

## Bulk and edge correlations in the compressible half-filled quantum Hall state

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(Received 15 December 1998)*

We study bulk and edge correlations in the compressible half-filled state [B. I. Halperin *et al.*, Phys. Rev. B **47**, 7312 (1993) and, E. Rezayi and N. Read, Phys. Rev. Lett. **72**, 900 (1994); **73**, 1052 (1994)] using a modified version of the plasma analogy. The corresponding plasma has anomalously weak screening properties, and as a consequence, we find that the correlations along the edge do not decay algebraically as in the Laughlin (incompressible) case, while the bulk correlations decay in the same way. The results suggest that due to the strong coupling between charged modes on the edge and the neutral fermions in the bulk, reflected by the weak screening in the plasma analog, the (attractive) correlation hole is not well defined on the edge. Hence, the system there can be modeled as a free Fermi gas of *electrons* (with an appropriate boundary condition). We finally comment on a possible scenario in which the Laughlin-like dynamical edge correlations may nevertheless be realized. [S0163-1829(99)00616-5]

### I. INTRODUCTION AND PRINCIPAL RESULTS

Laughlin's theory of the fractional quantum Hall effect<sup>1</sup> (QHE) was given in terms of wave functions of the ground state and quasihole excitation. Using a plasma analogy to calculate the static many-body correlators, which characterize these wave functions, he was able to advance a very successful physical picture of the electron system. The wave functions, describing the incompressible states, contain the Laughlin-Jastrow factor, which leads to special, later introduced, Girvin-MacDonald (GM) correlations in the bulk,<sup>2</sup> and Wen's correlations on the edge.<sup>3,4</sup> The Laughlin-Jastrow factor is ever present in QHE states—it exists even in the compressible half-filled state,<sup>5</sup> for which an explicit wave function has been proposed by Rezayi and Read (RR).<sup>6</sup> The question arises whether its manifestations, in terms of the above-mentioned correlations, survive in more general quantum Hall states and, in particular, in the compressible states. Why is this question important? The correlations that are embodied in the Laughlin-Jastrow factor lie at the heart of various quasiparticle pictures<sup>7-9</sup> (composite fermions, composite bosons) of the QHE in the bulk. From the theoretical viewpoint, it is interesting to understand the status of Bose condensation, implicit in the Laughlin-Jastrow factor,<sup>2,8</sup> in the compressible state. Related to this is the question to what extent Laughlin's quasihole construction in the compressible state (a zero of the wave function) can be considered as an elementary excitation of the system.

Experimentally, these correlations are in principle accessible by tunneling measurements. Indeed, recent edge-tunneling experiments by Grayson *et al.*<sup>10</sup> prompted the question of whether the Luttinger liquid picture,<sup>3,4</sup> which is characterized by Wen's correlations, is valid for general quantum Hall systems, including the compressible states. A

number of theoretical works<sup>11,12</sup> have attempted to explain the puzzling results of Ref. 10, in terms of charged excitations on the edge that are effectively decoupled from the bulk.<sup>13</sup>

In this paper we concentrate on the compressible QHE system at filling factor one-half. We assume that the RR wave function well describes the ground state of the system, even when we consider a system with an edge. Namely, we assume the composite fermion (or, more precisely, dipole) picture<sup>7,9</sup> to apply everywhere. We rederive the GM and Wen's correlations in the Laughlin state considering the leading-order contributions of a weak-coupling plasma approximation (see also Ref. 14). Then we consider the same correlations (appropriately redefined) in the RR state. In calculating these we use the same approach—a systematic expansion of a plasma free energy—with necessary modifications to include the Fermi sea correlations.<sup>15</sup> This introduces a statistical mechanics viewpoint of the problem, in terms of an anomalous, *weakly screening* plasma.

Applying the aforementioned procedure (and viewpoint) on the RR state, we find that Wen's correlations of the edge do not decay algebraically (at large distances) as in the Laughlin state. This excludes the possibility of existence of a subspace of charge-density waves on the edge (of the type found in the Laughlin state) that is decoupled from the rest of the excitations—i.e., the neutral bulk excitations.<sup>16</sup> The form of the obtained equal-time electron Green's function on the edge suggests that, in the first approximation, the physical picture of the RR edge is that of a Fermi gas of electrons. The bulk GM correlations, on the other hand, decay algebraically, in an almost identical way as in the Laughlin state.

Below, we detail the derivation of the correlators, in the bulk (Sec. II) and on the edge (Sec. III). A discussion of theoretical and experimental implications of the results is given in Sec. IV.

II. CORRELATIONS OF THE BULK

In this section, we employ the plasma analogy to derive the appropriately generalized GM correlator in the compressible RR state. To introduce the method, we first use it to derive the known result for the Laughlin state [Eq. (3) below].

A. Correlations of the bulk in the Laughlin state

In the Laughlin state, corresponding to filling factors  $1/m$  with  $m$  odd, the GM correlator<sup>2</sup> is defined as the density matrix

$$\rho(z, z') = \frac{N \int d^2z_2 \cdots \int d^2z_N \Psi_b(z, z_2, \dots, z_N) \times \Psi_b(z', z_2, \dots, z_N)}{\int d^2z_1 \cdots \int d^2z_N |\Psi_b|^2} \tag{1}$$

for the bosonic many-body function

$$\Psi_b = \prod_{i < j} |z_i - z_j|^m \exp\left(-\frac{1}{4} \sum |z_i|^2\right) \tag{2}$$

obtained from the Laughlin wave function by omitting the phases of the relative distances between any two electrons,  $(z_i - z_j)$ . As shown in Ref. 2, the asymptotic form of  $\rho(z, z')$  is

$$\rho(z, z') \sim |z - z'|^{-m/2}. \tag{3}$$

This correlator expresses a Bose condensation, with algebraic off-diagonal long-range order, of composite bosons—defined as electrons with  $m$  flux quanta attached. We now derive the above form using the weak-coupling plasma analogy.

