Coexistence of Composite Bosons and Composite Fermions in $\nu = \frac{1}{2} + \frac{1}{2}$ Quantum Hall Bilayers

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In bilayer quantum Hall systems at filling fractions near $\nu = 1/2 + 1/2$, as the spacing d between the layers is continuously decreased, intralayer correlations must be replaced by interlayer correlations, and the composite fermion (CF) Fermi seas at large d must eventually be replaced by a composite boson (CB) condensate or "111 state" at small d. We propose a scenario where CBs and CFs coexist in two interpenetrating fluids in the transition. Trial wave functions describing these mixed CB-CF states compare very favorably with exact diagonalization results. A Chern-Simons transport theory is constructed that is compatible with experiment.

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Bilayer quantum Hall systems show a remarkable variety of phenomena [1]. Perhaps the most studied case is when the electron density in each of the two layers is such that $\nu = n\phi_0/B = \frac{1}{2}$, where n is density, $\phi_0 = 2\pi\hbar c/e$ is the flux quantum, and B is the magnetic field perpendicular to the sample. At this filling fraction, it is known that at least two types of states can occur depending on the spacing d between the layers. For large d the two layers must be essentially independent $\nu = \frac{1}{2}$ states, which are thought to be well described as compressible composite fermion (CF) Fermi seas with strong intralayer correlations and no interlayer correlations [2]. For small enough values of d one should have an interlayer coherent "111 state" which can be described as a composite boson (CB) condensate with strong interlayer correlations and intralayer correlations which are weaker than that of the CF Fermi sea [1]. The nature of the transition between CFs and CBs is the focus of this Letter. (Throughout this Letter we will assume zero interlayer tunneling and assume the spins are fully polarized.)

Initial numerical work [3] suggested that the transition between the CF Fermi sea and the CB 111 state may be first order. (The transition has also been studied theoretically in Refs. [4,5].) However, experiments clearly show that interlayer correlations and coherence turn on somewhat continuously as d/ℓ_B is reduced [6,7] where $\ell_B = (\hbar e/Bc)^{1/2}$ is the magnetic length. Based on a picture of a first order transition and percolating puddles of one phase within the other, Ref. [8] predicts the drag resistivity tensor ρ^D should roughly obey the semicircle relation $(\rho^D_{xx})^2 + (\rho^D_{xy} + \pi\hbar/e^2)^2 = (\pi\hbar/e^2)^2$ which agrees reasonably well with experiment [7]. On the other hand, the first order transition model has trouble accounting for the strong interlayer correlations that appear to occur even deep into the putative Fermi liquid state [7]. We are thus motivated to look for a more continuous transition from the CF Fermi liquid to the CB 111 state. The picture we have in mind is a family of states interpolating between these end points where each state is specified by the number of CFs and the number of CBs with the total number of CBs plus CFs remaining fixed. A first order transition could also be described as a mixture of CBs and CFs but with phase separation of the two fluids. Here we instead consider states where the CF and CB fluids interpenetrate. We find that such mixed CF-CB states agree well with exact diagonalizations. Further, a Chern-Simons version of our mixed Bose-Fermi theory is consistent with experimental observation, predicting the above mentioned semicircle relation.

CF Fermi sea and 111 state.—Near $\nu = \frac{1}{2}$ in a single layer, the Jain CF [2] picture is given by attaching two zeros of the wave function to each particle. Thus we write the electron wave function as $\Psi = \mathcal{P}[\Phi_f[z_1, \bar{z}_1, \ldots, z_N, \bar{z}_N]\prod_{i<j}(z_i - z_j)^2]$ where here and elsewhere $\mathcal{P}$ represents projection onto the lowest Landau level, Gaussian factors $\exp[-\sum_i |z_i|^2/(4\ell_B^2)]$ will not be written explicitly, and $z_i = x_i + iy_i$ is the complex representation of the position. In the above equation Φ_f represents the fermionic wave function of the CFs in the effective magnetic field $\mathcal{B} = B - 2n\phi_0$ where n is the density. Generally, we will assume the CFs are weakly interacting so that the wave function Φ_f can be written as a single Slater determinant appropriate for noninteracting fermions in the effective field $\mathcal{B}$. At $\nu = 1/2$, $\mathcal{B}$ is zero and Φ_f represents a filled Fermi sea wave function.

