

Wave-functional approach to the bilayer $\nu=1$ system and a possibility for a double nonchiral pseudospin liquid

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We systematically discuss candidate wave functions for the ground state of the bilayer $\nu=1$ as the distance between the layers is varied. Those that describe increased intralayer correlations at finite distance show a departure from the superfluid description for smaller distances. They may support finite energy meron excitations and a dissipative collective mode in the place of the Goldstone mode of the ordered phase, i.e., describe a vortex metal phase, or imply even an incompressible, pseudospin liquid, behavior. Therefore they describe possible outcomes of quantum disordering at finite distance between the layers. The vortex metal phase may show up in experiments in the presence of disorder at lower temperatures and explain the observed “imperfect superfluidity,” and the pseudospin liquid phase may be the cause of the thermally activated (gapped) behavior of the longitudinal and Hall resistances at higher temperatures in counterflow experiments.

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I. INTRODUCTION

The bilayer $\nu=1$ quantum Hall (QH) system consists of two layers of two-dimensional (2D) electron gases, each with a filling factor $1/2$, that are brought together at distance comparable to the average distance between electrons inside layers and the tunneling is negligible. The physics of the bilayer has been the focus of much of experimental and theoretical work. We will mention only two major experimental findings. First in the experiment of Spielman *et al.*¹ there was a very pronounced (“spectacular”) zero-bias peak in the tunneling conductance between layers. Second in the experiments of Kellogg *et al.*² and Tutuc *et al.*,³ in the counterflow setting of the bilayer (where in each layer the current is of opposite sign than the other) the resulting longitudinal and Hall resistances were dropping to zero in the limit of zero temperature. Both types of experiments signal a superfluid behavior in the pseudospin (relative between layers) channel of the system. An elaborated theoretical work of Moon *et al.*⁴ described this behavior in the framework of an easy-plane ferromagnet (where the layer index represents the electron pseudospin degree of freedom) and ensuing XY model description. Therefore the superfluid behavior was expected. Usually it is described as a consequence of an excitonic condensate inside the bilayer where each exciton stems from the electron coupling to its correlation hole just opposite in the other layer. Nevertheless some discrepancies were noticed. In the first experiment of Spielman *et al.* the peak due to some unknown source of dissipation, is not as high and narrow as in the analog, Josephson tunneling experiments in superconductors. In the second experiment of Kellogg *et al.* and Tutuc *et al.* there were no Kosterlitz-Thouless transition signatures in the measured counterflow resistance. It seems merons (vortices) are liberated (due to some unknown cause) from their confinement in the expected superfluid down to very low temperatures.

These findings point out that a careful investigation of quantum fluctuations or quantum disordering at finite d (distance between the layers) is necessary. One way to approach this question is to adopt the Laughlin approach to the frac-

tional QH effect and look for approximate but very close to the ground states wave functions at finite d . That this is possible it was numerically demonstrated in Ref. 5 for not large d . The functions that approximate the true ground states incorporate the effects of quantum disordering by allowing the presence of composite fermions (CFs) next to composite bosons (CBs), another transformed electrons, characterizing the Bose condensate of the assumed superfluid. CFs connect in a special way to the condensate to ensure and maintain the rigidity of the Bose condensate.

In this work we will systematically discuss candidates that we expect would approximate very well the ground state wave functions at finite d . Some of them we expect would be in a competition with the ones that are ferromagnetically ordered and possess a Goldstone mode. Among them we will point out one that describes an incompressible (in all channels) state. If the state were a true ground state, it would be a ground state of topological phase that supports quasiparticles—merons with finite excitation energy. In fact this would be a realization of what is usually called a spin liquid phase with two kinds of semionic quasiparticles—i.e., a double spin liquid, with the only difference that here we work with pseudospin instead of spin. We will use interchangeably the words double pseudospin liquid and pseudospin liquid, for short, denoting the same phase. This gapped phase might be the one that appears in counterflow experiments causing activated (gapped) behavior of the longitudinal and Hall resistances for a range of higher temperatures. Another competing possibility with possibly finite excitation energy for merons is a compressible version in which quantum disordering has allowed weakly coupled meron-antimeron pairs, i.e., a vortex metal phase. This state supports a dissipative collective mode that comes in the place of the Goldstone mode of the ordered phase. The physics of this state should be relevant for the explanation of the dissipation effects in the experiments, in which disorder, at lower temperatures, would dissociate the expected closed loops of meron-antimeron pairs in the topological phase.

The paper is organized as follows: Sections II–IV contain results of our paper, and Secs. V and VI provide discussion

and conclusions. Section II is an introduction to the wave-functional approach to the bilayer, Sec. III in two different approaches with compatible conclusions describes the physics of a candidate wave function—vortex metal state, and Sec. IV introduces its modification due to CF pairing—a pseudospin liquid state.

II. WAVE-FUNCTIONAL APPROACH TO THE BILAYER $\nu=1$ SYSTEM

If the distance between the layers is of the order or less the magnetic length—average distance between electrons inside the layers, interlayer Coulomb interaction will force the system, at total filling factor one and in the conditions of QH effect, to form the (111) state. The state is a simple generalization of the single-layer filling-factor-one case, i.e., completely filled lowest Landau level (LLL) of one species electrons, precisely it is

$$\Psi_{111}(z_{\uparrow}, z_{\downarrow}) = \prod_{i < j} (z_{i\uparrow} - z_{j\uparrow}) \prod_{k < l} (z_{k\downarrow} - z_{l\downarrow}) \prod_{p, q} (z_{p\uparrow} - z_{q\downarrow}), \quad (1)$$

where $z_{i\uparrow}$ and $z_{i\downarrow}$ are two-dimensional complex coordinates of electrons in upper and lower layer, respectively, and we omitted the Gaussian factors. If layers are far apart each one will be a separate system at filling factor one-half. Numerous studies⁶ show that the ground state at that filling factor is a generalization of the Laughlin construction which includes a single Slater determinant of noninteracting particles in zero magnetic field, i.e.,

$$\Psi_{1/2}(w) = \mathcal{P} \left(\Phi_f(w, \bar{w}) \prod_{i < j} (w_{i\uparrow} - w_{j\uparrow})^2 \right), \quad (2)$$

where Φ_f is the determinant and \mathcal{P} represents projection to LLL. In Refs. 6 and 7 it was shown that relevant, underlying composite particles in this case are CFs. On the other hand, the relevant, weakly interacting composite particles in the (111) case are CBs.^{8,9}

