

## Edge states on a quantum Hall liquid-solid interface

Milica Milovanovic

*Department of Physics, The Technion, Haifa 32000, Israel  
and Department of Mathematics-Physics, Oranim-Haifa University, Tivon 36006, Israel*

Efrat Shimshoni

*Department of Mathematics-Physics, Oranim-Haifa University, Tivon 36006, Israel*

(Received 18 February 1998)

We study the edge states excitations of a droplet of quantum Hall liquid embedded in an electron solid. The presence of strong correlations between the liquid and solid sectors in the ground state is shown to be reflected in the density of states  $D(E)$ , associated with the excitations of the liquid-solid interface. We find that the prominent effect of these correlations is a suppression of  $D(E)$  with respect to its value [ $D_0(E)$ ] in the absence of the electron solid environment:  $D(E) \sim e^{-\alpha|E|} D_0(E)$ . The coefficient  $\alpha$  (which is shown to vanish for a perfectly regular distribution of electron sites in the solid), is evaluated for two different realizations of an irregular distribution. We conclude that probing this effect (e.g., in a tunneling experiment), can provide evidence for correlated liquid-solid mixture states in quantum dots, or disordered samples, in very strong magnetic fields. [S0163-1829(98)01032-7]

### I. INTRODUCTION AND PRINCIPAL RESULTS

The two-dimensional electron gas (2DEG) in strong perpendicular magnetic fields can form a variety of exotic quantum phases. In particular, in clean systems at moderately low filling fractions (close to  $\nu = 1/5$ ), the correlations that favor a fractional quantum Hall liquid (QHL) state compete with the crystalline order of a Wigner solid (WS).<sup>1,2</sup> This competition can induce transitions between the QHL state and the insulator, as has been observed experimentally.<sup>3</sup> In the presence of slowly varying disorder or a confining potential, the electronic ground state may develop a fractured order—namely, form a binary liquid-solid mixture. A liquid-solid separation possibly occurs also at higher filling fractions (in which the electrons in the solid regions form a glassy state controlled by short-range disorder). An experimental evidence for this scenario is provided by photoluminescence data.<sup>4</sup> In addition, a set of puzzling transport data<sup>5,6</sup> can be explained most naturally under the assumption of a macroscopic inhomogeneity.<sup>7</sup>

The interplay of QHL correlations and crystallization in the low  $\nu$  regime has been clearly demonstrated by Zheng and Fertig.<sup>2</sup> Using a variational calculation, they have shown that a Wigner lattice with an interstitial electron introduced via a Laughlin-like Jastrow factor, can be lower in energy compared to the perfect WS with the same total number of electrons. This implies that in a certain range of  $\nu$ 's, the crystal is unstable to a specific type of density fluctuations—preformed QHL droplets. In the presence of density fluctuations induced by a slowly varying external potential, it is reasonable to expect a nucleation of such interstitials in the higher-density regions. It is therefore suggestive that the ground state slightly below  $\nu = 1/5$  separates into QHL and WS sectors, which are correlated by a Jastrow factor to minimize the energy of electrons close to the liquid-solid interface [see Eq. (2)].

In the present paper we investigate the physical implica-

tions of liquid-solid mixture states, as reflected by the corresponding low-lying excitations. Similarly to a finite droplet of a primary QHL (of  $\nu = 1/m$  with  $m$  an odd integer), the gapless excitations are chiral edge states,<sup>8</sup> i.e., deformations of the boundary of the incompressible droplet, which travel in a definite direction along the boundary. However, in case the liquid droplet is embedded in an electron solid<sup>9</sup> (ES) rather than a vacuum, the nature of these excitations of the liquid-solid interface is affected by the correlations between the two sectors. In particular, high amplitude deformations of the interface are generally suppressed, since the liquid electrons are constrained by their tendency to avoid the proximity to localized sites of the ES as much as possible. This can lead to a *decay* of the density of states with increasing deviation from the Fermi level, as long as higher energy excitations (which involve, e.g., a reorganization of the electrons in the solid) are not yet activated.

To facilitate the derivation of this peculiar effect, we consider a simple geometry of a large, circular quantum dot, in which the electrons are assumed to form a disc of QHL surrounded by an ES (see Fig. 1). We evaluate the electron propagator and consequently the density of states for tunneling into the liquid-solid interface,  $D(E)$ . In the thermodynamic limit, we find

$$D(E) \sim e^{-\alpha|E|} D_0(E), \quad (1)$$

where  $D_0(E)$  corresponds to the ordinary edge state (on an interface between QHL and a vacuum), and the coefficient  $\alpha$  depends crucially on the distribution of localized sites in the ES sector. In particular, when these sites form a structure with a perfect crystalline order around the disk (and the lattice constant is commensurate with the circumference),  $\alpha = 0$ : in that case, the ES electrons merely deform the effective boundary of the liquid into a regular shape, as depicted in Fig. 1(a). In contrast, an irregular distribution of sites induces frustration, and thus a suppression of  $D(E)$ .