In Chern-Simons fermion theory [2,9], each electron is exactly transformed into a fermion bound to two flux quanta. At mean field level, the fermions see magnetic field $\mathcal{B} = B - 2n\phi_0$ which is zero at $\nu = \frac{1}{2}$. The effective (mean) electric field seen by a fermion is given analogously by $\boldsymbol{\mathcal{E}} = \mathbf{E} - 2\boldsymbol{\epsilon}\mathbf{J}$ where $\mathbf{E}$ is the actual electric field, $\mathbf{J}$ is the fermion current (which is equal to the electrical current), and $\boldsymbol{\epsilon} = (2\pi\hbar/e^2)\tau$ where $\tau = i\sigma_y$ is the 2 by 2 antisymmetric unit tensor (here σ_y is the Pauli matrix). Defining a transport equation for the weakly interacting fermions $\boldsymbol{\mathcal{E}} = \rho_f\mathbf{J}$ (where ρ_f is approximated as simple Drude or Boltzmann transport for fermions in

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zero magnetic field), we obtain the RPA expression for the electrical resistivity $\rho = \rho_f + 2\boldsymbol{\epsilon}$.

We now turn to the double layer systems and focus on filling fraction $\nu_1 = \nu_2 = \frac{1}{2}$. If the two layers are very far apart, then we should have two independent $\nu = \frac{1}{2}$ systems. Thus, we would have a simple CF liquid state in each layer with the total wave function being just a product of the wave functions for each of the two layers.

On the other hand, if the two layers are brought very close together, the intralayer and interlayer interactions will be roughly the same strength. In this case, it is known that the system will instead be described by the so-called 111 state. The wave function for this state is written as $\Psi = \prod_{i<j}(z_i - z_j)\prod_{i<j}(w_i - w_j)\prod_{i,j}(z_i - w_j)$, where the z_i's represent the electron coordinates in the first layer and the w_i's represent electron coordinates in the second layer. Here each electron is bound to a single zero of the wave function within its own layer as well as being bound to a single zero of the wave function in the opposite layer. By Fermi statistics, each electron must be bound to at least one zero within its own layer so the only additional binding here is interlayer. Thus, when d is small, the electron binds a zero in the opposite layer (to form a CB interlayer dipole), whereas when d is large the electron binds a zero within its own layer (forming a CF dipole [2]).

One can also write a Chern-Simons boson theory for the 111 state. Here, each electron is exactly modeled as a boson bound to one flux quantum where the bosons see the Chern-Simons flux from both layers. Thus, the effective magnetic (mean) field seen by a boson in layer α is given by $\mathcal{B}^\alpha = B - \phi_0(n^1 + n^2)$ with n^i being the density in layer α. Here, at total filling fraction of $\nu_1 + \nu_2 = 1$, the bosons in each layer see an effective field of zero and can condense to form a superfluid (or quantum Hall) state. The effective electric field seen by the bosons in layer α is similarly given by $\boldsymbol{\mathcal{E}}^\alpha = \mathbf{E}^\alpha - \boldsymbol{\epsilon}(\mathbf{J}^1 + \mathbf{J}^2)$ where $\mathbf{E}^\alpha$ is the actual electric field in layer α. This can be supplemented by a transport equation for the bosons in zero magnetic field $\boldsymbol{\mathcal{E}}^\alpha = \rho_b^\alpha \mathbf{J}^\alpha$. If the bosons are condensed, then we can set $\rho_b^\alpha = 0$. This results in the perfect Hall drag of the 111 state where $\mathbf{E}^1 = \mathbf{E}^2 = \boldsymbol{\epsilon}(\mathbf{J}^1 + \mathbf{J}^2)$.

