Status of the Fermi surface in mixed composite-boson–composite-fermion quantum Hall states

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We argue that the naively expected singularities of the Fermi surface, in the mixed composite boson-composite fermion states proposed [S.H. Simon *et al.*, Phys. Rev. Lett. **91**, 046803 (2003)] for the evolution of $\nu=1$ bilayer quantum Hall system with distance, are obliterated. Our conclusion is based on a careful analysis of the momentum distribution in $\nu=\frac{1}{2}$ single-layer composite-fermion state. We point out to a possibility of the phenomenon hitherto unknown outside Kondo lattice systems when, in a translationally invariant system, Fermi-liquid-like portion of electrons enlarges its volume.

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The nature and physics of the transition in the bilayer $\nu=1$ quantum Hall (QH) system[1] between the well-established phases: one characteristic for the distances between the layers of the order of or smaller than magnetic length, sometimes described as "111" state, and the other for larger distances, described by two separate Fermi-liquid-like states of composite fermions (CFs) recently attracted the attention of experimentalists[2] and is the focus of several theoretical papers.[3–6] Only Refs. 4 and 5 make a prediction for a coexistence region between two phases, with a unique property, semicircle law for the longitudinal and Hall drag resistance that was revealed in the experiments.[2] Reference 5 introduces a form of the ground state of the system that may continuously interpolate between the 111 state, usually described by composite bosons (CBs), and the two separate Fermi-liquid-like states of CFs. The ground state proved to be a good variational ansatz when compared with the exact solution in numerical studies.[5] The form of the variational state for certain distance between the layers may be described as one in which classically speaking some of the electrons are in the 111 state (they make CBs) and the others participate in two Fermi seas of CFs. Gradually the number of CFs increases as the distance becomes larger. Therefore the description easily accounts for the continuous nature of the transition as observed in the experiments.[2] On the other hand the proposal that came first, based on a phase separated picture,[4] in which percolating puddles of one phase are in the other, well enough exhibits the transport properties measured in the experiments. The advantage of the homogenous model,[5] which accounts for the same transport properties, is that it also accounts for the strong 111 (interlayer) correlations that occur even deep in the CF region.[2]

Here we study the Fermi surface singularities in the proposed wave functions.[5] Naively they are expected at the Fermi momenta directly related to the number of CFs in the particular partition of the overall number of electrons into CFs and CBs. The analysis begins with a careful study of the $\nu=\frac{1}{2}$ CF problem, so that the relationships found can be readily applied to the mixed state case. We found that the CF momentum distribution near the naively expected Fermi momenta depend analytically on the distance to the Fermi momenta, therefore showing no signature of the Fermi surfaces.

Soon after Halperin, Lee, and Read[7] proposed their theory for $\nu=\frac{1}{2}$ fractional QH effect Bares and Wen[8] considered fermions in low dimensions interacting via a long range $-2\pi/|\vec{q}|^2$ interaction. They used as a good ansatz for the ground state, a wave function of the Feenberg–Jastrow type:

$$\Psi_o(\{x\}) = \prod_{i<j} |x_i - x_j|^m \Psi_{FS}(\{x\}), \tag{1}$$

where Ψ_{FS} denotes a Slater determinant of filled Fermi sea of free single-particle states. If $m=2$ this construction is the Rezayi–Read[9] ground state, in the representation of CFs and when the projection to the lowest Landau level (LLL) is neglected, found to correctly captures the physics at $\nu=\frac{1}{2}$. By doing a calculation of a random phase approximation (RPA) type on Eq. (1) Bares and Wen found that the leading singularity of the momentum distribution near k_F, in two dimensions, is

$$\delta n_k \approx \frac{m}{2}\{n_k^o \ln|\delta k| - (1 - n_k^o)\ln|\delta k|\}, \tag{2}$$

where $\delta k=|\vec{\mathbf{k}}|-k_F$ and n_k^o denotes the free-Fermi-gas momentum distribution. They also remarked that if we interpret the rhs of Eq. (2) as the first term in an expansion in powers of m we can write (near k_F)

$$n_k = \frac{1}{2} + \frac{1}{2}\{n_k^o|k - k_F|^{m/2} - (1 - n_k^o)|k - k_F|^{m/2}\}. \tag{3}$$

What they did not emphasize is that if $m=2$ and although we have a Luttinger-liquid-type expansion near k_F[10] there is no nonanalytic behavior due to the odd power of $|k-k_F|$ and all trace of the Fermi surface has been eliminated.