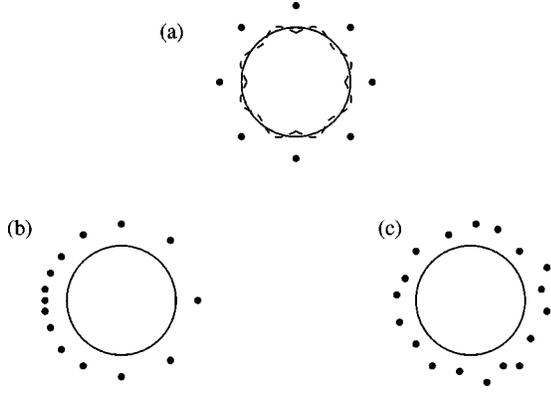


FIG. 1. A disk of QH liquid surrounded by an ES (the dots denote the sites at which the ES electrons are localized). Three different types of site configurations are sketched: (a) an ordered, commensurate chain (the effective boundary of the liquid in the presence of the ES is marked by a dashed line); (b) an inhomogeneous configuration; (c) a random distribution.

The most direct way to probe this effect is via tunneling into the 2DEG, e.g., using the technique developed by Ashoori:<sup>10</sup> the tunneling conductance is given by  $G(V) \sim D(eV)$  (where  $V$  is the voltage across the tunnel barrier). The suppression of  $G(V)$  at low  $V$  may lead to a non-monotonous behavior—at higher voltage bias, higher-energy excitations take over and induce an increase of  $G(V)$ . The effect is expected to become more pronounced with increasing inhomogeneity of the external potential. In particular, in a quantum dot where both the number of electrons and the confining potential can be controlled, the suppression of  $G(V)$  is expected to exhibit oscillations: the tunneling rate should be maximized when the control parameter enables a nearly regular configuration of sites in the ES sector.

In the following sections, we detail the derivation of the electron propagator along the liquid-solid interface (Sec. II), and the implied behavior of the density of states, Eq. (1) (Sec. III). In the latter, we consider two different realizations of the irregularity in site configuration: (a) a regular distortion of the circular symmetry, and (b) a symmetric random distribution. The corresponding expressions for the suppression time  $\alpha$  are given by Eqs. (38) [case (a)] and (43), (47) [case (b)].

## II. DERIVATION OF THE ELECTRON PROPAGATOR

As we explained in the introduction we expect that the ground state wave function that describes the QH droplet surrounded by an ES is essentially of the following form:

$$\Psi_{LS} = \mathcal{A} \left\{ \Psi_L(z_1, \dots, z_N) \Psi_S(w_1, \dots, w_M) \prod_{i,j} (z_i - w_j)^m \right\}, \quad (2)$$

where  $\Psi_L$  describes the liquid part,  $\Psi_S$  the solid part, and the last expression describes the Jastrow correlations of these two phases.  $\mathcal{A}$  denotes antisymmetrization over all electron coordinates.  $m$  corresponds to the filling factor  $\nu = 1/m$  of the QH liquid part:

$$\Psi_L(z_1, \dots, z_N) = \prod_{i < j} (z_i - z_j)^m \exp\left(-\frac{1}{4} \sum |z_i|^2\right). \quad (3)$$

The wave function

$$\Psi_S(w_1, \dots, w_M) = \exp\left(\frac{1}{2} \sum \bar{w}_i R_i - \frac{1}{4} \sum |w_i|^2\right) \quad (4)$$

describes ES electrons localized on the positions  $R_i, i = 1, \dots, M$ . (It is a multiple of lowest-Landau-level  $\delta$  functions.)

Our main assumption in the derivation of the electron propagator is that the edge of the QH droplet, coupled to ES, essentially behaves as a slightly modified Luttinger liquid. That assumption allows us to use a construction of one electron state on the edge similar to the one of the Luttinger liquid. At the end of the derivation we will be able to specify constraints on the configuration of the ES sites, such that the assumption is valid.