Transition wave functions.— At intermediate d we need to ask what the energetic price is for binding within the layer (to form a CF) versus out of the layer (to form a CB). Note that each fermion put into the Fermi sea costs successively more energy (each having a higher wave vector than the last). Thus, it might be advantageous for some of the fermions at the top of the Fermi sea to become unbound from their zeros within the layer and bind to the other layer—falling into the boson condensate. We imagine having some number N_f^α of electrons in layer α that act like CFs (filling a Fermi sea) and N_b^α that act like CBs (which can condense). Of course, we should have $N_f^\alpha + N_b^\alpha = N^\alpha$ the total number of electrons in layer α. We can write down mixed Fermi-Bose wave functions for double layer systems as follows:

$$\Psi = \mathcal{A}\mathcal{P}\Bigg[\Phi_f^1[z_1, \bar{z}_1, \ldots, z_{N_f^1}, \bar{z}_{N_f^1}]\Phi_f^2[w_1, \bar{w}_1, \ldots, w_{N_f^2}, \bar{w}_{N_f^2}]\prod_{i<j\le N_f^1}(z_i - z_j)^2\prod_{i<j\le N_f^2}(w_i - w_j)^2 \times \prod_{j<i;N_f^1<i}(z_i - z_j)\prod_{j<i;N_f^2<i}(w_i - w_j)\prod_{i,j;N_f^1<i}(z_i - w_j)\prod_{i,j;N_f^2<i;j\le N_f^1}(w_i - z_j)\Bigg]. \quad (1)$$

We have chosen to order the particles so that particles $i = 1, \ldots, N_f^\alpha$ are fermions and $i = N_f^\alpha + 1, \ldots, N^\alpha$ are bosons. The antisymmetrization operator $\mathcal{A}$ antisymmetrizes only over particle coordinates within each layer. Here, Φ_f^α is the CF wave function of N_f^α fermions in layer α. The first line of the wave function is thus the fermionic part, including the CF wave functions and also the Jastrow factors. Here the Jastrow factors bind two zeros to each fermion within a layer so that the wave function vanishes as z^2 as two fermions in the same layer approach each other (before antisymmetrization and projection). The second line of the wave function binds zeros to each boson such that the wave function vanishes as z as any particle (boson or fermion) approaches that boson from either layer. (The bosons are assumed condensed so the explicit boson wave function is unity.)

It is perhaps easier to describe these Jastrow factors in terms of a Chern-Simons description. Within such a description, we write expressions for the effective magnetic field $\mathcal{B}^\alpha$ seen by either species in layer α as

$$\mathcal{B}_f^\alpha = B - 2\phi_0 n_f^\alpha - \phi_0(n_b^1 + n_b^2), \quad (2)$$

$$\mathcal{B}_b^\alpha = B - \phi_0(n_b^1 + n_b^2 + n_f^1 + n_f^2). \quad (3)$$

While it may appear that such a Chern-Simons approach is ill-defined since one must make an arbitrary choice of which electrons are CBs and which are CFs, it turns out that this approach can be put on more rigorous grounds by using a variant of the two-fluid picture of superfluidity [10]. Describing a 111 state as a superfluid of CBs at $\mathbf{k} = \mathbf{0}$, any CBs taken out of the condensate to finite $\mathbf{k}$ form a "normal" fluid which is almost independent of the condensate. This normal fluid can then be independently Chern-Simons transformed again to form CFs. In this picture particles can scatter in and out of the condensate, changing from fermions into bosons or vice versa. This has an analogy in the wave function language where zeros of the wave function are transferred between electrons. (Such processes are ignored in the above simplified

Chern-Simons theory.) More rigorous details of this picture will be given in a future paper.