We can come to the same expressions employing the weakly screening plasma analogy,[11] which in considering quantum-mechanical expectations in the state, Eq. (1), mimics Laughlin's plasma approach.[12] In the Laughlin approach there is the perfect screening of the classical Coulomb plasma, when interaction $-2\pi/|\vec{k}|^2$ becomes screened as

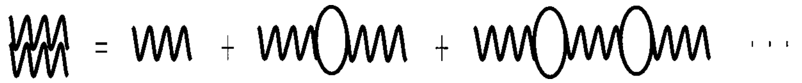

FIG. 1. Effective screened interaction.

$$\frac{-\dfrac{2\pi}{|\vec{k}|^2}}{1+\dfrac{2\pi\beta m^2}{|\vec{k}|^2}s_o(k)}, \tag{4}$$

where m is from $\nu=1/m$, filling factor; $\beta=2/m$ is the plasma inverse temperature, and $s_o(k)$ is the static structure factor of the noninteracting particles, in this case bosons, so that $s_o(k)=\rho$- particle density, and hence a perfect screening. More precisely it can be found[13] that the expansion in small m of classical statistic averages (to which quantum expectations correspond) is well defined, gives the results that can be found by other methods, and allows continuation to larger than $m=1$ values. In this context the screening is captured by the accustomed infinite summation of a geometric series described by Eq. (4) and symbolically can be represented by the sum of diagrams as in Fig. 1.

In the case of the weakly screening plasma analogy due to the presence of the free-fermion Slater determinant in Eq. (1), the first summation, Eq. (4), that is done while organizing diagrams, gets modified, having for $s_o(k)$ the static structure factor of free fermion gas, which in two dimensions for small k can be found to be $s_o^f(k)=\frac{3}{4}k_F/\pi^2 k$. This leads to not so perfect screening of the long-range interaction which becomes as $1/r$ instead of $\ln r$ in real space. The approach introduced parallels to the RPA calculation in Ref. 8 in getting Eq. (2) when Fig. 1 corresponds to a RPA summation with the value of the bubble equal to $s_o^f(k)\beta m^2$.

We want to see in more detail how the equal-time CF propagator can be found, and, possibly, which additional diagrams in its calculation would lead to the conjectured expression for the CF occupation number. It is instructive to first consider how we can get the equal-time CB correlator, i.e., Girvin–MacDonald correlations[14] in the Laughlin case using the diagramatic expansion.[13] As introduced by Girvin and MacDonald we in fact in the plasma language have to deal with two impurities of charge $m/2$ each, which do not interact directly. Therefore, we have

$$G_B(z,z') \sim |z-z'|^{-m/2}\frac{Z(z,z')}{Z(z,z)}, \tag{5}$$

where $Z(z,z')$ is the partition function of the classical 2D plasma with inverse temperature $\beta=2/m$, each particle with charge m, as before, and two impurities with charge $m/2$ each at the locations z and z'. [$Z(z,z)$ is the partition function with one impurity of charge m at an arbitrary location because the value of the partition function does not depend on z.] What we expect is that the ratio will have the following form,

$$\frac{Z(z,z')}{Z(z,z)} = \exp[-\beta\Delta f(z,z')], \tag{6}$$

where $\Delta f(z,z')$ represents the difference in the free energy between the two configurations. Indeed we can find doing the simple expansion in m that the term right after the first term (of value one) is

$$V_{\text{eff}}(|z-z'|) = \left(\frac{m}{2}\right)^2 \int \frac{d^2k}{(2\pi)^2}\exp[i\vec{k}(\vec{r}-\vec{r}\,')]\frac{-\dfrac{2\pi}{|\vec{k}|^2}}{1+\dfrac{2\pi\beta m^2}{|\vec{k}|^2}\rho}, \tag{7}$$

($\rho=1/2\pi m$), which represents an effective screened interaction between two impurities and extract to mimic Eq. (6), contributions of disconnected $V_{\text{eff}}(|z-z'|)$ parts that follow so that for the final expression we get

$$\frac{Z(z,z')}{Z(z,z)} = \exp\{V_{\text{eff}}(|z-z'|)\}. \tag{8}$$

Therefore we can conclude that in calculating $G_B(z,z')$ we have to exponentiate the value of the diagram shown in Fig. 2, and get, due to the screening, the famous algebraic decay.