Now let us start from the (111) case, increase the distance, and introduce one-half correlations in the (111) state in a minimal way, preserving (111) intercorrelations of newly introduced CFs with all other remaining CBs. This means that though we are perturbing the (111) state, we are assuming its inherent rigidity. The wave function that describes this is

$$\begin{aligned} \Psi_{bbf} = & \mathcal{P} \mathcal{A} \left(\prod_{i < j} (z_{i\uparrow} - z_{j\uparrow}) \prod_{k < l} (z_{k\downarrow} - z_{l\downarrow}) \prod_{p, q} (z_{p\uparrow} - z_{q\downarrow}) \right. \\ & \times \Phi_f(w_{\uparrow}, \bar{w}_{\uparrow}) \prod_{i < j} (w_{i\uparrow} - w_{j\uparrow})^2 \\ & \times \Phi_f(w_{\downarrow}, \bar{w}_{\downarrow}) \prod_{k < l} (w_{k\downarrow} - w_{l\downarrow})^2 \\ & \times \prod_{i, j} (z_{i\uparrow} - w_{j\uparrow}) \prod_{k, l} (z_{k\uparrow} - w_{l\downarrow}) \\ & \left. \times \prod_{p, q} (z_{i\downarrow} - w_{q\uparrow}) \prod_{m, n} (z_{m\downarrow} - w_{n\downarrow}) \right), \quad (3) \end{aligned}$$

where we omitted the Gaussian factors, \mathcal{P} denotes the projection to the LLL, \mathcal{A} denotes the antisymmetrization between Bose and Fermi variables in each layer separately, and Φ_f 's are Slater determinants of free waves. In the thermodynamic limit the relationship between the flux and number of particles is

$$\begin{aligned} N_{\Phi} = & N_{b\uparrow} + N_{b\downarrow} + N_{f\uparrow} + N_{f\downarrow} = 2N_{f\uparrow} + N_{b\uparrow} + N_{b\downarrow} \\ = & 2N_{f\downarrow} + N_{b\uparrow} + N_{b\downarrow} \end{aligned} \quad (4)$$

Consequently we must have $N_{f\uparrow} = N_{f\downarrow}$ but there is no constraint on the relative number of bosons.

In fact, once we adopt the hypothesis that with increasing d (distance) CFs are slowly nucleating we are left, because of the flux-counting arguments, with only two (simple, Laughlin) possibilities for interpolating ground state. The second possibility has instead of the last two lines in (3) for the intercorrelations between the Bose and Fermi parts, the following expression:

$$\prod_{i, j} (z_{i\uparrow} - w_{j\uparrow})^2 \prod_{k, l} (z_{k\downarrow} - w_{l\downarrow})^2. \quad (5)$$

These intercorrelations are of the Fermi part type, i.e., they are intralayer correlations, a consequence of possibly more important intralayer Coulomb interactions.

In this case we have

$$\begin{aligned} N_{\Phi} = & 2N_{f\uparrow} + 2N_{b\uparrow} = 2N_{f\downarrow} + 2N_{b\downarrow} = 2N_{f\uparrow} + N_{b\uparrow} + N_{b\downarrow} \\ = & 2N_{f\downarrow} + N_{b\uparrow} + N_{b\downarrow}, \end{aligned} \quad (6)$$

i.e., both Bose and Fermi numbers are constrained to $N_{f\uparrow} = N_{f\downarrow}$ and $N_{b\uparrow} = N_{b\downarrow}$.

In the small particle (5+5) numerical study in Ref. 5, the first possibility had much larger overlaps, presented in Ref. 5, with the true ground state as it was evolving with distance (and the ratio between bosons and fermions was changing) than the second possibility [Eq. (5)]. The overlaps of the first possibility were slowly decreasing with distance so that no definite conclusions can be drawn about the precise evolution of the system near the transition region. But for smaller d the relevance of the first possibility for the evolution was established.

We will not discuss the states that we naturally expect would interpolate between these two limits, small d [Eq. (3)] and near the transition [Eq. (5)] limit. In these states some of the fermions would connect via Bose (111) type intercorrelations to the Bose part and some via Fermi type.

It is no wonder that the first possibility is more relevant for smaller d . It also allows the imbalanced ($N_1 \neq N_2$) situation, which is required due to the theoretical and experimental insight gained^{1,4,10,11} about the existence of the Goldstone mode connected with the particle number difference. (The projected to definite particle number picture, which we here discuss, must incorporate that physics.) Later we will show that the state is incompressible (in the charge channel), at least in the scope of a suggested Chern-Simons picture.

A. Chern-Simons approach

In this section we will employ the classical CS approach, described in Ref. 8 and Ref. 7 with its usual RPA (random

phase approximation) to find out the linear response of the system that supports the ground state in Eq. (3). This approach is advanced in Ref. 12 but at the same time there, a conclusion is drawn that the classical CS approach in the RPA allows us with not much work to find out qualitatively the physics of the linear response. Here we have still another problem; there is no obvious way to implement antisymmetrization procedure of the wave function in Eq. (3). The antisymmetry of electronic wave functions comes naturally and automatically only in only CB or only CF theories. Therefore to simplify matters from the start we choose the classical CS approach and neglect the antisymmetrization requirement.

Without the antisymmetrization the wave function in (3) would be the ground state (in the RPA and when the projection to LLL is neglected) of the following CS theory:

$$\begin{aligned}
 \mathcal{L} = & \sum_{\sigma} \left(i\Psi_{\sigma}^{\dagger}(\partial_0 + ia_0^{\sigma} - iA_0 - i\sigma B_0)\Psi_{\sigma} \right. \\
 & + \frac{1}{2m} \sum_k \Psi_{\sigma}^{\dagger}(\partial_k + ia_k^{\sigma} - iA_k - i\sigma B_k)^2 \Psi_{\sigma} \left. \right) \\
 & + \sum_{\sigma} \left(i\Phi_{\sigma}^{\dagger}(\partial_0 + ia_0 - iA_0 - i\sigma B_0)\Phi_{\sigma} \right. \\
 & + \frac{1}{2m} \sum_k \Phi_{\sigma}^{\dagger}(\partial_k + ia_k - iA_k - i\sigma B_k)^2 \Phi_{\sigma} \left. \right) \\
 & + \sum_{\sigma} \frac{1}{2\pi^2} a_0^{\sigma} (\vec{\nabla} \times \vec{a}^{\sigma}) \\
 & - \frac{1}{2} \sum_{\sigma} \int d\vec{r}' \delta\rho_{\sigma}(\vec{r}) V_a(\vec{r} - \vec{r}') \delta\rho_{\sigma}(\vec{r}') \\
 & - \int d\vec{r}' \delta\rho_{\uparrow}(\vec{r}) V_e(\vec{r} - \vec{r}') \delta\rho_{\downarrow}(\vec{r}'), \quad (7)
 \end{aligned}$$

where σ is the layer index, taking \uparrow and \downarrow values as a variable, Ψ_{σ} and Φ_{σ} are CF and CB σ -layer fields, respectively, and V_a and V_e are intra- and inter-Coulomb interactions. $\delta\rho_{\sigma}$'s are given as sums of CBs and CFs, i.e., $\delta\rho_{\sigma} = \delta\rho_{\sigma}^F + \delta\rho_{\sigma}^B$, as the most natural choice for the electron density in this distinguishable picture. External fields, A and B , couple to charge and pseudospin (up minus down) degrees of freedom, respectively. The CS, gauge fields, a_{\uparrow} and a_{\downarrow} , are related to a as

$$a = \frac{a_{\uparrow} + a_{\downarrow}}{2},$$

and in this way, in the mean field approximation, reproduce the relations encoded in the ground state given by Eq. (3).