Since at $\nu = \frac{1}{2}$ the effective magnetic field seen by bosons or fermions is zero, we will take Φ_f^α to be a filled Fermi sea of N_f^α CFs in layer α. Similarly, the CBs in zero field condense into a $\mathbf{k} = \mathbf{0}$ ground state (as assumed in the wave function). If we have two layers of matched density we must have $N_f^1 = N_f^2$ in the ground state. However, the overall number of CFs versus CBs is a matter of energetics and may vary continuously as d changes. When $N_f = 0$ in both layers, this wave function is the 111 state, whereas $N_b = 0$ (or $N_f = N$) consists of two uncorrelated layers of CFs.

Numerical calculations.—We have considered a finite sized bilayer sphere with five electrons per layer and a monopole of flux $9\phi_0$ at its center. We first generate mixed CB-CF wave functions of the form of Eq. (1). The method of numerical generation is involved and will be discussed elsewhere. We note that in the Jastrow factors of Eq. (1) the fermions ($z_i, w_i \leq N_f$) experience one less flux quantum than the bosons ($z_i, w_i > N_f$). Thus, when the CBs are in zero effective magnetic field (so they can condense), the CFs experience a single flux quantum. Thus, Φ_f is modified to represent fermions on a sphere with a monopole of charge ϕ_0 in the center. We generate wave functions with 0, 1, 2, 3, 4, 5 fermions per layer (and correspondingly 5, 4, 3, 2, 1, 0 bosons per layer). Note that the 1 fermion state is identical to the 0 fermion state. However, all of the remaining states are linearly independent. Thus we have 5 states which we label $|0\ \text{fermions}\rangle$, $|2\ \text{fermions}\rangle$, $|3\ \text{fermions}\rangle$, $|4\ \text{fermions}\rangle$, and $|5\ \text{fermions}\rangle$. Of course, $|0\ \text{fermions}\rangle$ is the 111 state, and $|5\ \text{fermions}\rangle$ is two separate layers of composite fermions in the presence of one flux quanta. (In most of the wave functions, the fermionic ground state in a single layer [11] is at maximal angular momentum $L \neq 0$. We always couple these to produce a bilayer $L = 0$ wave function.)

We next perform an exact lowest Landau level diagonalization on the bilayer sphere using a pure Coulomb interaction $v_{11}(r) = e^2/r$ within a layer and $v_{12}(r) = e^2/\sqrt{r^2 + d^2}$ between the two layers where r is the chord distance between two points and d the interlayer spacing. In Fig. 1, we show the overlap (squared) of each of our trial wave functions with the exact ground state at each of the values of d/ℓ_b. We also show the relative energy of the various ground states as a function of d/ℓ_B. It is clear that the mixed CB-CF states have very high overlap with the exact ground state and very low energy in the transition region. This supports the picture of the transition from CF to CB occurring through a set of ground states with interpenetrating CF-CB mixtures. In Fig. 2 we show the interlayer and intralayer electron pair correlation functions (g_{12} and g_{11}). We see that the mixed CB-CF states allow us to interpolate between the types of correlations that exist in the limiting 111 and Fermi liquid states.