Because of the above assumption, it would be instructive first to briefly recapitulate the derivation of the (equal-time) propagator when the QH disk is surrounded by vacuum.<sup>8</sup> The derivation begins by considering  $m$ -Laughlin quasihole constructions, i.e.,

$$\prod_{i=1}^N (z_i - \xi)^m \Psi_L = \Psi_o(\xi), \quad (5)$$

where  $\xi$  lies outside of the system;  $|\xi| > R$ , and  $R = 2mN$  is the radius of the QH droplet.  $\Psi_L$  is the Laughlin wave function [Eq. (3)]. The first step towards the electron correlator is calculating the following scalar product of the state (5)

$$\mathcal{N}(\xi^*, \xi) = \frac{\langle \Psi_o(\xi) | \Psi_o(\xi) \rangle}{\langle \Psi_L | \Psi_L \rangle} \xi^{-2mN}. \quad (6)$$

The leading contribution in the  $R/|\xi|$  expansion can be obtained by the plasma analogy<sup>8</sup> or simply by considering the decomposition of coordinates into the center of mass,  $Z_{cm} = 1/N \sum_{i=1}^N z_i$ , and relative ones. When the numerator is approximated by

$$m \sum_k \ln |z_k - \xi|^2 \approx Nm \ln |\xi| - mN \frac{Z_{cm}}{\xi} - mN \frac{Z_{cm}^*}{\xi^*}, \quad (7)$$

the integration over the center of mass coordinate is decoupled from the other integrations, which do not depend on  $\xi$ . It yields the leading dependence on  $R/|\xi|$ :

$$\mathcal{N}_o(\xi^*, \xi) \approx \left(1 + m \frac{R^2}{|\xi|^2}\right). \quad (8)$$

In an average, macroscopic picture, we expect singular behavior as  $|\xi| \rightarrow R$ , with singularity at  $|\xi| = R$  (for the equal-time, equal-space correlator).<sup>8</sup> Then Eq. (8) can be rewritten as

$$\mathcal{N}_o(\xi^*, \xi) = \left(1 - \frac{R^2}{|\xi|^2}\right)^{-m} \quad (9)$$

and we will assume that it is valid also for  $|\xi| \sim R$  ( $|\xi| > R$ ). By doing this we neglect any finite size corrections (due to

finite  $R$ ) that might be present in  $\mathcal{N}_o$  as  $|\xi| \rightarrow R$ . To get the electron propagator we analytically continue the function  $\mathcal{N}_o(\xi^*, \xi)$  of the variable  $\xi$  to  $\mathcal{N}_o(\tilde{\xi}^*, \xi)$ , which depends on  $\tilde{\xi}$  and  $\xi$ . That allows us to take  $\xi$  and  $\tilde{\xi}$  to the edge of the system— $\xi = R \exp\{i2\pi(x/L)\}$  and  $\tilde{\xi} = R$ , without encountering the singularity at  $x=0$  (which determines the behavior of the function in its neighborhood). By taking  $\xi$  and  $\tilde{\xi}$  to the edge, we in fact describe a particle-hole excitation on the edge that goes into the electron propagator, and find that for  $x \ll L$  the propagator behaves as

$$G_e^o(x) = \mathcal{N}_o(\tilde{\xi}^*, \xi) \tilde{\xi}^{*(N-1)m} \xi^{(N-1)m} \sim \frac{1}{\left(\frac{x}{L}\right)^m} \exp\left[im\left(N - \frac{1}{2}\right)\frac{2\pi}{L}x\right], \quad (10)$$

where  $m(N - \frac{1}{2})(2\pi/L)$  in the exponential is the value of the generalized ( $m \neq 1$ ) ‘‘Fermi momentum.’’

Following a similar strategy, we address the case of the droplet surrounded by ES, described by the wave function (2). We consider the following state:

$$\Psi(\xi) = \prod_{i=1}^N (z_i - \xi)^m \Psi_{LS} \quad (11)$$

and the corresponding scalar product,

$$\mathcal{N}(\xi^*, \xi) = \frac{\langle \Psi(\xi) | \Psi(\xi) \rangle}{\langle \Psi_{LS} | \Psi_{LS} \rangle} \xi^{-2mN} \quad (12)$$

where

$$\begin{aligned} \langle \Psi(\xi) | \Psi(\xi) \rangle &= \int \prod_{i=1}^N d^2 z_i \int \prod_{j=1}^M d^2 w_j \exp\left\{ \sum_{i < j} 2m \ln |z_i - z_j| \right. \\ &+ \sum_{i=1}^N \sum_{j=1}^M 2m \ln |z_i - w_j| - \frac{1}{2} \sum_{i=1}^N |z_i|^2 \\ &\left. - \frac{1}{2} \sum_{j=1}^M |w_j - R_j|^2 + 2m \sum_{i=1}^N \ln |z_i - \xi| \right\}. \quad (13) \end{aligned}$$