We first rewrite the integrand as<sup>2</sup>

$$\begin{aligned} & \Psi_b(z, \dots, z_N) \times \Psi_b(z', \dots, z_N) \\ &= \exp\left(2m \sum'_{i < j} \ln|z_i - z_j|\right) \exp\left(+m \sum'_i \ln|z - z_i|\right) \\ & \quad + m \sum'_i \ln|z' - z_i| \exp\left(-\frac{1}{2} \sum'_i |z_i|^2 - \frac{1}{4}|z'|^2 - \frac{1}{4}|z|^2\right) \end{aligned} \tag{4}$$

and similarly the numerator. (The prime means that  $i = 1$  is excluded from the summations.) Using the Laughlin plasma analogy we can write  $\rho(z, z')$  as

$$\rho(z, z') = |z - z'|^{-m/2} \frac{Z(z, z')}{Z(z, z)} n, \tag{5}$$

where  $Z(z, z')$  is a partition function of a classical two-dimensional (2D) plasma with inverse temperature  $\beta = 2/m$ , each particle with charge  $m$ , and two impurities with charge  $m/2$  each, at the locations  $z$  and  $z'$ . [ $Z(z, z)$  is a partition function with one impurity of charge  $m$  at an arbitrary location, because the value of the partition function does not depend on  $z$ .]  $n$  is the average density of particles (equal to  $1/2\pi m$  in the usual units). To calculate the ratio of the two partition functions, we may expand the exponentials in the parameter  $m$ , which we will assume to be small. The expansion will generate terms that can be described by diagrams and corresponding rules.

As usual in this kind of expansion in the statistical mechanics analogue, the expansion of the denominator involves only connected diagrams. Each diagram consists of parts, herein called disconnected parts, which connect two impurities at  $z$  and  $z'$  but are otherwise disconnected among themselves. Then, the rules that correspond to each diagram in the expansion are as follows.

(1) Associate with each interaction line a two momentum satisfying momentum conservation at each internal vertex.

(2) Associate with each interaction line between particles  $-(2\pi\beta m^2)/|\mathbf{k}|^2$ , with each interaction line between a particle and an impurity  $-[2\pi\beta m(m/2)]/|\mathbf{k}|^2$ , with each interaction line between impurities  $-[2\pi\beta(m/2)^2]/|\mathbf{k}|^2$ , and with each internal vertex  $n$ .

(3) For each incoming (from  $z$ ) (which is also outgoing to  $z'$ ) momentum for each disconnected part, integrate as  $\int d^2k/(2\pi)^2 \exp(i\mathbf{k}(\mathbf{r} - \mathbf{r}'))$ , but for each internal momentum as  $\int d^2k/(2\pi)^2$ .

(4) Multiply with a symmetry factor (if any). The symmetry factor is an inverse of the number of ways that we can interchange a given number of identical parts of a given diagram and recover the same graph.

The diagrams that represent the interaction with the background are mutually canceled (as we checked for the first diagrams in the expansion), and we will not consider them. In our problem the density  $n$  is fixed and depends on the small parameter  $m$ . In order to get the correct order of the diagram (i.e., the power of  $m$ ) in the expansion, we must

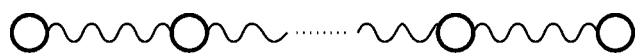


FIG. 1. The diagrams leading to screening of the interaction in the bulk.

take this into account. The lowest-order diagram has value one. The next in order are diagrams of the form shown in Fig. 1 and are of order  $m$ . We can easily sum them, and the result is

$$V_{\text{eff}}(|\mathbf{r}-\mathbf{r}'|) = \left(\frac{m}{2}\right)^2 \int \frac{d^2k}{(2\pi)^2} \exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')] \frac{-\frac{2\pi\beta}{|\mathbf{k}|^2}}{1 + \frac{2\pi\beta m^2}{|\mathbf{k}|^2} n}. \quad (6)$$

The sum represents an effective screened interaction between two impurities. The infinite summation of certain types of diagrams that diverge even more singularly as we increase the number of interaction lines is a well-known ansatz in the many-body theory of the Coulomb-interacting electron gas in three dimensions. This captures well the phenomenon of screening that is characteristic of long-range forces. In our case the infinite summation is even further enforced, given the fact that the diverging diagrams are of the same order in  $m$ .

We now rewrite the ratio of partition functions in Eq. (5) as

$$\frac{Z(z, z')}{Z(z, z)} = \exp[-\beta\Delta f(z, z')], \quad (7)$$

where  $\Delta f(z, z')$  represents the difference in the free energy between the two configurations of the impurities. The above exponential form can be obtained by summing the set of diagrams, whose disconnected parts are of the form shown in Fig. 1. Thus we find

$$\frac{Z(z, z')}{Z(z, z)} = \exp[V_{\text{eff}}(|\mathbf{r}-\mathbf{r}'|)]. \quad (8)$$

As  $|\mathbf{r}-\mathbf{r}'| \rightarrow \infty$  the ratio approaches unity, because  $V_{\text{eff}}$  is an effective screened interaction.<sup>17</sup> Hence, Eq. (5) reduces to the well-known expression for the GM correlator, Eq. (3).

This result is derived and found to have the same form for larger, physical  $m$ 's.<sup>2</sup> Therefore, it is possible to analytically continue the correlator obtained in the weak-coupling approach to larger  $m$ 's. Applying the same weak-coupling infinite summation, it can be shown that the continuation is valid also in the calculation of the static structure factor in the small-momentum limit (when corrections to the infinite summation are added, this includes also the term proportional to the fourth power of the momentum.<sup>14</sup>)

It is interesting to check what the weak-coupling approach yields for the distribution of the charge in the tail of the Laughlin quasihole excitation.<sup>1</sup> The quantity that describes this is<sup>1</sup>

$$g_{12}(|z_1 - w|) = \frac{N}{Z} \int d^2z_2 \cdots \int d^2z_N \left| \prod_{i=1}^N (z_i - w) \Psi \right|^2, \quad (9)$$

where  $\Psi$  and  $Z$  are the Laughlin wave function and its norm, respectively. In order to capture the physics of screening, we sum the same most important diagrams as before and approximate Eq. (9) by<sup>14</sup>

$$g_{12}(|z_1 - w|) = n + \int \frac{d^2k}{(2\pi)^2} \exp[i\mathbf{k}(\mathbf{r}_1 - \mathbf{w})] \times \frac{-\frac{2\pi\beta(m \times 1)}{|\mathbf{k}|^2} n}{1 + n \frac{2\pi\beta m^2}{|\mathbf{k}|^2}}. \quad (10)$$

As  $|z_1 - w| \rightarrow \infty$ , the function  $g_{12}(|z_1 - w|)$  should tend to the unperturbed density  $n$ , and it behaves as

$$g_{12}(|z_1 - w|) = n - m \text{const} \frac{1}{\sqrt{|z_1 - w|}} \exp\left(-\frac{|z_1 - w|}{r_D}\right), \quad (11)$$

where  $(1/r_D^2) = 2\pi\beta m^2 n = 2$ ,  $r_D$  being the Debye length.

