{H}(t) = \hat{H}_{\rm at} + \hat{H}_{\rm ph} + \hat{H}_{\rm p} + \hat{H}_{\rm c}(t)$.

2.2. Dark-state polaritons

Now, we focus on the dark-state polaritons in an ensemble of degenerate two-level atoms. Various features of the method in [9], which are obvious *per se* in the case of a simple Λ system, need to be properly adapted to the degenerate two-level system. The additional complexity of the system we investigate also yields some new inherent requirements.

First of all, we remove the time dependence from the Hamiltonian $\hat{H}(t)$ by performing the following unitary gauge transformation:

$$\hat{H}_{\rm T} = \hat{U}_{\rm c}(t)\hat{H}(t)\hat{U}_{\rm c}^\dagger(t) - \hbar\omega_{\rm c}\left(\sqrt{N}\hat{\mathbb{I}}_{\rm e}(k=0) + \sum_k \hat{a}_k^\dagger\hat{a}_k\right), \tag{7}$$

where

$$\hat{U}_{\rm c}(t) = \exp\left[{\rm i}\omega_{\rm c}t\left(\sqrt{N}\,\hat{\mathbb{I}}_{\rm e}(k=0) + \sum_k \hat{a}_k^\dagger\hat{a}_k\right)\right]. \tag{8}$$

Eventually, we restate the time-dependent Schroedinger equation $i\hbar\partial_t|\phi(t)\rangle = \hat{H}(t)|\phi(t)\rangle$ as

$$i\hbar\,\partial_t\big[\hat{U}_c(t)|\phi(t)\rangle\big] = \hat{H}_T\big[\hat{U}_c(t)|\phi(t)\rangle\big]. \quad (9)$$

Solutions of (9) can be obtained by finding the energy eigenstates of the time-independent Hamiltonian $\hat{H}_T$.

Assume that the atomic ensemble is initially prepared in the collective vacuum state with no probe photons $|\mathbf{g_0}, 0\rangle = |g_0\rangle \otimes |0\rangle \equiv \otimes_r |g_0\rangle_r \otimes |0\rangle$. Analogously with the Λ system case [8, 9], the atomic ground state $|g_0\rangle$ must be dark with respect to the control field, i.e.

$$\hat{V}_c^\dagger|g_0\rangle = 0, \qquad \text{or equivalently} \qquad |g_0\rangle \in \mathcal{H}_g^d. \quad (10)$$

Additional requirements on the state $|g_0\rangle$ will be specified later.

Dark-state polaritons are particular low energy, single probe photon driven, collective excitations that do not have a contribution of the excited atomic states. To obtain DSPs, we look for a polariton excitation operator $\hat{\phi}_k^\dagger$ such that in the low energy, single excitation case $\hat{\phi}_k^\dagger|\mathbf{g_0}, 0\rangle$ is an eigenstate of $\hat{H}_T$ with the energy $\hbar\omega(k)$. This leads to the following relation:

$$\big[\hat{H}_T, \hat{\phi}_k^\dagger\big] = \hbar\omega(k)\hat{\phi}_k^\dagger + \cdots, \quad (11)$$

where dots represent the terms that are omitted in the single excitation case and also terms that give zero when acting on the collective vacuum state $|\mathbf{g_0}, 0\rangle$. Note that, for notational simplicity, we keep in mind that all subsequent commutators always act on the state $|\mathbf{g_0}, 0\rangle$. In agreement with [8, 9], we neglect Langevin noise effects, which do not influence the adiabatic evolution of the DSPs.

Collective atomic excitations are driven by the probe photons. Hence, we begin by calculating the commutator

$$\big[\hat{H}_T, \hat{a}_k^\dagger\big] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k\sqrt{N}\,\hat{V}_p^\dagger(k). \quad (12)$$

The states that arise from the interaction with the probe field are the pure photon excitation $\hat{a}_k^\dagger|\mathbf{g_0}, 0\rangle$, and the collective atomic excitation $\hat{V}_p^\dagger(k)|\mathbf{g_0}, 0\rangle$, up to a normalization constant. Hence, in addition to $\hat{a}_k^\dagger$ the operator $\hat{V}_p^\dagger(k)$ is also a member of the polariton excitation operator $\hat{\phi}_k^\dagger$. Next, we determine the commutation relation

$$\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \sum_{k'}\hbar g_{k'}^*\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k'). \quad (13)$$