In the above formulas the antisymmetrization was neglected, which is possible due to the localized nature of the ES. Again, if for  $|\xi| \gg R$  and  $|w_j| \gg R$ ,  $j=1, \dots, M$ , the last sum in Eq. (13) is approximated as Eq. (7), and

$$\begin{aligned} &\sum_{i=1}^N \sum_{j=1}^M 2m \ln |z_i - w_j| \\ &\approx \sum_{j=1}^M 2mN \ln |w_j| - mNZ_{cm} \sum_{j=1}^M \frac{1}{w_j} - mNZ_{cm}^* \sum_{j=1}^M \frac{1}{w_j^*}, \quad (14) \end{aligned}$$

the integration over  $Z_{cm}$  yields the functional dependence of the integrals over  $z_i$ 's in Eq. (13) on  $\xi$  to leading order in  $R[\sum_{j=1}^M (1/w_j) + (1/\xi)]$ . It is of the following form:

$$\left( 1 + mR^2 \left| \sum_{j=1}^M \frac{1}{w_j} + \frac{1}{\xi} \right|^2 \right). \quad (15)$$

Now we will assume that the sites  $R_j, j=1, \dots, M$  are such that the integration over  $w_j$ 's is dominated by contributions for which

$$R^{2n} \left| \sum_{j=1}^M \frac{1}{w_j^n} + \frac{1}{\xi^n} \right|^2 < 1, \quad n=1, \dots, \infty. \quad (16)$$

Then, the result of the  $z$  integration can be expressed as an expansion in variables symmetric in  $\xi$  and  $w_j$ 's, defined as

$$x_n = \sum_{j=1}^M \frac{1}{w_j^n} + \frac{1}{\xi^n}, \quad n=1, \dots, \infty. \quad (17)$$

The first two terms of the expansion are given by the expression (15). As explained in the Appendix, the final  $w$  integration amounts to replacing  $w_j$  with  $\tilde{R}_j$ , defined in the Appendix, and variables (17) in the expansion yield

$$X_n \{R_j, j=1, \dots, M\} = \sum_{j=1}^M \frac{1}{\tilde{R}_j^n} + \frac{1}{\xi^n}. \quad (18)$$

At this point, we can see that in the case where the positions of ES electrons satisfy

$$\sum_{j=1}^M \frac{1}{\tilde{R}_j^n} = \sum_{j=1}^M \frac{1}{\tilde{R}_j^{*n}} = 0, \quad n=1, \dots, \infty \quad (19)$$

the problem reduces to the one of the droplet surrounded by vacuum, and the expansion should sum up to the Luttinger-liquid form (9). These conditions can be satisfied, e.g., when

$$\tilde{R}_j = \tilde{R} \exp\{i(\theta_j + \theta_o)\}, \quad (20)$$

where  $\theta_j = j\theta$ ,  $\theta = 2\pi/M$ , and  $\tilde{R}$  is a constant radius, i.e., when a commensurate chain of ES electrons surrounds the droplet [see Fig. 1(a)]. Note that our assumption of a small correction to the Luttinger-liquid behavior is justified, provided the configuration of ES sites is a small perturbation of one that satisfies Eq. (19).

To get the electron propagator, we first exponentiate the expression of the leading-order behavior for  $|\xi| \gg R$ ,  $|R_j| \gg R$ ,  $j=1, \dots, M$

$$\mathcal{N}(\xi^*, \xi) \approx \left( 1 - R^2 \left| \sum_{j=1}^M \frac{1}{\tilde{R}_j} + \frac{1}{\xi} \right|^2 \right)^{-m}. \quad (21)$$

Here we assume that even for  $R_j$ 's close to the droplet,  $\sum (1/\tilde{R}_j)$  is small with respect to  $(1/|\xi|) \sim (1/R)$  (which implies that the pole structure of the correlator is similar to the one of the Luttinger liquid with the pole slightly shifted from  $|\xi|=R$ ). Hence, we can also regard Eq. (21) valid for general  $R_j$ ,  $j=1, \dots, M$ , and  $\xi$  (with  $|R_j|, |\xi| > R$ ).  $\mathcal{N}(\xi^*, \xi)$  is then analytically continued to  $\mathcal{N}(\tilde{\xi}^*, \xi)$ ; taking  $\xi = R \exp\{i2\pi(x/L)\}$  and  $\tilde{\xi} = R$  gives for the electron equal-time propagator

$$\begin{aligned}
G_e(x) \sim & \exp\left\{im\left(N - \frac{1}{2}\right)2\pi\frac{x}{L}\right\} \left[ \exp\left\{i\pi\frac{x}{L}\right\} - \exp\left\{-i\pi\frac{x}{L}\right\} \right. \\
& \left. - \exp\left\{i\pi\frac{x}{L}\right\} \sum - \exp\left\{-i\pi\frac{x}{L}\right\} \right] \\
& \times \sum^* - \exp\left\{i\pi\frac{x}{L}\right\} \left| \sum \right|^2 \Big|^{-m} \\
& \text{where } \sum \equiv \sum_{j=1}^M \frac{R}{\tilde{R}_j}. \quad (22)
\end{aligned}$$