## B. Correlations of the bulk in the compressible half-filled state

The theory and physical picture of the filling fractions  $1/m$  where  $m$  is *even*, evolved from some Fermi condensation of charged (Chern-Simons) composite fermions (electrons with even number of flux quanta attached) to a well-defined Fermi condensation of dipole quasiparticles.<sup>9</sup> This emphasized the advantage of Read's picture,<sup>18</sup> which, from the beginning, takes into account the binding of electrons to (so-called) correlation holes. (Equivalently, the statement is that the zeros of the many-body functions are found at or near the electrons.) At even denominators the overall neutral composite object is a dipole (with Fermi statistics).

The ground-state wave function that corresponds to this picture is the RR wave function<sup>6</sup>

$$\Psi_{\text{RR}} = \mathcal{P}_{\text{LLL}}\{\det_{i,j}[\exp(i\mathbf{k}_i \mathbf{R}_j)] \Psi_L\} \quad (12)$$

with a Slater determinant of free waves that fill a Fermi sea, which when projected to the lowest Landau level (LLL) ( $\mathcal{P}_{\text{LLL}}$  stands for the projector) acts on  $\Psi_L$ —the Laughlin wave function. In Eq. (12) we wrote the determinant in terms of plane waves, which constitute a convenient basis for a system of free particles (in a rectangular geometry). The Laughlin wave function, on the other hand, is very often expressed in the rotationally symmetric gauge (corresponding to a rotationally symmetric geometry) amenable to the Laughlin plasma analogy. In order to facilitate our computations we will keep these two distinct geometry choices in the RR wave function. We justify this by the fact that, first, we will be interested in the (long-wavelength) properties of the system in the thermodynamic limit (when the boundary conditions should not matter), and second, each component of  $\Psi_{\text{RR}}$  will enter our calculations in the form of translationally invariant, geometry independent elements.

To illustrate<sup>6,9</sup> the dipole physics contained in Eq. (12), we note that the LLL projection translates factors of the form  $\exp[i(k\bar{z})/2]$ , where  $k = k_x + ik_y$  and  $z = x + iy$ , into the shift operator  $\exp[ik(\partial/\partial z)]$ , which acts on the original (before projection) holomorphic ( $z$  dependent) part of the wave function. This effectively means that each electron becomes dis-

placed from the position of its correlation hole by  $(-ik)$  (where  $k$  takes values from the Fermi sea), and therefore dipole moments are induced.

In the calculation of correlation functions the effects of the LLL projection can be taken into account by using the following identity

$$\int d^2 z \exp(-\frac{1}{2}|z|^2) \exp(i\mathbf{q}\mathbf{r}) \times \left[ \exp\left(-ik_1^* \frac{\partial}{\partial z^*}\right) \exp\left(-ik_1 \frac{z^*}{2}\right) F_1(z^*) \right] \times \left[ \exp\left(ik_2 \frac{\partial}{\partial z}\right) \exp\left(ik_2^* \frac{z}{2}\right) F_2(z) \right]$$

$$= \exp\left(-\frac{k_1^* q}{2}\right) \exp\left(+\frac{k_2 q^*}{2}\right) \exp\left(-\frac{k_2 k_1^*}{2}\right) \times \int d^2 z \exp(-\frac{1}{2}|z|^2) \exp(i\mathbf{q}\mathbf{r}) \exp(-i\mathbf{k}_1 \mathbf{r}) \times \exp(i\mathbf{k}_2 \mathbf{r}) F_1(z^*) F_2(z). \quad (13)$$

If we search for the long-distance behavior of the correlation functions, usually the calculations give the same result as obtained from the unprojected version of the RR function.

This is the case with the appropriately generalized GM correlations to the compressible case. The many-body wave function employed in the calculation of the density matrix [Eq. (1)] is

$$\Psi(z, z_2, \dots, z_N) = \sum_{\sigma \in \mathcal{S}_{N-1}} \text{sgn } \sigma \prod_{i=2}^N \exp[i(k_{\sigma(i)} z_i)/2] \prod_{i<j}' |z_i - z_j + ik_{\sigma(i)} - ik_{\sigma(j)}|^2 \times \prod_{i=2}^N |z - z_i - ik_{\sigma(i)}|^2 \exp\left(-\frac{1}{4} \sum_i |z_i|^2\right). \quad (14)$$

We now introduced a particle, with coordinate  $z$ , without the (projected) plane wave that enters the Fermi sea part, therefore without the fermionic statistics that characterizes the rest of the  $N-1$  particles. The rest of its correlations with other particles is the same as any other particle. Similarly to the Laughlin case, the phase part of the Jastrow-Laughlin factor with coordinate shifts is omitted. This, in the Chern-Simons picture, corresponds to attaching of two flux quanta [at distance  $(ik)$ ] to each electron.

Nevertheless, as can be shown, for the type of calculations that we do, the projection to the LLL does not affect the final result and, for the sake of simplicity, we will explain the method on the unprojected version for which

$$\Psi(z, z_2, \dots, z_N) = \prod_{i=2}^N |z - z_i|^2 \det[\exp(i\mathbf{k}_i \mathbf{r}_j)] \times \prod_{i<j} |z_i - z_j|^2 \exp\left(-\frac{1}{4} \sum_i |z_i|^2\right). \quad (15)$$