Note that $\sqrt{N}\big[\hat{A}_1(k), \hat{A}_2(k')\big] = \big[\hat{A}_1, \hat{A}_2\big](k + k')$ holds for any two atomic operators $\hat{A}_1$ and $\hat{A}_2$. The new operators, $(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)$ and $\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k')$, appearing in (13) yield the collective states via stimulated emission. The former can readily be included into the polariton excitation operator $\hat{\phi}_k^\dagger$. It creates the spatially dependent coherence among the atomic ground states $|g_0\rangle$ and $\hat{V}_c\hat{V}_p^\dagger|g_0\rangle$, i.e. the ground state coherence wave. When we commute the latter operator with $\hat{H}_T$, we get the operator $\hat{a}_{k''}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k')(\hat{V}_p\hat{V}_p^\dagger)(k' - k'')$. The emergence of such operators of increasing complexity continues and ends with $\hat{a}_{k^{(N)}}^\dagger\prod_{i=1}^N(\hat{V}_p\hat{V}_p^\dagger)(k^{(i)} - k^{(i-1)})$, where $k^{(0)} = k$. This case corresponds to a formidably complex DSP mode that is not tractable. Tractable modes are obtained by imposing one further requirement on the collective vacuum state. Namely, it is crucial that upon action $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle$ we end up with the state $|g_0\rangle$, i.e.,

$$\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle, \quad (14)$$

where $\lambda_p > 0$ is the corresponding eigenvalue. Thus, one obtains $(\hat{V}_p\hat{V}_p^\dagger)(k - k')|\mathbf{g_0}, 0\rangle = \lambda_p\sqrt{N}\delta_{k,k'}|\mathbf{g_0}, 0\rangle$, so that the relation (13) greatly simplifies to

$$\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \hbar g_k^*\lambda_p\sqrt{N}\hat{a}_k^\dagger. \quad (15)$$

To proceed further, we define the excited atomic state $|e\rangle = \hat{V}_p^\dagger|g_0\rangle/\sqrt{\lambda_p}$ associated with the action of the probe field. Clearly, it has the property $\hat{V}_p|e\rangle = \sqrt{\lambda_p}|g_0\rangle$ and it is an eigenstate of $\hat{V}_p^\dagger\hat{V}_p$, i.e. $\hat{V}_p^\dagger\hat{V}_p|e\rangle = \lambda_p|e\rangle$. The eigenstates $|g_0\rangle$ and $|e\rangle$ are 'tuned' to the polarization of the probe field. These are so-called polarization-dressed states, first introduced and used in [28, 29] for problems of interaction of resonant elliptically polarized light with atomic and molecular energy levels degenerate in angular momentum projections. Next, let us consider the commutators

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)\big] = -\hbar\Omega(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k), \quad (16)$$

and also

$$\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] = \hbar(\omega_{eg} - \omega_c)(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \sum_{k'}\hbar g_{k'}^*\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k - k'). \quad (17)$$

Similar to the discussion of the relation (13), in order to avoid the appearance of probe photons with all wavevectors, we require that $\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle \propto |g_0\rangle$. That can hold provided that

$$\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle \qquad \text{i.e. } \hat{V}_c^\dagger\hat{V}_c|e\rangle = \lambda_c|e\rangle, \quad (18)$$

where $\lambda_c > 0$ is the corresponding eigenvalue. Thus, the excited atomic state $|e\rangle$ is a common eigenstate of the operators $\hat{V}_p^\dagger\hat{V}_p$ and $\hat{V}_c^\dagger\hat{V}_c$. Under such a condition, the relation (16) becomes

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)\big] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \quad (19)$$

while (17) turns into

$$\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] = \lambda_c\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big], \quad (20)$$

where the last commutator is found in (15). Hence, under the previous conditions no new components of the polariton excitation operator $\hat{\phi}_k^\dagger$ appear. Stimulated emission, which is driven by the control field, transfers the atoms from the excited state $|e\rangle$ into the ground state $|f\rangle = \hat{V}_c|e\rangle/\sqrt{\lambda_c}$. The states $|g_0\rangle$ and $|e\rangle$ are coupled by the probe field, while the states $|e\rangle$ and $|f\rangle$ are coupled by the control field. Thus, for each eigenvalue pair (λ_p, λ_c) the three states $|g_0\rangle$, $|e\rangle$ and $|f\rangle$ form an independent Λ system that is related to one independent collective DSP mode. The number of such Λ systems, i.e. tractable DSP modes, can be at most equal to the total number of DSP modes, i.e. to the dimensionality of the dark space $\mathcal{H}_g^d$.