The antisymmetrization can be implemented by the following constraint:

$$\vec{S} \cdot \vec{S} = \frac{N_{\sigma}}{2} \left(\frac{N_{\sigma}}{2} + 1 \right), \quad (8)$$

for each layer separately so that N_{σ} denotes the number of electrons in the layer. \vec{S} denotes a generalized spin of the

layer obtained by integrating over the volume of the system of the following field density:

$$\hat{\Psi}_{\sigma}^{\dagger}(\vec{r}) \frac{\vec{\sigma}}{2} \hat{\Psi}_{\sigma}(\vec{r}).$$

$\hat{\Psi}_{\sigma}(\vec{r})$ denotes a spinor for which we have

$$\hat{\Psi}_{\sigma}(\vec{r}) = \begin{bmatrix} U_b^{\dagger} \Phi_{\sigma} \\ U_{f,\sigma}^{\dagger} \Psi_{\sigma} \end{bmatrix},$$

where U_b and $U_{f,\sigma}$ are the CS unitary transformations, i.e.,

$$\begin{aligned}
 U_b(r) = \exp \left(i \int d^2r' \arg(r - r') [\rho_{b\sigma}(r') + \rho_{b-\sigma}(r') \right. \\
 \left. + \rho_{f\sigma}(r') + \rho_{f-\sigma}(r')] \right) \quad (9)
 \end{aligned}$$

and

$$\begin{aligned}
 U_{f\sigma}(r) = \exp \left(i \int d^2r' \arg(r - r') \right. \\
 \left. \times [2\rho_{f\sigma}(r') + \rho_{b\sigma}(r') + \rho_{b-\sigma}(r')] \right), \quad (10)
 \end{aligned}$$

Φ_{σ} and Ψ_{σ} are the already introduced Bose and Fermi field, respectively, and $\vec{\sigma}$ are the usual Pauli matrices. The idea behind the constraint in Eq. (8) is simple; it uses the fact that spin $\frac{1}{2}$ particles, N_{σ} of them, must necessarily, in the states for which the constraint in Eq. (8) is true, i.e., have $\vec{S} \cdot \vec{S}$ at its largest possible value, be completely symmetric in the spin space and necessarily antisymmetric in the real space.

We still must fix S_z , which is the second constraint, i.e., the number difference between CBs and CFs in the layer, to project the artificially introduced spin $\frac{1}{2}$ problem to the reduced Hilbert space and our problem.

What we can immediately notice is that the constraint in Eq. (8) will introduce the terms that interchange bosons and fermions at different points instantaneously which conforms to our idea of indistinguishability. Although it is easy to formulate it is very hard to implement the constraint. Only, maybe, if we have overwhelming number of bosons or fermions we will be allowed to neglect the constraint as it is usual in the case of bose fluids with the natural decomposition into classical-macroscopic, condensate part and normal part.

Nevertheless there is a much deeper reason that allows us to neglect the antisymmetrization requirement. The reason is that, just like in a hierarchical construction and as it will be much more clear later, CFs represent meron excitations (meron-antimeron pairs, see the Appendix) that quantum disorder the (111) state. As it is usual when we discuss the dual picture of the FQHE,¹³ we do not extend the antisymmetrization requirement to the quasiparticle part of the electron fluid.

We want to prove that indeed the static density-density response that follows from the Lagrangian in Eq. (7) shows the incompressibility of the system with the ground state in

Eq. (3) in which the antisymmetrization is neglected. The charge of the system is related to $\delta\vec{a}$ as

$$\delta\rho = \delta\rho_{\uparrow} + \delta\rho_{\downarrow} = \frac{ik\delta a}{2\pi},$$

where, as we work in the transverse gauge, $|\delta\vec{a}| = \delta a$ is the transverse space component of the vector. For the response calculations (with $B_0 = B = 0$) we will adopt the conventions introduced in Ref. 7. Integrating out the quadratic terms in fermionic fields to the second order in gauge fields (RPA),⁷ then introducing density-angle variable for the bosonic fields and expanding the action in them neglecting the amplitude-density derivatives,⁸ we get

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \sum_{\sigma} \frac{1}{2} [K_{00}^{\sigma} (\delta a_0^{\sigma})^2 + K_{11}^{\sigma} (\delta a^{\sigma})^2] + \frac{1}{2\pi} \frac{1}{2} \sum_{\sigma} a_0^{\sigma}(-k) ika^{\sigma}(k) \\ & + \sum_{\sigma} \delta\rho^{B\sigma}(-k) [i\omega\Theta_{\sigma}(k) - \delta a_0^{\sigma}(k)] \\ & + \sum_{\sigma} (-) \frac{\bar{\rho}_b}{2m} [k^2\Theta_{\sigma}^2 + (\delta a)^2] \\ & - \frac{1}{2} \frac{k^2(\delta a)^2}{(2\pi)^2} V_c(k) - \frac{1}{2} V_s(k) \left| \delta\rho^{B\uparrow} - \delta\rho^{B\downarrow} + \frac{1}{2\pi} ika^s \right|^2, \end{aligned}$$

where $V_c(k) = \frac{V_a(k) + V_v(k)}{2}$, $V_s(k) = \frac{V_a(k) - V_v(k)}{2}$, $K_{00} = K_{00}^{\uparrow} = K_{00}^{\downarrow}$, and $K_{11} = K_{11}^{\uparrow} = K_{11}^{\downarrow}$ are density-density and current-current response functions of a free fermionic system, respectively,⁷ Θ_{\uparrow} , Θ_{\downarrow} are bosonic angle variables and $\bar{\rho}_b$ averaged bosonic density per layer. In the static ($\omega=0$) case we are left with the following charge part, decoupled from the pseudospin part in the RPA:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & K_{00}(\delta a_0)^2 + K_{11}(\delta a)^2 + \frac{1}{2\pi} a_0 ika \\ & + \delta\rho_c^b \delta a_0 - \frac{2\bar{\rho}_b}{2m} |\delta a|^2 - \frac{1}{2} \frac{k^2(\delta a)^2}{(2\pi)^2} V_c(k). \end{aligned}$$

The integration over $\delta\rho_c^b$ gives the constraint $a_0 = A_0$ and consequently the integration over δa gives for the (static) density-density response,

$$\frac{\left(\frac{1}{2\pi}\right)^2 k^2}{\frac{2\bar{\rho}_b}{m} + \frac{k^2 V_c(k)}{(2\pi)^2} - 2K_{11}},$$

where $K_{11} = -\frac{k^2}{12\pi m}$.⁷ As long as there is a finite density of bosons ($\bar{\rho}_b$) the system is quantum Hall, incompressible (and the above expression vanishes in $k \rightarrow 0$ limit). Even the presence of composite fermions, i.e., particles which we would naively consider to represent gapless degrees of freedom in the system, does not lead to compressibility. A small fluctuation in the charge density ($\delta\rho_f = \delta\rho_f^{\uparrow} + \delta\rho_f^{\downarrow}$) of the composite fermions lead to a fluctuation in the bosonic charge density for which we know we need a finite amount of energy to create.⁸