We next assume that the dominant correlations lie in the (Jastrow-Laughlin) differences and for the moment neglect the Slater determinant. The complete plasma analogy is again possible and, as explained above, the infinite summation of the diagrams of the type shown in Fig. 1 (for small  $m$ ) is relevant. In the presence of the determinant the first necessary correction to this picture is the introduction of a new vertex that captures also possible Fermi (exchange) correlations between two points in the coordinate space.<sup>14</sup> In the momentum space, this vertex then corresponds to the static structure factor of the free Fermi gas

$$s_0(\mathbf{q}) = n + n^2 \int d^2 r \exp(i\mathbf{q} \cdot \mathbf{r}) [g(|\mathbf{r}|) - 1], \quad (16)$$

where  $\mathbf{q} \neq \mathbf{0}$ , and the radial distribution function is

$$g(\mathbf{r}) = \frac{1}{n^2} \int_{\mathbf{k}_1 \in F.S.} \frac{d^2 k_1}{(2\pi)^2} \int_{\mathbf{k}_2 \in F.S.} \frac{d^2 k_2}{(2\pi)^2} \times \{1 - \exp[i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}]\} \quad (17)$$

(*F.S.* stands for the Fermi sphere). Symbolically, the new vertex is depicted in Fig. 2, as a sum of a direct and an exchange part, in which full lines represent Fermi particle lines. From Eq. (17), and the definition [Eq. (16)],

$$s_0(\mathbf{q}) - n = - \int_R \frac{d^2 k}{(2\pi)^2}. \quad (18)$$

Here  $R$  represents the area of overlap between two Fermi spheres as shown in Fig. 3, where the center of one of the two spheres is displaced by  $\mathbf{q}$  from the center of the other one. The value of  $s_0(\mathbf{q})$  is then given exactly by the shaded area in Fig. 3. The area is easily calculated for  $|\mathbf{q}|$  small and the result is

$$s_0(\mathbf{q}) = \frac{3}{4} \frac{k_f |\mathbf{q}|}{\pi^2}. \quad (19)$$

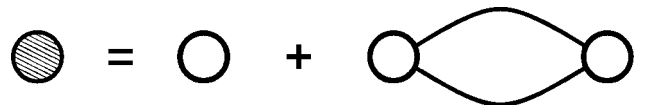


FIG. 2. The vertex  $s_0(\mathbf{q})$  in the bulk of a RR state.

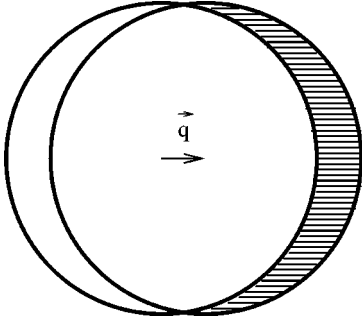


FIG. 3. The overlap of two shifted Fermi spheres.

With the necessary introduction of the new vertex  $s_0(\mathbf{q})$ , the interaction becomes less effectively screened than in the usual (Laughlin) case. It becomes

$$V_{\text{eff}}(|\mathbf{q}|) = \frac{-\frac{2\pi\beta m^2}{|\mathbf{q}|^2}}{1 + \frac{2\pi\beta m^2}{|\mathbf{q}|^2} s_0(\mathbf{q})}, \quad (20)$$

where the denominator can be interpreted as an anomalous dielectric constant of the corresponding modified plasma. In the coordinate space, at large distances,  $V_{\text{eff}} \sim 1/r$ , i.e., it is still long ranged and only partially screened.

Nevertheless, if (keeping this change in mind) we apply the same summations and arguments as in the Laughlin case, we come up with the same algebraic decay of the GM correlations as in that case.<sup>14</sup> This decay is slightly modified by the exponential [Eq. (8)] of the partially screened interaction (effectively a constant as in the Laughlin case at large distances).

The question that arises immediately is whether the analytical continuation to larger (physical)  $m$ 's is possible, and, moreover, whether the screening plasma approach is reliable in giving the leading behavior of the correlator. Because of the absence of a complete analogy with some physical, well-studied plasma, there are no available results for larger  $m$  to compare with. It was found<sup>14</sup> that the weakly screening plasma approach gives the right [valid also for large  $m$  (Ref. 19)] leading (small-momentum) behavior for the static structure factor of the compressible state and generates expected (odd) powers of momentum in the expansion. If we try to go beyond this approach, and look for small- $m$  (expected) corrections, it seems that they cannot be generated.<sup>14</sup> This is probably due to the nonanalyticities present in the compressible case (which were absent in the Laughlin case) that do not allow a perturbative treatment. Therefore we believe that the approach (essentially nonperturbative) can generate the correct (large- $m$ ) leading behavior for the correlations that we study. They are between points that are directly connected only to the charge (Jastrow-Laughlin) part of the wave function, and that immediately suggests an approach that captures screening for their calculation.

The weak-screening property of this modified plasma can be very well seen by considering a zero of the electron coordinates at a point  $w$ ,<sup>14</sup> which corresponds to the Laughlin quasihole in the incompressible case,

FIG. 4. The diagrams contributing to the calculation of  $g_{12}(|z_1 - w|)$  in the RR state.

$$\prod_{i=1}^N (z_i - w) \Psi_{\text{RR}}. \quad (21)$$

To simplify the calculation of the distribution of charge in the tail of this excited state, we will assume the unprojected version of the RR state in Eq. (9) (The use of the projected state involves some complications that are not essential and do not influence the final result.) Now the appropriate infinite sum of the modified plasma can be expressed, in terms of diagrams depicted in Fig. 4, with the shaded circle representing the new vertex, i.e., in this case

$$g_{12}(|z_1 - w|) = n + \int \frac{d^2k}{(2\pi)^2} \exp[i\mathbf{k}(\mathbf{r}_1 - \mathbf{w})] \times \frac{-\frac{2\pi\beta(m \times 1)}{|\mathbf{k}|^2} s_0(\mathbf{k})}{1 + s_0(\mathbf{k}) \frac{2\pi\beta m^2}{|\mathbf{k}|^2}}. \quad (22)$$

The most important contributions to  $g_{12}$  in the limit  $|z_1 - w| \rightarrow \infty$  come from nonanalyticities present in the integrand. They stem from the nonanalytic behavior of  $s_0$  at  $k=0$  and  $k=2k_f$ . Assuming that the small-momentum result [Eq. (19)] for  $s_0$  is valid for any  $k$  (analogously to the Thomas-Fermi approximation for the electron gas in three dimensions), we get the contribution from the  $k=0$  region

$$[g_{12}(|z_1 - w|) - n]_{T.F.} \propto -\frac{k_f}{m} \frac{1}{|z_1 - w|^3}. \quad (23)$$

The contribution from the  $k=2k_f$  region can be calculated to be

$$[g_{12}(|z_1 - w|) - n]_{F.O.} \propto -\frac{1}{k_f} \frac{1}{|z_1 - w|^3} \sin(2k_f r). \quad (24)$$

We may conclude from expressions (23) and (24), which summed up give the change in the distribution of the charge from the uniform ground-state contribution  $n$  (in the  $|z_1 - w| \rightarrow \infty$  limit), that the density far from the point  $w$  tends to  $n$  very slowly in comparison with the Laughlin case. The charge of this excited state (which may be argued to be  $1/m$  as for the Laughlin quasihole<sup>18</sup>) is spread over a much larger region than the one in the Laughlin case, due to the poor-screening properties of the modified plasma.