In the limit  $x/L \ll 1$  that corresponds to the short-distance behavior, the electron propagator can be expressed as

$$G_e(x) \sim \exp\left[im\left(N - \frac{1}{2}\right)2\pi\frac{x}{L}\right] \frac{1}{(ix + C_1x - C_2)^m}, \quad (23)$$

where

$$C_1 = \frac{2 \operatorname{Im}\{\Sigma\}}{2 - |\Sigma|^2} \quad (24)$$

and

$$C_2 = \frac{L}{\pi} \frac{(2 \operatorname{Re}\{\Sigma\} + |\Sigma|^2)}{2 - |\Sigma|^2}. \quad (25)$$

( $\operatorname{Re}\{\Sigma\}$  and  $\operatorname{Im}\{\Sigma\}$  are the real and imaginary parts of  $\Sigma$ , respectively.) Note that  $|\Sigma|$  serves as a small parameter that determines the deviation from a standard Luttinger-liquid behavior [Eq. (10)].

### III. TUNNELING DENSITY OF STATES

In the previous section we calculated the equal-time propagator. In order to get the tunneling density of states, we need the time-dependent propagator  $G_e(x=0, t)$ . In general, this requires a knowledge of the energies of excited states of the system. For ES configurations that satisfy conditions (19) [and hence have the Luttinger-liquid correlations (10)], we expect that the ground state is one of the edge states of a QH droplet surrounded by vacuum; the excited states (which may also be interpreted as edge states of that droplet) have energies linear in momentum, measured from the new ground state. The linear dispersion is expected on general grounds, as the first-order expansion in small momenta, and not precluded by any (symmetry, etc.) argument.<sup>11</sup> Then, to get  $G_e(x=0, t)$  we should merely substitute the coordinate  $x$  with  $vt$  (where  $v$  is the velocity of the drift motion of electrons on the boundary<sup>8</sup>). The sign of time should be specified, as we will explain and elaborate below. Assuming that the linear dispersion is valid also in the case of a small deviation from Eq. (19) (i.e., for  $|\Sigma| \ll 1$ ), we may apply the same substitution in Eq. (23) to get  $G_e(x=0, t)$ .

We recall the definition of the (equal-space) fermionic Green's function, in the field-theoretic notation:

$$G_e(x=0, t>0) = -i\langle 0 | \Psi(0, t) \Psi^\dagger(0, 0) | 0 \rangle, \quad (26)$$

and

$$G_e(x=0, t<0) = i\langle 0 | \Psi^\dagger(0, 0) \Psi(0, t) | 0 \rangle \quad (27)$$

where  $\Psi^\dagger$  and  $\Psi$  are the electron and hole creation operators, respectively. If we assume that, in our system,

$$\langle 0 | \Psi(0, t) \Psi^\dagger(0, 0) | 0 \rangle = \langle 0 | \Psi^\dagger(0, t) \Psi(0, 0) | 0 \rangle \quad (28)$$

holds, as it does in the case of the standard Luttinger liquid, going from formula (27) to formula (26) involves only time translation ( $T \rightarrow T - t$ ) and time inversion ( $T \rightarrow -T$ ) (and the overall change of the sign). In our case, the particle coordinates  $\xi = R \exp\{ix(2\pi/L)\}$  and  $\tilde{\xi} = R$  become under the substitution ( $x \rightarrow vt$ )  $\xi(t) = R \exp\{ivt(2\pi/L)\}$  and  $\tilde{\xi}(0) = R$ .  $\xi(t)$  and  $\tilde{\xi}(0)$  denote the hole and electron coordinates, respectively. If we reconsider the correlator  $\mathcal{N}(\tilde{\xi}^*, \xi)$  with the substitutions, we will get for the hole propagator (27) in the short-time limit:

$$G_e(0, t<0) \propto \frac{1}{(ivt + C_1vt - C_2)^m}. \quad (29)$$

On the other hand, to get the electron propagator for  $t>0$  [Eq. (26)] we should perform the time translation and inversion. This amounts to an exchange of  $\xi(t)$  and  $\tilde{\xi}(0)$ , which leads to the following short-time behavior:

$$G_e(0, t>0) \propto \frac{1}{(-ivt + C_1vt - C_2)^m}. \quad (30)$$

If we consider  $t$  as a complex variable and assume  $|\Sigma| \ll 1$ , we may approximate the positions of poles in Eqs. (29) and (30) as  $t_1 \approx -i(C_2/v)$ , and  $t_2 \approx i(C_2/v)$ , respectively. Then, the tunneling density of states  $D(E)$  is given by

$$\begin{aligned}
D(E) \sim & \operatorname{Re} \left( \int_{-\infty}^0 dt \exp\{iEt\} \frac{1}{(it - C_2/v)^m} \right. \\
& \left. + \int_0^{\infty} dt \exp\{iEt\} \frac{1}{(it + C_2/v)^m} \right), \quad (31)
\end{aligned}$$

where  $E$  is the energy measured from the Fermi energy,  $\hbar = 1$ . As a final result we get

$$D(E) \propto \exp\{-\alpha|E|\} D_o(E), \quad \alpha \equiv \frac{|C_2|}{v}, \quad (32)$$

where  $D_o(E)$  is the Luttinger-liquid tunneling density of states. Therefore, to lowest order in  $|\Sigma|$ , the dominant modification to the standard Luttinger behavior is the exponential suppression, at a time scale  $\alpha$  of order  $\sim (L/v)|\Sigma|$ . It should be stressed that  $\alpha$ , and hence  $D(E)$ , is a local quantity (adiabatically varying around the disk) that describes the tunneling density of states of an electron at distance  $\sim R$  from the origin, and at angle  $\pi(x/L) \ll 1$  to the reference point. Below we calculate  $\alpha$  for the two different types of imperfect site configurations depicted in Figs. 1(b) and 1(c).

#### A. Inhomogeneous configuration of sites

We model the inhomogeneous configuration [Fig. 1(b)] by considering  $M = 2n + 2$  ES sites, at a distance  $\tilde{R}$  from the origin, where two of them are exactly on the opposite sides

of the droplet, i.e., the sum of their phases is  $\exp\{i\pi\}+1=0$ . The rest  $2n$  electrons are positioned at the angles  $\theta_j$  and  $-\theta_j$  ( $1 \leq j \leq n$ ), where

$$\theta_j = \theta_j + \epsilon \cos\{\delta(j-1)\}, \quad (33)$$

$\theta = (\pi/n + 1)$  and  $\delta = (\pi/2n)$ .  $\epsilon > 0$  represents a small deviation from the perfect chain distribution, which is modulated, as we move from the reference point at angle  $\gamma = 0$ . We then get  $\Sigma = \Sigma_0$ , where

$$\begin{aligned} \Sigma_0 \equiv & \frac{R}{\bar{R}} \sum_{j=1}^n \exp\{i\theta_j + i\epsilon \cos[\delta(j-1)]\} \\ & + \frac{R}{\bar{R}} \sum_{j=1}^n \exp\{-i\theta_j - i\epsilon \cos[\delta(j-1)]\}. \end{aligned} \quad (34)$$

For  $\epsilon$  small  $\Sigma_0$  can be approximated as

$$\begin{aligned} \Sigma_0 \sim & i\epsilon \frac{R}{\bar{R}} \sum_{j=1}^n \exp\{i\theta_j\} \cos\{\delta(j-1)\} \\ & - i\epsilon \frac{R}{\bar{R}} \sum_{j=1}^n \exp\{-i\theta_j\} \cos\{\delta(j-1)\} \end{aligned} \quad (35)$$

and for  $n \gg 1$  this yields

$$\Sigma_0 = -\frac{8n\epsilon R}{3\pi \bar{R}}. \quad (36)$$

If the reference point is at an arbitrary angle  $\gamma \neq 0$ , the sum becomes

$$\Sigma = \Sigma_0 \exp\{-i\gamma\}. \quad (37)$$

Inserting in Eqs. (25) and (32), we then get

$$\alpha = \frac{L}{\pi v} \left| \frac{2\Sigma_0 \cos\{\gamma\} + |\Sigma_0|^2}{2 - |\Sigma_0|^2} \right|. \quad (38)$$

Therefore, at  $\gamma = \pi/2$ ,  $\alpha$  becomes  $\alpha \propto \epsilon^2$ , at  $\gamma = 0$  it is  $\alpha \propto (\epsilon - \text{const} \epsilon^2)$ ,  $\text{const} > 0$ , and, at  $\gamma = \pi$ , it is  $\alpha \propto (\epsilon + \text{const} \epsilon^2)$ . This means that around  $\gamma = 0$  and  $\gamma = \pi$  deviations from the perfect chain case are stronger, and the suppression of the density of states is stronger in the denser region ( $\gamma = \pi$ ), but only to second order in  $\epsilon$ .