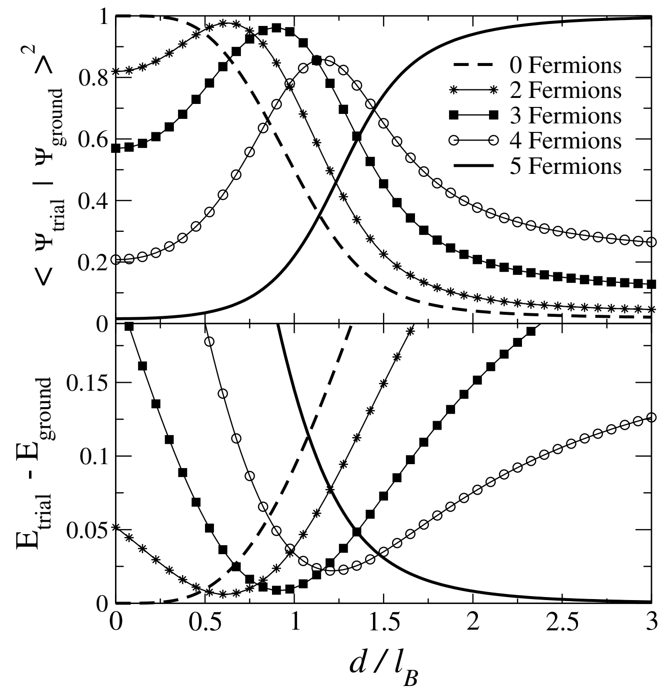

FIG. 1. (Top) Overlap squared of trial wave functions with exact ground state as a function of layer spacing d/ℓ_B. (Bottom) Energy of trial states minus energy of the ground state (in units of e^2/ℓ_B) as a function of d/ℓ_B. Calculations are for 5 electrons per layer on a bilayer sphere with flux $9\phi_0$. At small d/ℓ_B the 111 state ($|0\ \text{fermions}\rangle$) has the highest overlap and the lowest energy. As the spacing increases, we go through a sequence of states $|2\ \text{fermions}\rangle$, $|3\ \text{fermions}\rangle$, and $|4\ \text{fermions}\rangle$ until at large d/ℓ_B $|5\ \text{fermions}\rangle$ (the CF Fermi sea) has the highest overlap and lowest energy.

Chern-Simons RPA.—We can calculate the resistivities and drag resitivities of these mixed Bose-Fermi states by using the Chern-Simons RPA approach. Analogous to Eqs. (2) and (3) we can write effective electric (mean) fields seen by the bosons or fermions in layer α:

$$\mathcal{E}_f^\alpha = \mathbf{E}^\alpha - 2\boldsymbol{\epsilon}\mathbf{J}_f^\alpha - \boldsymbol{\epsilon}(\mathbf{J}_b^1 + \mathbf{J}_b^2), \tag{4}$$

$$\mathcal{E}_b^\alpha = \mathbf{E}^\alpha - \boldsymbol{\epsilon}(\mathbf{J}_b^1 + \mathbf{J}_b^2 + \mathbf{J}_f^1 + \mathbf{J}_f^2), \tag{5}$$

where $\mathbf{J}_{f(b)}^\alpha$ is the Fermi (Bose) current in layer α with the total current in layer α given by $\mathbf{J}^\alpha = \mathbf{J}_b^\alpha + \mathbf{J}_f^\alpha$.

We supplement Eqs. (4) and (5) with transport equations for the fermions (bosons) in each layer $\mathcal{E}_{f(b)}^\alpha = \rho_{f(b)}^\alpha \mathbf{J}_{f(b)}^\alpha$. Finally, for a drag experiment we fix $\mathbf{J}^2 = 0$ and fix $\mathbf{J}^1$ finite. We then solve for $\mathbf{E}^1$ and $\mathbf{E}^2$ yielding the in layer resistivity ($\mathbf{E}^1 = \rho^{11}\mathbf{J}^1$) and the drag resistivity ($\mathbf{E}^2 = \rho^D \mathbf{J}^1$). Assuming layer symmetry so $\rho_{b(f)}^\alpha$ is not a function of α, the results of such a calculation yield $\rho^{11} = (G + H)/2$ and $\rho^D = (G - H)/2$ where $G = (\rho_b^{-1} + \rho_f^{-1})^{-1} + 2\boldsymbol{\epsilon}$, and $H^{-1} = \rho_b^{-1} + (\rho_f + 2\boldsymbol{\epsilon})^{-1}$. At filling fraction $\nu_1 = \nu_2 = \frac{1}{2}$ both ρ_b and ρ_f are diagonal (since CBs and CFs are in zero effective field). Thus, there are