### III. CORRELATIONS OF THE EDGE

#### A. Edge correlations in the Laughlin case

In Ref. 4, Wen showed how calculation of the equal-time correlator along the edge in the Laughlin case can be reduced to the problem of finding the electrostatic energy of placing an impurity outside the Laughlin plasma. For the sake of completeness and easy reference for our calculation, we will,

in brief, repeat his arguments. We then demonstrate, that the result is recovered in a weak-coupling expansion.

*Review of Wen's procedure.* We consider a disk of the Laughlin plasma, with a fixed radius  $R$ , at fixed filling factor  $1/m$ . As we increase the number of particles  $N$ , the density will increase (with appropriate change in the magnetic field  $B$  to keep  $1/m$  constant) and the description that neglects details of the order of a magnetic length would be more and more accurate, and the Laughlin plasma will behave as a metal (with its screening properties).

To calculate the edge correlator, we envision placing an impurity of charge  $m$  outside the disk of such a plasma, at a distance  $z$  where  $|z| \gg R$  (so that the details of the edge do not matter), and consider the ratio  $Z_I/Z$ , in which

$$Z_I(z, \bar{z}) = \int \prod d^2 z_i \exp \left( \sum_{i < j} 2m \ln |z_i - z_j| \right) \times \exp \left[ + \sum_{k=1}^N \left( -\frac{1}{2} |z_k|^2 + 2m \ln |z - z_k| \right) \right] \quad (25)$$

and

$$Z = \int \prod d^2 z_i \exp \left[ \sum_{i < j} 2m \ln |z_i - z_j| + \sum_{k=1}^N \left( -\frac{1}{2} |z_k|^2 \right) \right]. \quad (26)$$

From the first quantization (quantum-mechanical) point of view the ratio is the one-particle (electron) density at point  $z$ . On the other hand, from the point of view of the plasma analog,  $\ln Z_I/Z$  is the electrostatic energy required to transfer the impurity from infinity to the point  $z$ . This energy can be expressed as

$$\ln \frac{Z_I}{Z} = mN2 \ln |z| - m \ln \left( 1 - \frac{R^2}{|z|^2} \right) + O(1/N). \quad (27)$$

The first contribution is the electrostatic energy between the total charge  $N$  and the impurity, where in the first approximation the plasma droplet is assumed undeformed by the presence of the impurity. The second contribution describes the most important part of the deformation that occurs: the image charges of the impurity.<sup>20</sup> The rest of the contributions are expected to be of order  $1/N$  or less (due to the form of the first contributions, analyticity in  $N$  is expected).

To find out the electron correlator between points  $z_1$  and  $z_2$  (on the edge), Wen first noticed that the expression on the right-hand side of Eq. (27) is holomorphic in  $z$  and antiholomorphic in  $\bar{z}$  (outside the system), and therefore can be analytically continued, i.e.,

$$\ln \frac{Z_I(z_1, z_2)}{Z} \approx mN \ln(z_1 \bar{z}_2) - m \ln \left( 1 - \frac{R^2}{z_1 \bar{z}_2} \right). \quad (28)$$

$z_1$  and  $z_2$  can be considered to be even on the edge if the final result of the analytical continuation exists, i.e., if it is finite. This excludes the points  $z_1 = z_2$  on the edge ( $|z_1| = |z_2| = R$ ), where the above expression is logarithmically singular. Then, if  $z_1 = R \exp[i(y/R)]$  and  $z_2 = R$ , the electron correlator is (in the disk geometry)

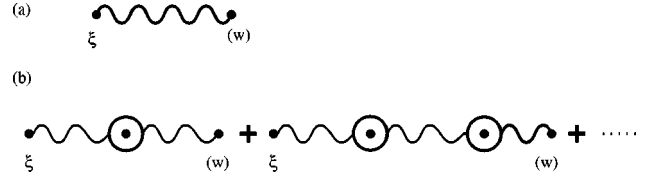


FIG. 5. (a) The leading contribution to the electrostatic interaction of a charged impurity with the plasma. (b) Diagrams that are not included in Eq. (27).

$$\langle L | \Psi^\dagger(z_1) \Psi(z_2) | L \rangle = \frac{Z_I(z_1, \bar{z}_2)}{Z} \times \exp(-\frac{1}{4} |z_1|^2) \exp(-\frac{1}{4} |z_2|^2). \quad (29)$$

In the limit  $y/R \ll 1$ , where circular and rectangular geometries are indistinguishable, this becomes

$$\langle L | \Psi^\dagger(z_1) \Psi(z_2) | L \rangle \sim \frac{1}{y^m}, \quad (30)$$

which coincides with the correlations on the edge obtained in the (more familiar) bosonization approach.