### B. Random distribution of sites

We consider a configuration of sites obtained by a random distortion of the perfect chain Eq. (20) [see Fig. 1(c)]. A general distortion modifies both the radius and phase of the  $\tilde{R}_j$ 's; here we discuss the effect of each type of randomness separately.

In a phase-distorted chain at radius  $\bar{R}$ , the site  $\tilde{R}_j$  is given by

$$\tilde{R}_j = \bar{R} \exp\{i(j\theta + \delta_j)\}, \quad (39)$$

where  $\theta = 2\pi/M$  and  $\delta_j$  is a random variable. Averaging over the distribution of  $\delta_j$ 's, we obtain

$$\langle \Sigma \rangle = \sum_{j=1}^M \left\langle \frac{R}{\tilde{R}_j} \right\rangle = \frac{R}{\bar{R}} \langle \exp\{-i\delta\} \rangle \sum_{j=1}^M \exp\{-i(j\theta)\} = 0. \quad (40)$$

The lowest-order contribution to  $C_2$  [Eq. (25)] is then  $\sim \langle |\Sigma|^2 \rangle$ , where

$$\langle |\Sigma|^2 \rangle = \frac{R^2 M^{1/2}}{\bar{R}^2} \sum_{j=1}^M (2 - 2\langle \exp\{i\delta\} \rangle). \quad (41)$$

For a Gaussian distribution  $P(\delta) = (1/\sqrt{2\pi}\sigma) e^{-(\delta^2/2\sigma^2)}$ , we get

$$\langle |\Sigma|^2 \rangle = \frac{R^2}{\bar{R}^2} M (1 - \exp\{-\sigma^2/2\}) \approx \frac{R^2}{\bar{R}^2} \frac{M\sigma^2}{2}, \quad (42)$$

where the last approximation holds for  $\sigma \ll 1$ . Substituting in Eqs. (25) and (32), this yields

$$\alpha \approx \frac{L}{2\pi v} \frac{R^2}{\bar{R}^2} \frac{M\sigma^2}{2}. \quad (43)$$

We next consider a radius distortion of Eq. (20) of the form

$$\tilde{R}_j = (\bar{R} + r_j) \exp\{i(j\theta)\}, \quad (44)$$

where  $r_j$  is a random variable, subject to a distribution of width  $\sigma_r \ll \bar{R}$ . Again, the first order in  $\Sigma$  vanishes upon averaging:

$$\langle \Sigma \rangle = \frac{R}{\bar{R}^2} \langle r \rangle \sum_{j=1}^M \exp\{-i(j\theta)\} = 0. \quad (45)$$

For a symmetric distribution of  $r_j$ 's,  $\langle r \rangle = 0$  and we get

$$\langle |\Sigma|^2 \rangle = \frac{R^2}{\bar{R}^4} \sum_{j=1}^M \langle r^2 \rangle = \frac{R^2}{\bar{R}^4} M \sigma_r^2. \quad (46)$$

We then get an expression for  $\alpha$  which is quite similar to Eq. (43):

$$\alpha \approx \frac{L}{2\pi v} \frac{R^2}{\bar{R}^4} M \sigma_r^2. \quad (47)$$

### ACKNOWLEDGMENTS

We thank E. Akkermans, R. Ashoori, Y. Avron, J. Feinberg, Y. Meir, and S. Sondhi for fruitful discussions. This work was partly supported by Grant No. 96-00294 from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel, and the Technion-Haifa University Collaborative Research Foundation. M.M. also acknowledges support from the Fund for Promotion of Research at Technion, and the Israeli Academy of Sciences.

### APPENDIX

To evaluate  $\langle \Psi(\xi) | \Psi(\xi) \rangle$  [Eq. (13)], one needs to solve integrals of the form

$$I = \int d^2w f(w, w^*) |w|^{2mN} e^{-(1/2)|w|^2} e^{(1/2)wR_j^*} e^{(1/2)w^*R_j}, \quad (\text{A1})$$

where  $w$  is the coordinate of the  $j$ th ES electron (the index  $j$  being omitted), and  $f(w, w^*)$  can be expressed as a power series in  $R/w, R/w^*$ :

$$f(w, w^*) = \sum_{n,k=0}^{\infty} a_{nk} \left(\frac{R}{w}\right)^n \left(\frac{R}{w^*}\right)^k. \quad (\text{A2})$$

Equation (A1) is then recast as

$$I = \sum_{n,k=0}^{\infty} a_{nk} R^{n+k} I_{mN-n, mN-k},$$

$$I_{p,l} \equiv \int d^2w w^p w^{*l} e^{-(1/2)|w|^2} e^{(1/2)wR_j^*} e^{(1/2)w^*R_j}. \quad (\text{A3})$$