Now, we collect the necessary commutation relations

$$\left[\hat{H}_T, \hat{a}_k^\dagger\right] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k\sqrt{N}\hat{V}_p^\dagger(k), \quad (21a)$$

$$\left[\hat{H}_T, \hat{V}_p^\dagger(k)\right] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar g_k^*\lambda_p\sqrt{N}\hat{a}_k^\dagger - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c), \quad (21b)$$

$$\left[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)\right] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \quad (21c)$$

so that the polariton excitation operator is of the form

$$\hat{\phi}_{nk}^\dagger = \alpha_{nk}\hat{a}_k^\dagger + \beta_{nk}\frac{\hat{V}_p^\dagger(k)}{\sqrt{\lambda_p}} + \gamma_{nk}\frac{(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)}{\sqrt{\lambda_p\lambda_c}}, \quad (22)$$

where the band index n enumerates the different polariton species. Orthonormal collective excitations $|\boldsymbol{g_0}, 1_k\rangle$, $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k - k_c), 0\rangle$ result from the action of the operators $\hat{a}_k^\dagger$, $\hat{V}_p^\dagger(k)/\sqrt{\lambda_p}$ and $(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)/\sqrt{\lambda_p\lambda_c}$ on the collective vacuum state $|\boldsymbol{g_0}, 0\rangle$, respectively,

$$|\boldsymbol{g_0}, 1_k\rangle = \bigotimes_r |g_0\rangle_r \otimes |1_k\rangle, \quad (23a)$$

$$|\boldsymbol{e}(k), 0\rangle = \frac{1}{\sqrt{N}}\sum_r \mathrm{e}^{\mathrm{i}kr}|e\rangle_r \bigotimes_{r'\neq r} |g_0\rangle_{r'} \otimes |0\rangle, \quad (23b)$$

$$|\boldsymbol{f}(k - k_c), 0\rangle = \frac{1}{\sqrt{N}}\sum_r \mathrm{e}^{\mathrm{i}(k-k_c)r}|f\rangle_r \bigotimes_{r'\neq r} |g_0\rangle_{r'} \otimes |0\rangle. \quad (23c)$$

Note that the collective states $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k - k_c), 0\rangle$ are entangled. This enables the usage of the polariton state

$$|\phi_{nk}\rangle = \alpha_{nk}|\boldsymbol{g_0}, 1_k\rangle + \beta_{nk}|\boldsymbol{e}(k), 0\rangle + \gamma_{nk}|\boldsymbol{f}(k - k_c), 0\rangle \quad (24)$$

as a resource for quantum information processing [2].

We determine the c-numbers α_{nk}, β_{nk} and γ_{nk} by inserting (22) into (11) and make use of (21). This leads to three self-consistency equations that we can represent in the basis $\{|\boldsymbol{g_0}, 1_k\rangle, |\boldsymbol{e}(k), 0\rangle, |\boldsymbol{f}(k - k_c), 0\rangle\}$ as

$$\begin{bmatrix} \omega_k - \omega_c & -\tilde{g}_k^*\sqrt{N} & 0 \\ -\tilde{g}_k\sqrt{N} & \omega_{eg} - \omega_c & -\tilde{\Omega} \\ 0 & -\tilde{\Omega}^* & 0 \end{bmatrix}\begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix} = \omega_n(k)\begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix}, \quad (25)$$

where $\tilde{g}_k = g_k\sqrt{\lambda_p}$ and $\tilde{\Omega} = \Omega\sqrt{\lambda_c}$. Our effective Hamiltonian in (25) is similar to the one in [9], but with a major difference. The effective coupling constant $\tilde{g}_k$ and the effective Rabi frequency $\tilde{\Omega}$ differ from the corresponding one in [9] because of the inclusion of the eigenvalues λ_p and λ_c. The mentioned difference clearly arises as a consequence of the degenerate two-level atomic system.