*Derivation of the electrostatic energy of the edge impurity in the Laughlin case using the weak-coupling plasma expansion.* According to Wen's idea, in order to find the equal-time electron correlator, it is sufficient to compute the electrostatic energy of an impurity of charge  $m$  at a point  $z$  outside the Laughlin plasma. We now describe the diagrammatic solution of this statistical mechanics problem. To simplify the calculation, we consider a plasma that extends over the half-plane  $x \leq 0$  instead of a disk (in the thermodynamic limit, the choice of geometry is immaterial); the impurity coordinate is  $z = \xi$  (along the positive  $x$  axis). The derivation of this electrostatic energy, using the weak-coupling plasma expansion, parallels that of the density in the bulk; i.e., calculating the electrostatic energy of a particle interacting with a negative background—the rest of the particles.<sup>21</sup> In the present case the system is not infinite in the  $x$  direction, and that introduces a new type of vertex in the diagrammatic expansion. The vertex connecting two interaction lines of momenta  $q_i, q_f$  in the  $x$  direction, which in the infinite case is

$$n \delta(q_i - q_f) \quad (31)$$

(where  $n$  is the density), is replaced in the half-plane case by

$$\frac{n}{2\pi} \left[ \frac{1}{-i(q_i - q_f)} + \pi \delta(q_i - q_f) \right], \quad (32)$$

i.e., proportional to the Fourier transform of theta function,

$$\int_{-\infty}^0 \exp[i(q_i - q_f)x] dx = \int_{-\infty}^{+\infty} \theta(-x) \exp[i(q_i - q_f)x] dx = \frac{1}{-i(q_i - q_f)} + \pi \delta(q_i - q_f). \quad (33)$$

The diagrams that are leading in the small- $m$  expansion, and are of order  $m$ , are given in Figs. 5(a) and 6. The dia-

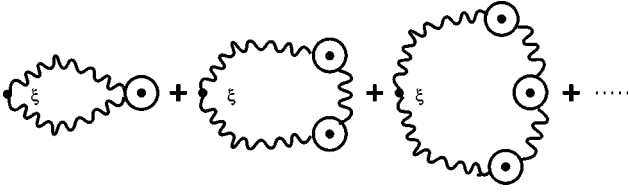


FIG. 6. Diagrammatic representation of the image charge energy term. The vertex in the half-plane case [corresponding to Eq. (32)] is denoted by a dotted circle.

gram in Fig. 5(a), in which the points  $w$  in the half-plane are integrated over, corresponds to the first (direct term) in Wen's expansion, Eq. (27). It is also proportional to the size of the system and strictly speaking diverges in the case of half-plane system. [This divergence does not matter and can be handled by considering a rectangular system with sizes  $L_x$  and  $L_y$  much longer than the distance ( $\xi$ ) of the impurity from the  $y$  axis.]

The diagrams of the type depicted in Fig. 5(b) are not included [by using a screened instead of the bare interaction in Fig. 5(a)], although they are of order  $m$  as well. As we remarked earlier in this section, the diagrams that should be taken into account are of the same form as the ones that we select to play the role of positive background (i.e., those that cure divergences in the expansion with the two-particle interaction) in the infinite system case. In that case, the diagram of the form in Fig. 5(a) cancels all divergences when the interaction line does not connect to any other interaction line. When the proper selection is done, and all diagrams that cure divergences are present, the complete partition function is well defined and a constant. Similarly with impurities and in the semi-infinite case, if all due interactions (additional diagrams) are included in the partition function (including the interaction of impurities with positive background), it becomes a constant (due to the screening property of plasma). The partition function  $Z_I$  in Wen's derivation is not complete, and therefore the part on the right-hand side of Eq. (27) is not a constant and can be associated with the interaction of the impurity with "negative background."

The diagrams in Fig. 6 are all relevant and deserve special attention. Their value (at least in the long-distance limit) can be calculated by solving an integral equation for an effective vertex  $V(q_i, q_f)$ ,

$$V(q_i, q_f) = \frac{n}{2\pi} \left\{ \left[ \frac{1}{-i(q_i - q_f)} + \pi \delta(q_i - q_f) \right] + \int dk \left[ \frac{1}{-i(q_i - k)} + \pi \delta(q_i - k) \right] \times \frac{-4\pi m}{(q^2 + k^2)} V(k, q_f) \right\}. \quad (34)$$

This equation can be schematically introduced as in Fig. 7, where we denoted only momenta along the  $x$  direction. The momentum  $q$  along the  $y$  direction is the same on every line as in the infinite-plane case. Then the contribution of all diagrams in Fig. 6, summarized by the diagram on the left-hand side of Fig. 7, can be expressed as

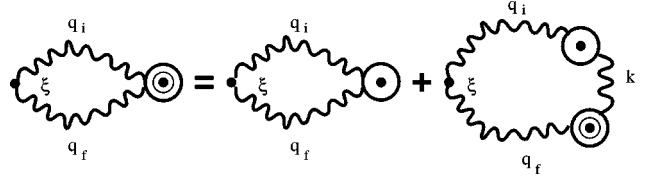


FIG. 7. The infinite sum of diagrams included in Fig. 6, represented as an integral equation for the effective vertex  $V(q_i, q_f)$  (dotted double circle).

$$\frac{1}{2} \int \frac{dq}{(2\pi)} \int \frac{dq_i}{(2\pi)} \int dq_f \exp[-i(q_i - q_f)\xi] \times \frac{-4\pi m}{(q^2 + q_i^2)} V(q_i, q_f) \frac{-4\pi m}{(q^2 + q_f^2)}. \quad (35)$$

The solution to the equation, given in the long-distance approximation, can be found in Appendix A. It reproduces the electrostatic energy of the impurity and its image charge in the half-plane case, corresponding to the leading contribution to the second term in Eq. (27) when the disk is considered to be large [ $R \gg (|z| - R)$ ].

## B. Edge correlations in the compressible half-filled state

Now we switch to the calculation of the edge correlations in the RR case using the diagrammatic method. We first consider the unprojected RR state, and as a basis of free waves that enter the Slater determinant, we choose

$$\frac{\exp(ik_y y)}{\sqrt{2\pi}} \times \frac{\cos(k_x x)}{\sqrt{\pi}}$$

or

$$\frac{\exp(ik_y y)}{\sqrt{2\pi}} \times \frac{\sin(k_x x)}{\sqrt{\pi}}, \quad (36)$$

where  $k_x$  and  $k_y$  take values from a Fermi box (not sphere) in the  $k$  space. As in the Laughlin case, we assume that the radius of the Laughlin disk is very large in comparison with the distance (along the edge) over which we measure correlations. So, effectively, we again consider the half-plane problem for which, on the other hand, the basis choices [Eq. (36)] are also appropriate; there is no discrepancy between geometries of the Laughlin-Jastrow and free-wave part in the ground state, as in the full-plane case. In Eq. (36) the coordinate  $x$  is measured from the edge of the half-plane, i.e., a tangent to the large disk. If, somehow, the charge and neutral (fermionic) part decouple on the edge, the choices [Eq. (36)] are quite natural, because they satisfy the requirement that the (neutral) current normal to the boundary is zero, i.e., that the fermionic number is conserved.