III. A STATE WITH POSSIBLY DECONFINED MERONS

The second possibility, state

$$\begin{aligned} \Psi_{bff} = & \mathcal{P}\mathcal{A} \left(\prod_{i<j} (z_{i\uparrow} - z_{j\uparrow}) \prod_{k<l} (z_{k\downarrow} - z_{l\downarrow}) \prod_{p,q} (z_{p\uparrow} - z_{q\downarrow}) \right) \\ & \times \Phi_f(w_{\uparrow}, \bar{w}_{\uparrow}) \prod_{i<j} (w_{i\uparrow} - w_{j\uparrow})^2 \Phi_f(w_{\downarrow}, \bar{w}_{\downarrow}) \\ & \times \prod_{i<j} (w_{i\downarrow} - w_{j\downarrow})^2 \prod_{i,j} (z_{i\uparrow} - w_{j\uparrow})^2 \prod_{k,l} (z_{k\downarrow} - w_{l\downarrow})^2 \end{aligned} \quad (11)$$

should be relevant for the transition into two decoupled Fermi seas region. In the following section we will explore its properties; in the first part we will show that in this state possibly a finite energy is needed to excite meron, and in the second part we will show that the state is compressible in the pseudospin channel and nearly supports a gapless pseudospin mode. The first property tells us that in this state merons may be deconfined relative to the (111) and Eq. (3) states in which merons as vortices of the pseudospin superfluid are confined. If it were not for the second property, the state would possibly describe a quantum Hall, topological phase with four kinds of gapped, meron quasiparticle excitations.

A. The screening of meron

In the following we will show that the screening charges of the meron excitation in the plasma analogy of the state in Eq. (11) are localized, not of long range, and may not lead to the usual logarithmic divergence with the size of the system of the energy required to excite a meron.

Namely we will use an effective expression from Ref. 14 for the meron excitation that in the (relative change in) density calculations for the meron state leads to results valid in the long wavelength limit. The effective expression is the one reduced from the well-known expression in the (111) state,⁴ and in Ref. 14 it was shown that leads to the logarithmic divergence in the energy to excite meron in both (111) and the first mixed state [Eq. (3)] that describe the pseudospin condensate at small d . Therefore the effective expression gives the expected behavior of a meron excitation in these states. As in the Laughlin quasihole construction, the meron effective construction multiplies the ground state, Ψ_o . Explicitly the construction is

$$\prod_i \frac{z_{i\uparrow} - w}{|z_{i\uparrow} - w|} \exp\left(\sum_i \frac{C}{2|z_{i\uparrow} - w|}\right) \Psi_o, \quad (12)$$

where C is a constant, found to be equal to $C=0.80$ in the case of the (111) state. We emphasize that Eq. (12) is an effective expression, modeled for the task of density calculations, and in no way an expression that would be valid in the short distance limit or stand for a meron construction in the lowest Landau level.

In the plasma language of Ref. 14, and as an interpretation of the squared norm of the construction [Eq. (12)] we have that an impurity at point w connects via the interaction $C/|z_{i\uparrow}|$ to the \uparrow particles of the plasma defined by the state, Ψ_o . To obtain the charge contributions (\uparrow and \downarrow) far away

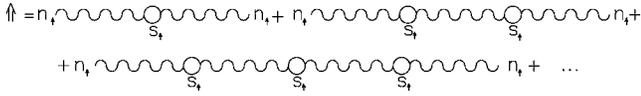


FIG. 1. The infinite sum of Eq. (13)

from the center of excitation in the second state [Eq. (11)], i.e., if we take in Eq. (11) that Ψ_o is the second state, we use the same type of approximations explained in Refs. 15 and 14. We assume that the classical partition function of the classical system that is defined by the square norm of the second state has the property of screening, and therefore that the essential physics including the description of the screening charges of impurities can be found by summing so-called chain diagrams. The contributions to the screening charges of an impurity can be easily visualized as a type of chain diagrams shown in Figs. 2 and 3 which connect impurity (w) to the probing point (r) on the right-hand side. In the figures the straight line denotes the $\frac{1}{r}$ interaction, i.e., $\frac{1}{q}$ in the momentum space by which the meron in Eq. (12) connects to \uparrow particles, the wiggly line denotes the $\ln\{r\}$ ($\sim \frac{1}{q}$) interaction, i.e., the Coulomb interaction of 2D plasma, and there are two kinds of vertices. For bosonic quasiparticles we have a vertex that is equal to their densities, n_\uparrow or n_\downarrow , but for fermionic quasiparticles the value of the vertex is the static structure factor of free Fermi gas, i.e., $s_\uparrow(q)$ or $s_\downarrow(q)$ for which we have $s_\uparrow(q)=s_\downarrow(q)\sim q$ in the long wavelength limit. To calculate the screening charges let us introduce two infinite sums; the one for \uparrow particles is depicted in Fig. 1 and equals

$$\uparrow\uparrow = \frac{4s_\uparrow V^2 n_\uparrow^2}{1-2s_\uparrow V} \xrightarrow{q\rightarrow 0} -2Vn_\uparrow^2, \quad (13)$$

where $V\sim \frac{1}{q^2}$, and similarly we have $\downarrow\downarrow$ for \downarrow particles.

Using these infinite sums we are able to write the effective interaction between n_\uparrow vertices in a compact form,

$$\begin{aligned} \mathcal{V}(n_\uparrow, n_\uparrow) &= \frac{n_\uparrow V n_\uparrow}{1-V(n+\uparrow+\downarrow)} + \frac{\uparrow\uparrow}{1-\frac{\uparrow\uparrow}{n_\uparrow}} \\ &+ \frac{n_\uparrow V \uparrow\uparrow + \uparrow\uparrow V n_\uparrow}{1-V(n+\uparrow+\downarrow)} \frac{1}{1-2\frac{\uparrow\uparrow}{n_\uparrow}} \xrightarrow{q\rightarrow 0} -n_\uparrow. \end{aligned} \quad (14)$$

In Eq. (14) we also stated the value of the interaction in the long wavelength limit. The introduction of the infinite sums comes naturally because of the type of intercorrelations that exist in the electronic wave function. In this way we can express the effective interaction between a bosonic and fermionic \uparrow vertex, $\mathcal{V}(n_\uparrow, s_\uparrow)$, as

$$\mathcal{V}(n_\uparrow, s_\uparrow) = n_\uparrow \frac{2s_\uparrow V}{1-2s_\uparrow V} + \mathcal{V}(n_\uparrow, n_\uparrow) \frac{2s_\uparrow V}{1-2s_\uparrow V} \xrightarrow{q\rightarrow 0}, \quad (15)$$

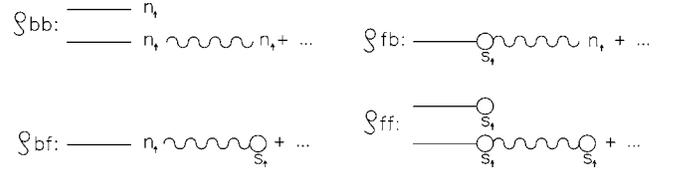


FIG. 2. The up density screening.