The dark-state polaritons are obtained as one of the solutions of the eigenproblem (25). The other two solutions are bright-state polaritons, similarly as in [9]. Exactly at the Raman resonance, $\omega_k = \omega_c$, there is an eigenvector $\propto \left[-\frac{\tilde{\Omega}}{\tilde{g}_k\sqrt{N}}, 0, 1\right]$. This eigenvector has no contribution of the excited atomic states and represents a stable dark-state polariton that is insensitive to incoherent decay processes acting on the excited atoms. Expansion around the resonance $\omega_k \sim \omega_{eg}$ and $\omega_c \sim \omega_{eg}$ yields a linearized solution for the dark-state polaritons

$$\omega(k) = \frac{|\tilde{\Omega}|^2}{|\tilde{g}_k|^2N + |\tilde{\Omega}|^2}(\omega_k - \omega_c), \quad (26a)$$

$$\alpha_k = -\frac{\tilde{\Omega}}{\tilde{g}_k\sqrt{N}}\gamma_k, \qquad \beta_k = -\frac{\tilde{\Omega}(\omega_k - \omega_c)}{|\tilde{g}_k|^2N + |\tilde{\Omega}|^2}\gamma_k. \quad (26b)$$

An interesting property of the DSP solution is that it only depends on the Raman detuning $\omega_k - \omega_c$ of the coupling fields and on the coupling parameters $\tilde{g}_k$ and $\tilde{\Omega}$. It does not depend on the energy spacing ω_{eg} of the underlying degenerate two-level system.

The algorithm for finding tractable DSP modes in a degenerate two-level system can be summarized as:

(1) determine the dark space $\mathcal{H}_g^d$ for the operator $\hat{V}_c^\dagger$;
(2) find all states $|g_0\rangle$ from $\mathcal{H}_g^d$ and pairs of eigenvalues (λ_p, λ_c) such that $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle$ and $\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle$ hold;
(3) for every such pair of eigenvalues obtain DSPs $|\psi_k(\lambda_p, \lambda_c)\rangle$ from (24) and (26).

3. Dark-state polaritons in rubidium vapor

In this section we apply the general formalism to the rubidium vapor. Control and probe fields couple the hyperfine levels $5S_{1/2}, F_g = 2$ and $5P_{1/2}, F_e = 1$ of ^{87}Rb. The atomic lowering operators of the control and probe fields are, respectively,

$$\hat{V}_c = \hat{\mathbf{V}} \cdot \mathbf{e}_c, \qquad \hat{V}_p = \hat{\mathbf{V}} \cdot \mathbf{e}_p, \quad (27)$$

where $\mathbf{e}_c$ and $\mathbf{e}_p$ are polarizations of the fields. The vector operator $\hat{\mathbf{V}}$ is defined by [28, 30, 31]

$$\hat{\mathbf{V}} = (-1)^{F_e+J_g+I+1}\sqrt{(2F_e+1)(2J_g+1)}\begin{Bmatrix} J_e & J_g & 1 \\ F_g & F_e & I \end{Bmatrix} \times \sum_{q=-1}^{1}\sum_{m_g,m_e}\langle F_g, m_g|F_e, m_e; 1, q\rangle|F_g, m_g\rangle\langle F_e, m_e|\mathbf{e}_q^*, \quad (28)$$

where $I = 3/2$ is the nuclear quantum number of ^{87}Rb, $\{:::\}$ is the Wigner $6j$-symbol and $\langle F_g, m_g|F_e, m_e; 1, q\rangle$ is the Clebsch–Gordan coefficient that connects the excited level state $|F_e, m_e\rangle$ to the ground level state $|F_g, m_g\rangle$ via polarization $\mathbf{e}_q^*$,

$$\mathbf{e}_{\pm 1} = \mp\frac{1}{\sqrt{2}}(\mathbf{e}_x \pm \mathrm{i}\,\mathbf{e}_y), \qquad \mathbf{e}_0 = \mathbf{e}_z, \quad (29)$$

given in some orthonormal basis of polarization vectors. We choose the coordinate system such that the fields propagate along the z axis, and define a basis of Zeeman states relative to this quantization axis. The bases of the individual Hilbert spaces $\mathcal{H}_e$ and $\mathcal{H}_g$ are

$$\mathcal{E} = \{|1, -1\rangle_e, |1, 0\rangle_e, |1, 1\rangle_e\}, \quad (30a)$$

$$\mathcal{G} = \{|2, -2\rangle_g, |2, -1\rangle_g, |2, 0\rangle_g, |2, 1\rangle_g, |2, 2\rangle_g\}. \quad (30b)$$