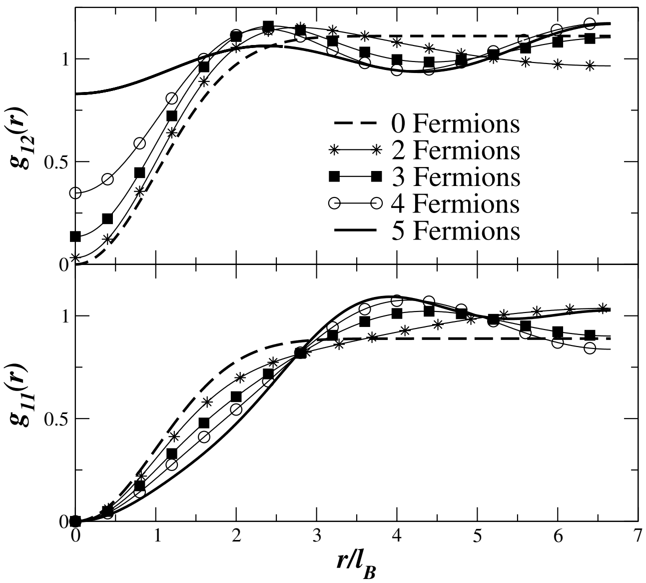

FIG. 2. Interlayer (top) and intralayer (bottom) electron pair correlation functions g_{12} and g_{11} as a function of (arc) distance r for each of the trial wave functions (5 electrons per layer). Going from the Fermi liquid state ($|5$ fermions$\rangle$) to the 111 state ($|0$ fermions$\rangle$) the short range intralayer correlations are reduced and the interlayer correlations build up. In the Fermi liquid state two $L \neq 0$ single layer states are combined to form an $L = 0$ bilayer state causing a deviation of g_{12} from unity.

only two free parameters ($\rho_{b,xx}$ and $\rho_{f,xx}$) and four measurable quantities (ρ_{xx}, ρ_{xy}, ρ^D_{xx}, ρ^D_{xy}) which enable us to derive experimental predictions, such as $\rho^D_{xy} + \rho^{11}_{xy} = 4\pi\hbar/e^2$ [12].

When d is large, we assume that there are very few CBs, and thus $\rho_{b,xx}$ should be large (at least at finite temperature or with disorder). Furthermore, from the experimentally measured CF resistivity, it is clear that $\rho_{f,xx}$ is small ($\ll 2\pi\hbar/e^2$), at least at large d. As d is reduced, presumably the density of CBs increases and $\rho_{b,xx}$ drops until the CBs condense at some critical density and $\rho_{b,xx} = 0$ (yielding perfect Hall drag as in the 111 state). Simultaneously, as d decreases, $\rho_{f,xx}$ presumably increases, but only mildly [9], and may remain small even when the CBs condense. Only when the density of CFs is near zero should $\rho_{f,xx}$ diverge. In the limit that $\rho_{f,xx} \ll 2\pi\hbar/e^2$ it is easy to derive the above mentioned semicircle law $(\rho^D_{xx})^2 + (\rho^D_{xy} + \pi\hbar/e^2)^2 \approx (\pi\hbar/e^2)^2$ from our above expression for ρ^D. In addition, in this limit one can derive $\rho^{11}_{xx} \approx \rho^D_{xx}$ which is also reasonably consistent with published data [7].

One can ask whether at zero temperature CBs (or, almost equivalently, coherence [13]) start being replaced by CFs for any small but finite d, or whether there is a nonzero d below which no CFs exist. Similarly, one could ask whether some CBs remain up to infinite d or whether there is a critical d above which there are none. These two questions remain topics of current research.

Our mixed CB-CF theory can be generalized for unequal densities as well as for filling fractions away from $\nu_1 = \nu_2 = \frac{1}{2}$. Further, one may be able to treat the effects of an in-plane magnetic field. Indeed, such an approach was already used in Ref. [14] to understand bilayer tunneling experiments in tilted fields.

In summary, we have constructed a theory describing the crossover between a CF liquid at large d to the 111 CB state at small d which can be thought of as a two-fluid model. Comparisons of trial wave functions to exact diagonalization are very favorable, and the corresponding Chern-Simons transport theory appears to be in reasonable agreement with experimental data.

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