with 0 as the value in the long wavelength limit. Similarly we have for the effective interaction between fermionic \uparrow vertices,

$$\mathcal{V}(s_\uparrow, s_\uparrow) = (s_\uparrow)^2 \frac{2V}{1-2s_\uparrow V} + \mathcal{V}(s_\uparrow, n_\downarrow) \frac{2s_\uparrow V}{1-2s_\uparrow V} \xrightarrow{q\rightarrow 0} -s_\uparrow(q). \quad (16)$$

Now we can combine all these effective interaction expressions to find out the screening charges with up pseudospin of the meron excitation in Eq. (12). The first contribution (ρ_{bb} in Fig. 2) is due to the meron connection to \uparrow Bose quasiparticles and, if we denote the direct interaction to \uparrow Bose quasiparticles by $V_m \sim \frac{1}{q}$, it is

$$V_m + V_m \frac{\mathcal{V}(n_\uparrow, n_\downarrow)^{q\rightarrow 0}}{n_\uparrow} \rightarrow 0. \quad (17)$$

Then the second contribution (ρ_{bf} in Fig. 2) is through the direct interaction to up Bose quasiparticles that ends up with up fermionic quasiparticles; it is

$$V_m \frac{\mathcal{V}(n_\uparrow, s_\downarrow)^{q\rightarrow 0}}{s_\uparrow} \rightarrow 0. \quad (18)$$

The third contribution (ρ_{fb} in Fig. 2) comes from the direct connection to up fermionic quasiparticles that ends up with up Bose quasiparticles. This contribution is similar to the previous one and has the same limit. The last, fourth contribution (ρ_{ff} in Fig. 2) connects through up Fermi quasiparticles again up Fermi quasiparticles, and equals

$$V_m + V_m \frac{\mathcal{V}(s_\uparrow, s_\downarrow)}{s_\uparrow}. \quad (19)$$

This contribution is again equal to zero according to Eq. (16) in the long wavelength limit.

For the meron screening by down quasiparticles we need $\mathcal{V}(n_\uparrow, n_\downarrow)$ interaction, and similarly to the same pseudospin interaction we get

$$\mathcal{V}(n_\uparrow, n_\downarrow) = \frac{n_\uparrow V n_\uparrow}{1-V(n+\uparrow+\downarrow)} + \frac{n_\uparrow V \downarrow\downarrow + \downarrow\downarrow V n_\downarrow}{1-V(n+\uparrow+\downarrow)} \frac{1}{1-2\frac{\otimes}{n_\sigma}}, \quad (20)$$

where \otimes is $\uparrow\uparrow$ or $\downarrow\downarrow$ and n_σ is n_\uparrow or n_\downarrow whether we attach \otimes to the right-hand or left-hand side of a diagram, respectively. In this way we get, in the long wavelength limit,

\mathcal{S}_{bb} : — n_\uparrow wavy n_\downarrow + ...

\mathcal{S}_{bf} : — n_\uparrow wavy n_\downarrow wavy \mathcal{S}_\uparrow + ...

\mathcal{S}_{fb} : — \mathcal{S}_\uparrow wavy n_\uparrow wavy n_\downarrow + ...

\mathcal{S}_{ff} : — \mathcal{S}_\uparrow wavy n_\uparrow wavy n_\downarrow wavy \mathcal{S}_\downarrow + ...

FIG. 3. The down density screening

$$\mathcal{V}(n_\uparrow, n_\downarrow) \xrightarrow{q \rightarrow 0} 0. \quad (21)$$

Because of this, any contribution of the down screening charges in which participate Bose quasiparticles is equal to zero. In fact $\mathcal{V}(n_\uparrow, n_\downarrow)$ participate in each of four possible contributions, see Fig. 3, leading us to the conclusion that both ρ_\uparrow and ρ_\downarrow screening charges tend to zero in the long wavelength limit. Therefore the screening charges are short ranged and localized just as in the case of any one-component quantum Hall system—a topological phase.

A comment is in order here. Although our plasma approach is straightforward and leads clearly to the localized meron screening charges it does not automatically lead to an overall finite meron excitation energy because we can draw a conclusion only for the charging part of the energy [charge (\uparrow and \downarrow) difference squared]. But it certainly signals a possibility for meron deconfinement.

B. The pseudospin degrees of freedom

Now, in the scope of already introduced Chern-Simons theory, we want to investigate the pseudospin channel of the state in Eq. (11). In particular we want to know whether the pseudospin degrees of freedom are compressible and support a gapless mode. For this we need a variant of the Chern-Simons theory introduced in Eq. (7) for the first state introduced in Eq. (3). In this case we have four gauge fields $a_i^{F\sigma}, a_i^{B\sigma}; i=0, 1, \sigma = \uparrow, \downarrow$,

$$\begin{aligned} \vec{\nabla} \times \vec{a}^{F\sigma} &= 2\pi(2\rho^{F\sigma} + 2\rho^{B\sigma}), \\ \vec{\nabla} \times \vec{a}^{B\sigma} &= 2\pi(2\rho^{F\sigma} + \rho^{B\uparrow} + \rho^{B\downarrow}), \end{aligned} \quad (22)$$

acting on Ψ_σ and Φ_σ , Fermi and Bose fields, respectively. In fact, as can be easily seen, we have only three independent gauge fields $a_c = \frac{a^{B\uparrow} + a^{B\downarrow}}{2} = \frac{a^{F\uparrow} + a^{F\downarrow}}{2}$, $a_{fs} = \frac{a^{B\uparrow} - a^{B\downarrow}}{2}$, and $a_s = \frac{a^{F\uparrow} - a^{F\downarrow}}{2}$, and, therefore,

$$\frac{ika_1^{fs}}{2\pi} = \rho^{F\uparrow} - \rho^{F\downarrow} \quad (23)$$

and

$$\frac{ika_1^s}{2\pi} = \rho^\uparrow - \rho^\downarrow. \quad (24)$$

Then, similarly as before, we can write down the effective Lagrangian for the pseudospin part,

$$\begin{aligned} \mathcal{L}_{ps} &= \frac{1}{2\pi} a_0^s ika_1^{fs} + \frac{1}{2\pi} a_0^{fs} ik(a_1^s - a_1^{fs}) \\ &\quad - \frac{1}{2} \left(\frac{k}{2\pi} \right)^2 V_s |a_1^s|^2 + |a_0^s|^2 K_{00} + |a_1^s|^2 K_{11} + \delta\rho_s B_0 \\ &\quad + \frac{1}{4} \frac{m\omega^2}{\bar{\rho}_b k^2} |\delta\rho_s^b|^2 - a_0^{fs} \delta\rho_s^b - \frac{\bar{\rho}^b}{m} |a_1^{fs}|^2. \end{aligned} \quad (25)$$