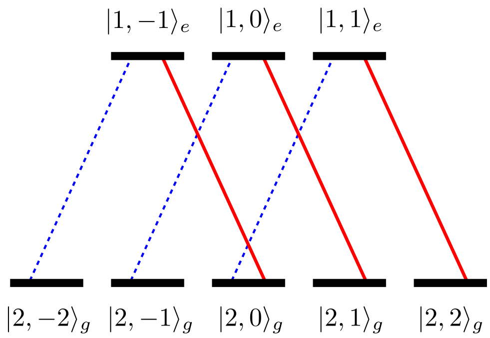

Figure 2. Zeeman sublevel scheme of the transition $F_g = 2 \to F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote σ^- transitions coupled by the control field while dashed lines denote σ^+ transitions coupled by the probe field.

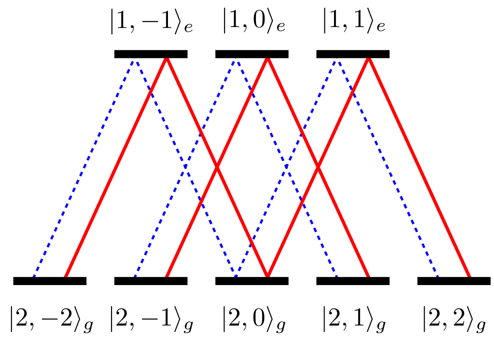

Figure 3. Zeeman sublevel scheme of the transition $F_g = 2 \to F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote control field linearly polarized along the y axis while dashed lines denote probe field linearly polarized along the x axis.

We will show that according to the appropriate choice of the polarizations of the coupling fields, one or two DSP modes can be obtained.

3.1. Case of orthogonal circular polarizations

Let the control field couple σ^- transitions, while the probe field couples σ^+ transitions, i.e. $\mathbf{e}_c = \mathbf{e}_{+1}$ and $\mathbf{e}_p = \mathbf{e}_{-1}$ (see figure 2). The lowering operators of the coupling fields, $\hat{V}_c$ and $\hat{V}_p$, are represented in the basis $\mathcal{E} \cup \mathcal{G}$ with the matrices

$$\mathbf{V}_c = \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{31a}$$

$$\mathbf{V}_p = \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2\sqrt{3}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{31b}$$

where zeros $\mathbf{0}_{m,n}$ denote rectangular $m \times n$ null matrices. Ground level dark space determined from the null space of $\mathbf{V}_c^\dagger$ is

$$\mathcal{H}_g^d = \{|2,-2\rangle_g, |2,-1\rangle_g\}. \tag{32}$$

Both dark states are appropriate as the initial state $|g_0\rangle$. Below we tabulate the corresponding states and eigenvalues of the Λ system:

	$\lvert g_0\rangle$	$\lvert e\rangle$	$\lvert f\rangle$	λ_p	λ_c
I	$\lvert 2,-2\rangle_g$	$\lvert 1,-1\rangle_e$	$\lvert 2,0\rangle_g$	1/2	1/12
II	$\lvert 2,-1\rangle_g$	$\lvert 1,0\rangle_e$	$\lvert 2,1\rangle_g$	1/4	1/4,

that lead to two DSP modes:

$$\omega^{\mathrm{I}}(k) = \frac{|\Omega|^2}{6\,|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{33a}$$

$$|\psi_k^{\mathrm{I}}\rangle \propto -\frac{\Omega}{\sqrt{6}\,g_k\sqrt{N}}|\boldsymbol{g}_0^{\mathrm{I}}, 1_k\rangle + |\boldsymbol{f}^{\mathrm{I}}(k-k_c), 0\rangle - \frac{2\sqrt{3}\,\Omega\,(\omega_k - \omega_c)}{6\,|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^{\mathrm{I}}(k), 0\rangle, \tag{33b}$$

$$\omega^{\mathrm{II}}(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{34a}$$

$$|\psi_k^{\mathrm{II}}\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0^{\mathrm{II}}, 1_k\rangle + |\boldsymbol{f}^{\mathrm{II}}(k-k_c), 0\rangle - \frac{2\,\Omega\,(\omega_k - \omega_c)}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^{\mathrm{II}}(k), 0\rangle. \tag{34b}$$

We see that for orthogonal circular polarizations of the coupling fields, the maximal number of tractable DSP modes exists. This is the generic case, because relevant independent Λ system(s) can be easily recognized.