Similarly, applying the same type of approximation we can get in the CF case

$$G_F(z,z') \sim G_F^o(z,z')\exp\left\{\left(\frac{m}{2}\right)^2\int\frac{d^2k}{(2\pi)^2}\left(\frac{2\pi\beta}{|\vec{k}|^2} - \frac{2\pi\beta}{|\vec{k}|^2+2\pi\beta m^2 s_o^f(k)}\right)\exp[i\vec{k}(\vec{r}-\vec{r}\,')]\right\}, \tag{9}$$

where (the screening bubble is proportional to the static structure factor of free fermions and) $G_F^o(z,z')$

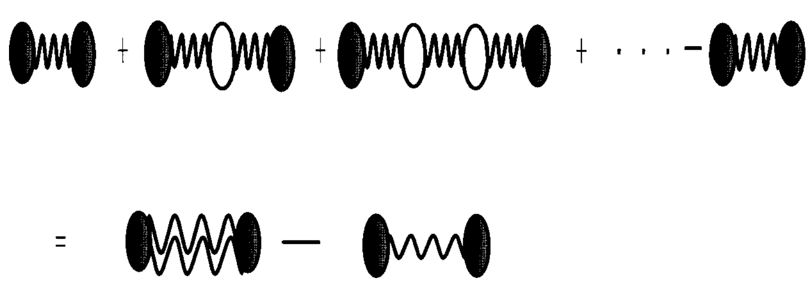

FIG. 2. Effective diagram contribution.

$=\Sigma_{|\vec{k}|<k_F} e^{i\vec{k}(\vec{r}-\vec{r}')}$ is the equal-time correlator of free Fermi gas. To fix the normalization we demand that the total number of CFs is the same as of noninteracting particles so that

$$\sum_{\vec{k}} \int d^2 r G_f^o(\vec{r}) e^{-i\vec{k}\vec{r}} = N = \sum_{\vec{k}} \int d^2 r G_F(\vec{r}) e^{-i\vec{k}\vec{r}}, \quad (10)$$

and $G_F^o(0)=G_F(0)$ follows. Therefore,

$$G_F(z,z') = G_F^o(z,z')\exp\left[\left(\frac{m}{2}\right)^2 \int \frac{d^2k}{(2\pi)^2}\left(\frac{2\pi\beta}{|\vec{k}|^2} - \frac{2\pi\beta}{|\vec{k}|^2 + 2\pi\beta m^2 s_o^f(k)}\right)\{\exp[i\vec{k}(\vec{r}-\vec{r}')] - 1\}\right]. \quad (11)$$

And indeed by taking $n_k=\int e^{-i\vec{k}\vec{r}} G_F(\vec{r})$ and considering the first nontrivial contribution in the expansion of the exponential in Eq. (11) we get

$$\delta n_k = \left(\frac{m}{2}\right)^2 \int \frac{d^2q}{(2\pi)^2}\left(\frac{2\pi\beta}{|\vec{q}|^2} - \frac{2\pi\beta}{|\vec{q}|^2 + 2\pi\beta m^2 s_o^f(q)}\right)[n_{k-\vec{q}}^o(1 - n_k^o) - n_k^o(1 - n_{k-\vec{q}}^o)], \quad (12)$$

exactly the same expression as Eq. (86) of Ref. 8. Once we specify that k is near k_F, assume a flat Fermi surface and neglect the contribution of the (weakly) screened interaction in Eq. (12) we can get, as in Ref. 8, the leading singularity in n_k given by Eq. (2).

But unfortunately after a suitable regrouping of free-Fermi-gas occupation numbers we can prove that the second nontrivial contribution in the expansion of the exponential (with the neglect of the screened interaction) is equal to zero. That does not mean that the conjectured contribution [Eq. (3)] is absent. Here, we have very likely the situation that due to the nonanalytic nature of the attempted expansion in the CF case we cannot generate corrections to the terms linear in m. That conclusion supports also the finding[13] that when the same expansion was applied in the calculation of the static structure factor of the CF state a first correction to the RPA result could not be generated although it was expected on the grounds that the correction would have made the infered LLL-projected static structure factor positive definite what by its definition it should be.