We first integrate out a_0^{fs} , which gives us the expected constraint on the pseudospin density of bosons, $\delta\rho_s^B$. Using this constraint and then integrating out a_1^{fs} and a_1^s we get for the pseudospin density-density response the following expression:

$$\mathcal{L}^{\text{eff}} = \frac{\frac{1}{2} \left(\frac{k}{2\pi} \right)^2 B_0^2}{W^4 + \left(\frac{k}{2\pi} \right)^2 V_s - 2K_{11} - \frac{1}{2} \frac{m\omega^2}{\bar{\rho}_b (2\pi)^2}}, \quad (26)$$

where

$$W^4 = - \frac{\left(\frac{1}{2} \frac{m\omega^2}{\bar{\rho}_b (2\pi)^2} \right)^2}{\left(\frac{k}{2\pi} \right)^2 \frac{1}{2K_{00}} - \frac{1}{2} \frac{m\omega^2}{\bar{\rho}_b (2\pi)^2} + \frac{2\bar{\rho}^b}{m}}. \quad (27)$$

First in the limit $\omega \rightarrow 0$ (and then $k \rightarrow 0$) we see that the system is compressible. Second, taking into account that

$$K_{11} \approx - \frac{k^2}{12\pi m} + i \frac{2\bar{\rho}_f \omega}{k_F k}, \quad (28)$$

we see that the pseudospin gapless mode, in the case of the (111) state simply $\omega^0 = \sqrt{\frac{2V_s}{m}} \bar{\rho}_b k$, does not exist as an eigenmode in this case. It is nearly so if we have in mind that the fraction of bosons in this state is to be considered small, and so is the imaginary part of K_{11} when for the frequency we consider the one that takes to zero the real part of the denominator in \mathcal{L}^{eff} .

In fact also for the state in Eq. (3) we find that there is a pseudospin eigenmode with a dispersion relation $\omega = \omega^0(k) + ick^3$ where c is a constant. Therefore it is slightly dissipative what we do not expect from the Goldstone mode.⁴ In the following section we will consider the variational constructions, Eq. (3) and Eq. (11), with p pairing of composite fermions, introduced in Ref. 16, that cures the dissipation problem of the construction in Eq. (3) and leads, as we will show, to incompressible behavior also in the pseudospin channel of the state in Eq. (11).

IV. COMPOSITE FERMION PAIRING AND A POSSIBILITY FOR A PSEUDOSPIN LIQUID

In Ref. 16 p -wave CF pairing was proposed as a way of lowering ground state energy of the state in Eq. (3). That was explicitly shown on a basis of Monte Carlo calculations in which the paired states are excellent variational ansatzes with respect to the true ground states. In the context of the phenomenological Chern-Simons theories based on the proposed wave functions it is not hard to show that in the case of p -wave CF pairing the state in Eq. (3) acquires a pseudospin mode without an imaginary term which must be then a Goldstone mode predicted by the theory of the ordered state for small d .⁴

The calculation begins by noting that p -wave pairing of CFs is simply a condensation into “11-1” state by the way of the Cauchy identity,

$$\prod_{i<j} (w_{i\uparrow} - w_{j\uparrow}) \prod_{k<l} (w_{k\downarrow} - w_{l\downarrow}) \prod_{p,q} (w_{p\uparrow} - w_{q\downarrow})^{-1} = \det \left(\frac{1}{w_{p\uparrow} - w_{q\downarrow}} \right). \quad (29)$$

In fact, as stated in Ref. 16, the p pairing they found is with the pairing function $g(w) \sim \frac{1}{w}$, i.e., a $p_x - ip_y$ instead of $p_x + ip_y$ pairing, and in the context of the Chern-Simons theory that means that the gauge fields on CFs acquire negative sign, so that in the end we have

$$\frac{ik}{2\pi} a_1^{F\sigma} = \frac{ik}{2\pi} a_1^{B\sigma} = \rho_{\sigma}^F + \rho_{-\sigma}^F + \rho_{\sigma}^B + \rho_{-\sigma}^B. \quad (30)$$

Then it is easy to show, similarly as before, by integrating out $\delta\rho_s^F$ and $\delta\rho_s^B$, in the pseudospin channel, that the Goldstone mode has dispersion $\omega(k) = \sqrt{\frac{2V_s}{m}(\bar{\rho}_b + \bar{\rho}_f)} \cdot k$. We get the same result if we consider $p_x + ip_y$ pairing instead of $p_x - ip_y$.

On the other hand, if we consider the state in Eq. (11) with Fermi-type intercorrelations between the Fermi and Bose part and introduce the $p_x - ip_y$ pairing between CFs, in the CS language we have

$$\frac{ik}{2\pi} a_1^{F\sigma} = \rho_{\sigma}^F + \rho_{-\sigma}^F + 2\rho_{\sigma}^B \quad (31)$$

and

$$\frac{ik}{2\pi} a_1^{B\sigma} = \rho_{\sigma}^B + \rho_{-\sigma}^B + 2\rho_{\sigma}^F, \quad (32)$$

i.e., $\frac{ik}{2\pi} a_1^{fs} = \rho_1^F - \rho_2^F$ and $\frac{ik}{2\pi} a_1^s = \rho_1^B - \rho_2^B$. Then the pseudospin part of the effective Lagrangian is

$$\begin{aligned} \mathcal{L}_{ps}^{\text{eff}} = & \frac{1}{2\pi} a_0^s i k a_1^{fs} + \frac{1}{2\pi} a_0^s i k a_1^s + \frac{1}{4} \frac{m\omega^2}{\bar{\rho}_B k^2} |\delta\rho_s^B|^2 - a_0^s \delta\rho_s^B \\ & - \frac{\bar{\rho}^B}{m} |a_1^{fs}|^2 + \frac{1}{4} \frac{m\omega^2}{\bar{\rho}_F k^2} |\delta\rho_s^F|^2 - a_0^s \delta\rho_s^F - \frac{\bar{\rho}^F}{m} |a_1^s|^2 \\ & - \frac{1}{2} \left(\frac{k}{2\pi} \right)^2 V_s |a_1^s + a_1^{fs}|^2 - \frac{ik}{2\pi} (a_1^s + a_1^{fs}) B_{\sigma}. \end{aligned} \quad (33)$$

In few steps, by reducing $\mathcal{L}_{ps}^{\text{eff}}$ into an effective, quadratic expression in B_{σ} we can find out the density-density correlator. It vanishes in the $k \rightarrow 0$ limit and signals that the state in Eq. (11) in which CFs pair is an *incompressible* state. It is our expectation that in this state exist four kinds of merons, characteristic also to the ordered pseudospin state. Therefore the state, if a ground state of an electron system, should represent a quantum Hall, i.e., in the low-energy limit, a topological phase.