First, we consider the correlations of the object introduced in Eq. (21) to which, due to the correspondence of its construction to the one of the Laughlin quasihole, we will refer to as a quasihole. This, of course, does not entail that the quasihole is a well-defined object—eigenstate of the Hamiltonian, as in the case of the Laughlin quasihole. It might be such (on the edge) if we find that its correlations are of the same type as in the Laughlin case [Eq. (30)], and therefore

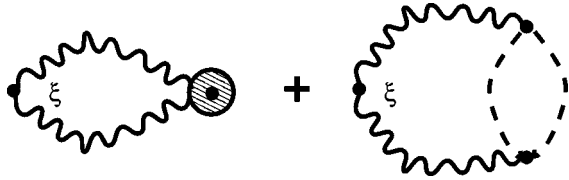


FIG. 8. The modified half-plane vertex in the RR state; the second diagram represents the exchange contribution.

the charge degrees of freedom (on the edge) in the RR state can be described in the Luttinger liquid framework (or, microscopically, by the possible states of quasiholes). Again, as in Sec. III A, to mimic the charge part of the electron, in this case, we consider the correlations of the object constructed by putting  $m$  ( $m=2$ ) quasiholes at the same place. Then, to find out if there is a departure from the Laughlin case, we consider the density of this object outside the half-plane system described by the RR state.

In the language of the modified plasma, we are placing a charged impurity (not directly connected to the plane-waves part) outside the system and checking whether the image charge physics still holds. Due to the poor screening in this modified plasma, the charge induced by the external impurity does not accumulate near the edge (within a microscopic screening length) as in the ideal plasma. Rather, the induced charge is expected to slowly decay towards the interior of the system. To get a handle on the form of the electrostatic energy associated with this effect, we can employ the Thomas-Fermi approximation to compute the induced charge, given the dielectric properties of the modified plasma derived in the Sec. II [Eq. (20) and the subsequent discussion]. The calculation is summarized in Appendix B. We find that unlike the Laughlin (ideal) plasma case, the leading behavior of the “image charge” electrostatic energy is a constant, rather than a logarithmically singular term. We now derive this result systematically in the diagrammatic expansion framework.

We first must find an effective vertex that corresponds to Eq. (32) in the Laughlin case and is represented by the diagrams in Fig. 6. That will be done at the same level of approximations as in the case of the bulk correlations. Explicitly, to find the value of the effective vertex, we consider the two contributions, direct and exchange, depicted symbolically in Fig. 8, in the simplest diagram with only two interaction lines. (The use of the dotted lines is to emphasize that we are now in the half-plane case.)

The direct contribution (unapproximated) for the first choice of basis for the Fermi sea in Eq. (36) is equal to

$$\begin{aligned}
 & \frac{1}{2} \int d^2 z_1 \ln^2 |z - z_1| (2m)^2 n \frac{1}{N} \sum_{\mathbf{k} \in F.S.} \cos^2(k_x x_1) \\
 &= \frac{n}{2} (4\pi m)^2 \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 q}{(2\pi)^2} \frac{1}{p_x^2 + p_y^2} \frac{1}{q_x^2 + q_y^2} \\
 & \quad \times \exp[-i(q_x - p_x)z] (2\pi) \delta(q_y - p_y) \\
 & \quad \times \int dx_1 \exp[i(q_x - p_x)x_1] \frac{1}{N} \sum_{\mathbf{k} \in F.S.} \cos^2(k_x x_1),
 \end{aligned} \tag{37}$$

where the summation in  $\mathbf{k}$  runs over the Fermi box. This summation can be rewritten as

$$\frac{1}{N} \sum_{\mathbf{k} \in F.S.} \cos^2(k_x x_1) = \frac{1}{2} + \frac{1}{N} \sum_{\mathbf{k} \in F.S.} \frac{\cos(2k_x x_1)}{2}. \tag{38}$$

The second part can be neglected, because it leads to an effective smearing of both the  $\delta$  function  $\delta(p_x + q_x)$  and the pole  $\sim 1/(p_x + q_x)$  that we would get if it were a constant. The first part is the most important and singular in the infrared limit, which dominates the infinite summation above. Therefore the direct contribution to the effective vertex is

$$\frac{1}{2} \frac{n}{2\pi} \left[ \frac{1}{-i(q_x - p_x)} + \pi \delta(q_x - p_x) \right], \tag{39}$$

i.e., half of the vertex in the Laughlin case.

Applying similar arguments, that is keeping the most important terms that contribute to the value of the diagram in the long-distance limit, we find that the exchange contribution to the effective vertex is

$$\left(-\frac{1}{4}\right) \frac{n}{2\pi} [\pi \delta(p_x - q_x)]. \tag{40}$$

We get the same contributions, Eqs. (39) and (40), for the second choice of basis in Eq. (36). Therefore, the effective vertex in the RR case that parallels Eq. (32) in the Laughlin case is

$$\frac{1}{2} \frac{n}{2\pi} \left[ \frac{1}{-i(q_x - p_x)} + \frac{\pi}{2} \delta(p_x - q_x) \right]. \tag{41}$$

It is not a simple multiple of the Laughlin vertex; because of the exchange contribution, the solution of a new integral equation, corresponding to Eq. (34) with the new “bare” vertex, will not yield the leading logarithmic behavior characteristic of the Laughlin case, which can be translated into the algebraic decay of the quasihole correlator. Namely, if  $a \neq 1$  (for the definition of  $a$  see Appendix A), the solution of the integral equation in the long-distance limit is

$$\begin{aligned}
 V(q_i, q_f) \approx & -\frac{(q^2 + q_i^2)}{b} \delta(q_i - q_f) \\
 & + \frac{1}{-i(q_i - q_f)} \frac{c}{(\pi a c b)^2} (q^2 + q_i^2)(q^2 + q_f^2).
 \end{aligned} \tag{42}$$