Therefore, very likely the exponential prescription [used to get Eq. (3)] is a valid one although there is no straightforward expansion to prove it. Once we accept the prescription we are left to wonder where is the expected nonanalyticity at $\nu=1/m$, $m/2=$*odd integer* [see Eq. (3) for that case]. A trace of the real Fermi-surface nonanalyticity at $m/2=$*odd integer* may be seen in the expansion only if we take into account the screened interaction (second part) in Eq. (12). As an effective contribution from this part we have

$$\delta n_k = \frac{1}{2\pi m s_o}[|\delta k|\ln|\delta k| n^o(k) - [1 - n^o(k)]|\delta k|\ln|\delta k|], \quad (13)$$

where $s_o = \frac{3}{4}k_F/\pi^2$. If we apply the exponential prescription again, taking also into account this second contribution, we have, with $1/2\pi m s_o \equiv c$ and, for $m/2=1$, for the contribution in the vicinity of k_F:

$$n_k = \frac{1}{2} + \frac{1}{2}[|\delta k|\exp\{c|\delta k|\ln|\delta k|\} n^o(k) - [1 - n^o(k)] \times |\delta k|\exp\{c|\delta k|\ln|\delta k|\}]. \quad (14)$$

Here a (weak) nonanalyticity is retained. Namely, in Eq. (14) we have singular (at k_F) the second derivative of $|\delta k|\exp\{c|\delta k|\ln|\delta k|\}$ with respect to $|\delta k|$. Therefore, a trace of the Fermi surface at $\nu=1/m$, $m/2=1$, is present because of the found nonanalytic behavior. (Such a behavior exists also for $m/2=odd>1$ but is weaker having singular higher derivatives.)

In the following we will give an example where aforementioned mechanism for getting the Fermi surface (nonanalyticity) does not work due to strong correlations of the CFs with other particles of the system. This is the case of the mixed CB–CF quantum Hall states proposed in Ref. 5 to describe the evolution of the bilayer $\nu=1$ QH system with distance between layers.

If we neglect the LLL projection again and assume that for our purposes we can also neglect the overall antisymmetrization between CB and CF parts that makes the mixed state completely antisymmetric and an electronic wave function, we can write it in the quasiparticle representation as

$$\begin{aligned}\Psi_o(z_\uparrow,z_\downarrow,w_\uparrow,w_\downarrow) &= \Pi_{i<j}|z_{i\uparrow} - z_{j\uparrow}|^n \Pi_{k<l}|z_{k\downarrow} - z_{l\downarrow}|^n \Pi_{p<q}|z_{p\uparrow} - z_{q\downarrow}|^n \\ &\Pi_{i,j}|z_{i\uparrow} - w_{j\uparrow}|^n \Pi_{k,l}|z_{k\uparrow} - w_{l\downarrow}|^n \Pi_{p,q}|z_{p\downarrow} - w_{q\uparrow}|^n \Pi_{r,s}|z_{r\downarrow} - w_{s\downarrow}|^n \\ &\Pi_{i<j}|w_{i\uparrow} - w_{j\uparrow}|^m \Psi_{FS}(\{w_\uparrow\}) \Pi_{k<l}|w_{k\downarrow} - w_{l\downarrow}|^m \Psi_{FS}(\{w_\downarrow\}),\end{aligned} \quad (15)$$

where $z_\uparrow$ and $z_\downarrow$ denote CB coordinates and $w_\uparrow$ and $w_\downarrow$ denote CF coordinates with arrows specifying to which layer quasiparticles belong. The total numbers of bosons are equal as well the total numbers of fermions, and $n=m/2$ $=$odd integer.

We would like to find out (the asymptotic behavior of) the equal-time correlator of a CF (belonging to one of the layers). It is not hard to conclude that in this case with assumptions similar to the ones done in the single-layer case, we get $G_F(w,w')$ by simply taking for the value of the "polarization" bubble:

$$\beta m^2 s_o^f(k) + \beta n^2 \rho_b, \quad (16)$$

instead of $\beta m^2 s_o^f(k)$ only in Eq. (11), where ρ_b denotes the total (up plus down) density of bosons and in $s_o^f(k)$ we have to take $k_F=\sqrt{4\pi\rho_f}$ where ρ_f is the density of fermions of one layer only. In this case we work with a completely screened interaction between the two impurities which does not produce nonanalytic contributions.

Therefore we find that at the total fillings of bilayer at which we can expect Bose–Fermi mixed states, $\nu = 2/m$; $m = 2, 6, \ldots$ the naively expected Fermi surface(s) cannot exist due to our analysis. This outcome reminds us of the similar disappearance of the small (naively) expected Fermi momentum in the Kondo lattice systems[15,16] due to the Luttinger theorem.[17] In the case considered in this paper we do not know for sure if we deal with (overall) Fermi-liquid-like states and a complete analogy (in which CBs and CFs play the roles of of localized spin $1/2$ local moments and conduction electrons, respectively) is still missing. Further insights into the physics of the mixed states are necessary.

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