3.2. Case of orthogonal linear polarizations

Now we analyze the case of the control field polarization along the y axis and the probe field polarization along the x axis, i.e. $\mathbf{e}_c = \mathbf{e}_y$ and $\mathbf{e}_p = \mathbf{e}_x$ (see figure 3). The matrices representing the atomic lowering operators $\hat{V}_c$ and $\hat{V}_p$ in the basis $\mathcal{E} \cup \mathcal{G}$ are

$$\mathbf{V}_c = i \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} \\ 0 & \frac{1}{2\sqrt{2}} & 0 \\ 0 & 0 & \frac{1}{2} \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{35a}$$

$$\mathbf{V}_{\rm p} = \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ \frac{1}{2} & 0 & 0 & \\ 0 & \frac{1}{2\sqrt{2}} & 0 & \\ -\frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} & \mathbf{0}_{5,5} \\ 0 & -\frac{1}{2\sqrt{2}} & 0 & \\ 0 & 0 & -\frac{1}{2} & \end{bmatrix}. \tag{35b}$$

In this case, the ground level dark space is

$$\mathcal{H}_{\rm g}^{\rm d} = \left\{ -\frac{1}{\sqrt{2}}|2,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|2,1\rangle_{\rm g}, \right.$$
$$\left. \frac{1}{\sqrt{8}}|2,-2\rangle_{\rm g} - \frac{\sqrt{3}}{2}|2,0\rangle_{\rm g} + \frac{1}{\sqrt{8}}|2,2\rangle_{\rm g}, \right\}, \tag{36}$$

but only the first dark state satisfies all necessary conditions for the vacuum state of the tractable mode. The states and eigenvalues of the corresponding Λ system are

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|2,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|2,1\rangle_{\rm g}, \\ |e\rangle &= |1,0\rangle_{\rm e}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|2,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|2,1\rangle_{\rm g}, \\ \lambda_{\rm p} &= 1/4, \qquad \lambda_{\rm c} = 1/4. \end{aligned} \tag{37a}$$

We identify one DSP mode

$$\omega(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_{\rm c}), \tag{38a}$$

$$|\psi_k\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0, 1_k\rangle + |\boldsymbol{f}(k-k_{\rm c}), 0\rangle$$
$$- \frac{2\,\Omega(\omega_k - \omega_{\rm c})}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}(k), 0\rangle, \tag{38b}$$

while the other one is non-tractable.

From the above examples, it can be seen that the choice of the polarization of the coupling fields yields entirely different DSP modes. This is reflected in the composition of the DSP state as well as in the polariton dispersion relation. Note that different polariton dispersion relations would lead to distinct slow light group velocities. In section 4 we outline one possible application of DSP modes in degenerate two-level systems for frequency and/or linear polarization conversion.

4. Frequency and polarization conversion

Let us consider the DSP modes that can be formed from the states within $5S_{1/2}, F_{\rm g} = 1$ hyperfine level of ^{87}Rb atoms, when the control and the probe field have orthogonal linear polarizations. There are three relevant atomic transitions:

(a) $5S_{1/2},\ F_{\rm g} = 1 \to 5P_{1/2},\ F_{\rm e} = 1,$

(b) $5S_{1/2},\ F_{\rm g} = 1 \to 5P_{3/2},\ F_{\rm e} = 1,$

(c) $5S_{1/2},\ F_{\rm g} = 1 \to 5P_{3/2},\ F_{\rm e} = 0.$

The first belongs to the D_1 line. The last two belong to the D_2 line and can be rendered non-overlapping by using ultracold rubidium atoms.