Even in existing numerical work we find a support for our expectation. In Ref. 17, in the data that represent the excitation spectrum of the bilayer $\nu=1$ system at $d=1.5l_B$ (l_B is the magnetic length), i.e., in the transition region, obtained by exact diagonalization on a torus, we can see signatures of the nearby topological phase. Namely, in the low-lying spectrum dissociated from the Goldstone mode excitations, exists a fourfold degenerate energy level that we expect represents expected four ground states on the torus of the topological theory.¹⁸ (We do not interpret these states as ordered spiral states as in Ref. 17 because no nearby, low-lying excitations can be seen in the existing data.) Because of the importance of the intralayer correlations in the transition region, the mixed state with Fermi-type intercorrelations between the Bose and Fermi part should compete with the true ground state and is very relevant to the physics of the region. The topological theory in question is $U_2(1) \otimes U_2(1)$ because it supports two kinds of semionic quasiparticles.¹⁸ (Meron fractional statistics is semionic.) On the other hand, the counterflow experiments^{2,3} do not support a “perfect” superfluid scenario with a Kosterlitz-Thouless transition. Instead the data on the counterflow longitudinal resistance show an activated behavior, for a range of higher temperatures, very similar to the usual data of a quantum Hall phase. Therefore it might be that due to an increased importance of the intralayer correlations in the counterflow experiments, the topological phase stabilizes and with it a gapped behavior even in Hall resistance.^{2,3} Because of the nonchiral flow of the currents in the experiment the relevant topological theory should be $U_2(1) \otimes U_2(1)$ double nonchiral pseudospin liquid. The theory is invariant under combined time reversal and Z_2 , an exchange of layer indices, operations implied by the experimental setup. Furthermore it is well known that the zero-bias peak in the tunneling conductance is not as of the usual Josephson effect in superconductors due to dissipation. The reason for this should also be found in the physics of the state in Eq. (11) that incorporates the effect of the increased intralayer correlations in the transition region in which the experiment occurred.

V. DISCUSSION: PHASE DIAGRAM FROM THE WAVE-FUNCTIONAL APPROACH

When we take into account what we found out in the preceding sections, in the scope of the wave-functional approach, the phase diagram, in the absence of disorder, may well have an intermediate phase between the superfluid phase and the two decoupled Fermi-liquid-like phases. The

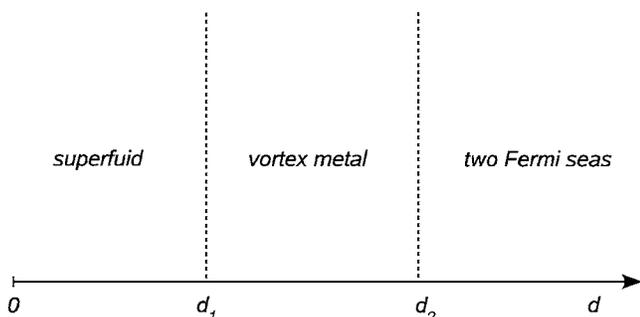


FIG. 4. The possible phase diagram of the bilayer as a function of the distance between layers in the presence of disorder at zero temperature.

wave-functional approach tells us (Secs. III and IV) that in the intermediate phase the pseudospin stiffness may go to zero (meron deconfinement) but the density of composite bosons stays finite. The density of bosons disappears at the transition to the two decoupled CF Fermi seas.

This scenario of the influence of quantum disordering in a superfluid is often described as a result of creation of vortex-antivortex pairs (loops) that cause phase fluctuations. The phase fluctuations may cause the disappearance of the superfluid long range order, but may leave the ordering amplitude (boson density) nonzero. In the Appendix we will argue that creation of (2+2) CFs in the (111) condensate of CBs as in the construction in Eq. (3) can be viewed as creation of two closely spaced pairs of merons. Each pair consists of two merons of the same vorticity but opposite charge. On the other hand, the creation of (2+2) CFs in the condensate of CBs as in the construction in Eq. (11) leads to meron pairs that are not closely spaced inside each pair. [The construction in Eq. (11) suits the increasing intralayer correlations with d .] This motivates a picture of the intermediate phase as either a condensate of closed meron-antimeron loops of any size—a pseudospin liquid¹⁸ (Sec. IV) or a soup of dissociated meron-antimeron pairs (loops), i.e., a vortex metal (Sec. III). The presence of disorder may stabilize the vortex metal phase as in Fig. 4, or maybe a phase separated version.¹⁹

VI. CONCLUSION

By systematically investigating possible candidates for ground state wave functions for the bilayer $\nu=1$ system we reached the conclusion that the ordered state that supports a Goldstone mode in the pseudospin channel for general d can be described as a mixture of CBs and p -paired CFs with dominant Bose (111) type intercorrelations between the two components. Because of the increasing intralayer correlations with d , a state that competes with the ordered one can be described as a mixture of CBs and p -paired CFs with dominant Fermi-type intercorrelations between two components. This state that describes a topological phase with four gapped meron quasiparticles may cause (already observed) activated (decaying exponentially with a gap) behavior of the counterflow longitudinal and Hall resistances at higher temperatures. At lower temperatures, because of the presence of disorder, a compressible in the pseudospin channel version of

this state, a vortex metal, may come out and cause the observed phenomenology of the bilayer “imperfect superfluid.”

There is a vast literature on the bilayer. We are planning a separate paper on the features of the vortex metal phase and comparisons to the previous, experimental and theoretical work. Here we will mention only theoretical work that like ours stresses the importance of the interactions and quantum disordering. In Ref. 20 an excitonic square lattice solid phase was proposed. Here we discussed only translationally invariant, homogenous possibilities and inhomogenous states, Ref. 20, may also come as candidates. (The inhomogenous, phase separated version of the mixed CB-CF states was first proposed in Ref. 21 before the homogenous kind.⁵) In Ref. 22 interactions only drive a phase transition into a state with algebraic (quasi-)long-range superfluid order. The numerics, without impurities and not biased by boundary conditions and special number of electrons to the Wigner crystal formation, Ref. 17, tells us that only the true superfluid order exists in the pertinent experimental region. Therefore we want to stress again what our work suggests. Quantum disordered states (“double pseudospin liquid”) are nearby the true superfluid ground states for distance between the layers pertinent to the experiments. We need also impurities to stabilize a version of these disordered states, “vortex metal state,” at lower temperatures. Quantum disordering caused by interactions is necessary but not a sufficient condition for the explanation of the dissipative phenomenology of the experiments.

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APPENDIX

In this Appendix we will give arguments that the four (2+2) CF inclusion into the (111) state, i.e.,

$$\begin{aligned} & \mathcal{S}_\uparrow\{(z_{1\uparrow} - z_{2\uparrow})[\exp(i\vec{k}\vec{z}_{1\uparrow}) - \exp(i\vec{k}\vec{z}_{2\uparrow})]\} \\ & \times \mathcal{S}_\downarrow\{(z_{1\downarrow} - z_{2\downarrow})[\exp(i\vec{k}\vec{z}_{1\downarrow}) - \exp(i\vec{k}\vec{z}_{2\downarrow})]\}\Psi_{111}, \end{aligned} \quad (\text{A1})$$

where \mathcal{S}_\uparrow and \mathcal{S}_\downarrow are symmetrizers inside each layer, and we omitted for the sake of simplicity the overall projection to the LLL, corresponds to two meron pairs, of opposite (up and down) vorticity. Inside each pair merons have the same vorticity but opposite charge. Therefore the two meron constructions that can act on the (111) state and produce such a pair is [see Eq. (12)]