Below we show that provided  $(mN-n), (mN-k) \gg 1$ ,

$$I_{mN-n, mN-k} \approx I_{mN, mN} \tilde{R}_j^{-n} (\tilde{R}_j^*)^{-k},$$

$$\frac{\tilde{R}_j}{R_j} = \frac{\tilde{R}_j^*}{R_j^*} = e^{\alpha(|R_j|)}, \quad \text{where} \quad \alpha(|R_j|) \approx \frac{R^2}{|R_j|^2}; \quad (\text{A4})$$

note that  $\exp\{\alpha(|R_j|)\} \rightarrow 1$  for  $(R/R_j) \ll 1$ . In addition, we note that since the integration over  $w$  is dominated by  $w \sim R_j$ , and  $R_j > R$ , the series expansion Eq. (A2) can be cut at some  $n_c, k_c$  such that  $|R/R_j|^{n_c}, |R/R_j|^{k_c} \ll 1$ , yet  $(mN-n_c), (mN-k_c) \gg 1$ . Consequently, we obtain

$$I \approx f(\tilde{R}_j, \tilde{R}_j^*) I_{mN, mN}, \quad (\text{A5})$$

which implies that the  $\{w_j\}$  integrations over the terms  $x_n$  [Eq. (17)] yield Eq. (18).

We now derive the approximation Eq. (A4)—the central result of this Appendix. The integrals  $I_{p,l}$  [Eq. (A3)] can be expressed as

$$I_{p,l} = 2^{p+l} \frac{\partial^p}{\partial R_j^{*p}} \frac{\partial^l}{\partial R_j^l} \int d^2w e^{-(1/2)|w|^2} e^{(1/2)wR_j^*} e^{(1/2)w^*R_j}$$

$$= 2^{p+l} \frac{\partial^p}{\partial R_j^{*p}} \frac{\partial^l}{\partial R_j^l} [4\pi e^{\frac{1}{2}|R_j|^2}]. \quad (\text{A6})$$

A straightforward application of the derivatives then yields

$$I_{p,l} = 4\pi e^{(1/2)|R_j|^2} R_j^p R_j^{*l} S(p,l),$$

where

$$S(p,l) = \sum_{i=0}^{\min\{p,l\}} \frac{p!l!}{i!(p-i)!(l-i)!} \left(\frac{2}{|R_j|^2}\right)^i. \quad (\text{A7})$$

We next assume that the above sum is dominated by  $1 \ll i \ll p, l$ , so that the factorials are well approximated by Stirling's formula:

$$S(p,l) \approx \sum_i s_i,$$

$$s_i = \left[ \frac{ple}{i} \frac{2}{|R_j|^2} \left(1 - \frac{i}{p}\right) \left(1 - \frac{i}{l}\right) \right]^i \sqrt{\frac{pl}{2\pi i(p-i)(l-i)}}. \quad (\text{A8})$$

The sum is then replaced by an integral

$$S(p,l) \approx \frac{1}{\sqrt{2\pi}} \int dx e^{\phi(x)},$$

$$\phi(x) = x \left\{ \ln \left[ \left( \frac{2ple}{|R_j|^2 x} \right) \left(1 - \frac{x}{p}\right) \left(1 - \frac{x}{l}\right) \right] \right\} + O(\ln(x)), \quad (\text{A9})$$

which can be solved in a saddle-point approximation. The saddle-point equation  $\phi'(x) = 0$  implies

$$\frac{2ple}{|R_j|^2 x} \left(1 - \frac{x}{p}\right) \left(1 - \frac{x}{l}\right) = \exp \left\{ 1 + \frac{x}{p-x} + \frac{x}{l-x} \right\}, \quad (\text{A10})$$

and hence

$$S(p,l) \approx \sqrt{\frac{1}{\phi''(x_s)}} \exp \left\{ x_s \left( 1 + \frac{x_s}{p-x_s} + \frac{x_s}{l-x_s} \right) \right\}, \quad (\text{A11})$$

where  $x_s$  is the solution of Eq. (A10). For  $(R/R_j) \ll 1$  (where  $p, l \ll Nm = R^2/2$ )

$$x_s \approx \frac{2pl}{|R_j|^2} \ll p, l, \quad \phi''(x_s) \approx \frac{1}{x_s}, \quad (\text{A12})$$

and we get

$$S(p,l) \approx \sqrt{x_s} e^{x_s} = \sqrt{\frac{2pl}{|R_j|^2}} \exp \left\{ \frac{2pl}{|R_j|^2} \right\}. \quad (\text{A13})$$

Noting that  $p, l \sim R^2/2$ , Eq. (A13) implies Eq. (A4), Q.E.D.

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