In the case of orthogonal linear polarizations $\mathbf{e}_{\rm c} = \mathbf{e}_x$ and $\mathbf{e}_{\rm p} = \mathbf{e}_y$ of the fields that are resonant to the D_1 line transition (a), we have

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ |e\rangle &= |1,0\rangle_{\rm e}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ \lambda_{\rm p} &= 1/12, \qquad \lambda_{\rm c} = 1/12. \end{aligned} \tag{39a}$$

When considering the D_2 line transition (b) with the same polarizations of the coupling fields as in the previous case, $\mathbf{e}_{\rm c} = \mathbf{e}_x$ and $\mathbf{e}_{\rm p} = \mathbf{e}_y$, we find

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ |e\rangle &= |1,0\rangle_{\rm e}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ \lambda_{\rm p} &= 5/24, \qquad \lambda_{\rm c} = 5/24. \end{aligned} \tag{40a}$$

Finally, for the *swapped linear polarizations*, $\mathbf{e}_{\rm c} = \mathbf{e}_y$ and $\mathbf{e}_{\rm p} = \mathbf{e}_x$, of the fields coupling the D_2 line transition (c), we have

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ |e\rangle &= |0,0\rangle_{\rm e}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\rm g} + \frac{1}{\sqrt{2}}|1,1\rangle_{\rm g}, \\ \lambda_{\rm p} &= 1/6, \qquad \lambda_{\rm c} = 1/6. \end{aligned} \tag{41a}$$

Note, if the polarizations of the fields had not been swapped, the states $|g_0\rangle$ and $|f\rangle$ would have been interchanged.

As can be seen from (39) to (41), the DSP modes are formed from the same states $|g_0\rangle$ and $|f\rangle$ in all three cases, but the considered transitions and polarizations of the coupling fields are different. This provides the possibility for frequency [32, 18] and/or polarization conversion [33] of linearly polarized light. First, one can store a pulse of the probe light polarized along the y axis into the atomic coherence among the states $|g_0\rangle$ and $|f\rangle$ using the transition (a) and the control field polarized along the x axis. The retrieval process, using the transition (b) and the control field polarized along the x axis, would release the pulse at a different frequency, but of the same optical quantum state and polarization along the y axis as the original probe pulse. However, the pulse retrieved using the transition (c) and the control field polarized along the y axis would be in the same optical quantum state as the original probe pulse, but of different carrier frequency and linear polarization along the x axis, i.e. *orthogonal to the original one*. Moreover, this realization does not suffer from losses in the retrieved pulse, since the ratios of the probe and control Clebsch–Gordan coefficients are the same among all three transitions [33].

5. Conclusion

To sum up, we have investigated the formation of dark-state polaritons in an ensemble of degenerate two-level atoms with ground state Hilbert space $\mathcal{H}_g$ and excited state Hilbert space $\mathcal{H}_e$, where $\dim \mathcal{H}_g \geq \dim \mathcal{H}_e$ holds. We elaborated an algorithm, which is a generalization of the Sawada–Brout–Chong approach [9, 25]. Under suitable conditions, the polariton mode dispersion relation and composition can be stated in a closed form. Such DSPs do not depend on the energy spacing of the two-level system, but rather on the Raman detuning of the coupling fields. For each polariton mode, the effective field coupling parameters depend on the appropriate eigenvalues of the atomic operators $\hat{V}_p^\dagger \hat{V}_p$ and $\hat{V}_c^\dagger \hat{V}_c$ that determine the eigenproblem for the polariton species. The application of the general procedure is given for ^{87}Rb atomic transition $F_g = 2 \rightarrow F_e = 1$ of the D_1 line. Two cases of polarizations of the control and probe field are analyzed, when the two fields have orthogonal circular polarizations and when both are linearly polarized in the orthogonal directions. In the former case, two DSP modes are identified, while in the latter case, only one DSP mode can be determined. The formation of the modes as well as their dispersion relation critically depend on the polarizations chosen. Possible application of DSP modes in ultracold ^{87}Rb atoms for frequency and/or linear polarization conversion without losses in the retrieved pulse is presented. Our algorithm can be extended to degenerate systems with more levels and might have applications in quantum information processing as a building block for a preparation and read out schemes with the DSPs as qubit states.

Acknowledgments

This work was supported by the Ministry of Education and Science of the Republic of Serbia, under Grants Nos III45016 and OI171038 and also by Scopes JRP IZ7370_127942.

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