$$\begin{aligned} \hat{S}^\uparrow(w_1)\hat{S}^\uparrow(w_2) &\equiv \prod_i \frac{z_{i\uparrow} - w_1}{|z_{i\uparrow} - w_1|} \exp\left(-\sum_i \frac{C}{2|z_{i\uparrow} - w_1|}\right) \\ &\times \prod_j \frac{z_{j\uparrow} - w_2}{|z_{j\uparrow} - w_2|} \exp\left(\sum_j \frac{C}{2|z_{j\uparrow} - w_2|}\right), \end{aligned} \quad (\text{A2})$$

and analogously for the down pair. These expressions are effective, i.e., valid in the long distance approximation. Nevertheless in this Appendix we will take that they are qualitatively correct even for shorter distances. Then in the quasiparticle (“fractional statistics”) representation²³ the wave function that describes the two merons in Eq. (A1) is

$$\Psi(w_1, w_2) = (w_1 - w_2)^{-1/2} f(w_1, w_2), \quad (\text{A3})$$

where due to the mutual semionic statistics between quasiparticles we have the difference, $(w_1 - w_2)$, to the power $-\frac{1}{2}$, and $f(w_1, w_2)$ is a symmetric function of coordinates, in our case,

$$f(w_1, w_2) = [\exp(ik\vec{w}_1) - \exp(ik\vec{w}_2)] \frac{(\vec{w}_1 - \vec{w}_2)}{|\vec{w}_1 - \vec{w}_2|}. \quad (\text{A4})$$

There are no Gaussian factors because, when calculated, the interaction of a meron with a positive background of the corresponding plasma,

$$\int d^2z \frac{1}{|z - w|} \exp\left(-\frac{|z|^2}{2}\right), \quad (\text{A5})$$

is a bounded function of w , and when exponential gives a factor that weakly depends on w .

To get the wave function in terms of electronic coordinates we must calculate²³

$$\begin{aligned} &\int d^2w_1 \int d^2w_2 \frac{1}{|w_1 - w_2|} f(w_1, w_2) \hat{S}^\uparrow(w_1) \hat{S}^\uparrow(w_2) \\ &\times \int d^2w_3 \int d^2w_4 \frac{1}{|w_3 - w_4|} f(w_3, w_4) \hat{S}^\downarrow(w_3) \hat{S}^\downarrow(w_4) \Psi_{111}. \end{aligned} \quad (\text{A6})$$

The combined exponentials in the two meron construction can be expanded as in the following:

$$\begin{aligned} &\exp\left(\sum_i \frac{C}{2|z_{i\uparrow} - w_2|}\right) \exp\left(-\sum_i \frac{C}{2|z_{i\uparrow} - w_1|}\right) \\ &= 1 + \frac{C}{2} \sum_i \frac{(\vec{z}_i - \vec{w}_1) \vec{\delta}}{(\vec{z}_i - \vec{w}_1)^2 |\vec{z}_i - \vec{w}_1|} \\ &\quad - \frac{C}{2} \sum_i \left(\frac{\vec{\delta}^2}{(\vec{z}_i - \vec{w}_1)^2 |\vec{z}_i - \vec{w}_1|} + 3 \frac{(\vec{z}_i - \vec{w}_1) \vec{\delta}}{(\vec{z}_i - \vec{w}_1)^4 |\vec{z}_i - \vec{w}_1|} \right) \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{2!} \left(\frac{C}{2}\right)^2 \sum_i \frac{\{(\vec{z}_i - \vec{w}_1) \vec{\delta}\}^2}{(\vec{z}_i - \vec{w}_1)^6} + \frac{1}{2!} \left(\frac{C}{2}\right)^2 \\ &\times \sum_{i \neq j} \frac{(\vec{z}_i - \vec{w}_1) \vec{\delta} \cdot (\vec{z}_j - \vec{w}_1) \vec{\delta}}{(\vec{z}_i - \vec{w}_1)^2 |\vec{z}_i - \vec{w}_1| (\vec{z}_j - \vec{w}_1)^2 |\vec{z}_j - \vec{w}_1|} + o(|\delta|^3), \end{aligned} \quad (\text{A7})$$

where $\vec{\delta} = \vec{w}_2 - \vec{w}_1$, and for the sake of clarity we suppressed the up arrow. To the same order of accuracy we can rewrite the above expression as

$$\begin{aligned} &1 + \frac{C}{2} \sum_i \frac{(\vec{z}_i - \vec{w}_2) \vec{\delta}}{(\vec{z}_i - \vec{w}_2)^2 |\vec{z}_i - \vec{w}_2|} \\ &+ \frac{1}{2!} \left(\frac{C}{2}\right)^2 \sum_i \frac{(\vec{z}_i - \vec{w}_1) \vec{\delta} \cdot (\vec{z}_i - \vec{w}_2) \vec{\delta}}{(\vec{z}_i - \vec{w}_2)^2 |\vec{z}_i - \vec{w}_1| (\vec{z}_i - \vec{w}_1)^2 |\vec{z}_i - \vec{w}_2|} \\ &+ \frac{1}{2!} \left(\frac{C}{2}\right)^2 \sum_{i \neq j} \frac{(\vec{z}_i - \vec{w}_1) \vec{\delta} \cdot (\vec{z}_j - \vec{w}_2) \vec{\delta}}{(\vec{z}_i - \vec{w}_2)^2 |\vec{z}_i - \vec{w}_1| (\vec{z}_j - \vec{w}_1)^2 |\vec{z}_j - \vec{w}_2|}. \end{aligned} \quad (\text{A8})$$

The contribution of the first two terms must be negligible or zero after the integration over \vec{w}_1 and \vec{w}_2 which brings averages of random phases with respect to electron distributions encoded in Ψ_{111} . The contribution of the third term, after picking singularities at z_i , is identical to zero, but the contribution of the fourth term, after picking up singularities at $z_{i\uparrow}$ and $z_{j\uparrow}$ is

$$f(z_{i\uparrow}, z_{j\uparrow}) |z_{i\uparrow} - z_{j\uparrow}| \prod_{k \neq i} \frac{(z_{k\uparrow} - z_{i\uparrow})}{|z_{k\uparrow} - z_{i\uparrow}|} \prod_{l \neq j} \frac{(z_{l\uparrow} - z_{j\uparrow})}{|z_{l\uparrow} - z_{j\uparrow}|}. \quad (\text{A9})$$

for the up part of the construction in Eq. (A6). We get a similar contribution for the down construction, and therefore, up to some phase factors, we get Eq. (A1). The phase factors are there to ensure the right flux count through the system, the problem we neglected by writing our wave functions [Eq. (3) and Eq. (11)] in the thermodynamic limit.

By examining (2+2) CF construction in which Fermi-type intercorrelations are dominant, i.e., the one as the construction in Eq. (11) for (2+2) CFs, we can find out that the small distance approximation as in Eq. (A8) is not applicable. Moreover its form suggests that much higher order terms of the expansion in Eq. (A7) are relevant, and therefore two merons are well separated in this construction.

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