In our case  $b = -4\pi m$ ,  $a = \frac{1}{2}$ , and  $c = \frac{1}{2}(n/2\pi)$ . The contribution from the  $\delta$  function to the electrostatic energy is

$$-m \ln \frac{\Lambda}{q_c}, \tag{43}$$

where  $q_c$  is the infrared cutoff. There is no obvious way to cancel the  $q_c$  dependence; i.e., in the case of the modified plasma, we must keep the size of the system finite, and the distance of the impurity should be considered smaller than the size of the system, in the calculations. (Note that this appears to indicate that the expansion in  $N$  is not analytic as in the Laughlin case.) The contribution of the second term can be written as



$$m \text{ const} \times (\Lambda - q_c) q_c \times (q_c \xi) + f(\Lambda, q_c) + O[(q_c \xi)^2], \quad (44)$$

where  $\text{const} > 0$  and  $f$  is an algebraic function of  $\Lambda$  and  $q_c$ . In principle, other contributions, of order higher in  $m$ , from a more detailed solution of the integral equation can be calculated. We expect that their dependence on  $\xi$  will be of the form  $(\xi q_c)^n$  or  $(1/\xi)^n$ , where  $n$  takes on positive integral values, ( $\xi < 1/q_c$ ), and will not change the leading behavior [Eq. (43)] (in which we are interested the most). In the scope of our approach, which takes  $m$  small (and assumes the possibility of an analytical continuation to higher  $m$ ), it is hard to estimate the true coefficients in front of the powers of  $\xi$ , due to the requirement to know them to all orders in  $m$ . Also, there might be relevant contributions from other diagrams (in the small- $m$  expansion), which we did not consider. But as we assume that the plasma correlations (although modified) are dominant for the calculation of the quasihole correlator, we do not expect that there will be any change in the leading behavior described by Eq. (43).

The above calculations imply that the overlap between two quasihole excitations on the edge does not depend on the distance between them; it is a constant, but decreases with the size of the system. This might be understood, taking into account that the quasiholes in the RR state are not well-defined, well-localized objects in the bulk (see the end of Sec. II B) and certainly not on the edge where the screening seems to be even weaker than in the bulk.

Once we take this point of view that, in fact, the states described by the Laughlin quasihole construction on the edge are extended, a special care must be taken concerning their normalization. In general, the normalization is expected to depend on the size of the system [as in the case of the free waves (in the noninteracting system)]. Therefore, the first contribution to the plasma electrostatic energy [Eq. (43)] (that through the infrared cutoff depends on the size of the system) might be a consequence of an incomplete normalization of the quantum-mechanical correlator at the beginning of our calculation. If this term is included (in the normalization) from the beginning, the value of the correlator at large distances (in our approximation) approaches unity.<sup>22</sup>

To find out the electron correlator, we must take into account the correlations that come from the neutral (plane-wave) part of the RR function, alone. These are not included in the preceding (modified plasma) calculations, which gave the correlator of the quasihole, the object that (in our approximation) carries the charge part of the electron. The neutral contribution is expected to be of the form

$$\sim g \frac{\sin(k_y^F y)}{y}, \quad (45)$$

where  $g$  is a coefficient that depends on the boundary conditions. When combined with the charge correlator, it produces the usual (physical) decay of the electron correlator with the distance. Except for the dependence on the size of the system, the electron correlations on the edge are as if the system was a free (two-dimensional) Fermi gas of *electrons*. It can be shown that the same long-distance behavior of the correlations follows from the LLL projected wave function [Eq. (12)].

#### IV. DISCUSSION AND CONCLUSIONS

If we assume that, indeed, the whole description of the edge of the compressible state is equivalent to that of a free Fermi gas, we can try to predict the occupation numbers (probability density) of electrons near the edge. Then the second choice for the boundary condition in Eq. (36) is more appropriate because the probability density  $[\rho(x)]$  should vanish at some point near the edge ( $x \sim 0$ ). The resulting probability-density distribution

$$\rho(x) \sim \left[ k_F - \frac{\sin(2k_F x)}{2x} \right] \quad (x < 0) \quad (46)$$

is very similar to the smooth function that one can get extrapolating the data that describe the occupation numbers for electron near the edge in (finite-system) exact-diagonalization studies,<sup>23</sup> and the observed oscillations might be identified as the Friedel oscillations. Also, with the above assumption, the density of states for electron tunneling into the compressible edge would be similar to the one for tunneling into a Fermi liquid (metal). This is consistent with our intuitive expectations given the compressible nature of the system, if, loosely speaking, the characteristic energies for the motion of the charge and neutral (Fermi) part are comparable.

We believe that it would be possible to construct an effective (1+1)-dimensional theory along the edge, which has the same correlations that we expect, taking the coordinate normal to the edge where it corresponds to time, i.e.,  $x \equiv vt$ , and translating our diagrammatic calculations into an effective interaction between a neutral and charged part. This would yield a model for the suppression of the correlation of the chiral boson theory<sup>3,4</sup> (charge part), which assumes that its neutral and charge components move with the same velocity ( $v_c = v_n = v$ ) along the edge. If the model is generalized to the one for which  $v_c \gg v_n$ , at sufficiently large momenta high energies where the exchange part of the interaction is suppressed (due to a reduced overlap of the two one-dimensional spheres), we expect that the chiral boson correlations will be released. Therefore, the difference in the dynamics of the charge and neutral part appear to be a necessary condition for the decoupling of the edge and bulk (the charge and neutral part) at high-enough energies, as seen in experiments.<sup>10</sup> (For a similar explanation of the simultaneous suppression of the neutral part see Ref. 12.) The “true” (low-energy) correlations should reflect the compressible nature of the system.

In contrast with the edge problem, the bulk correlations in the compressible case that we considered seem to be similar to the ones in the incompressible case. The GM correlations are almost identical, and, due to the finite screening, the quasiholes (correlation holes) have a chance to be considered as well-defined (albeit very extended) objects (like Skyrmions when the compressible degree of freedom—spin—is included in the incompressible problem<sup>24</sup>). This, intuitively, gives additional support to the quasiparticle pictures of the bulk that we have by now. On the other hand, the edge correlations differ completely from the ones in the incompressible case. In incompressible states the edge physics is a reflection of the bulk physics, and the same quasiparticle picture of the bulk is possible on the edge. In the compress-

ible case, and, in the plasma analogy, due to the very weak screening on the edge, we probably cannot talk about existence of the correlation hole that, in Read's picture of the bulk, attracts an electron and creates a weakly-interacting composite object—a Fermi quasiparticle. In the scope of our approach, and in the first approximation, electrons are unbounded, and the edge of the compressible state appears to be similar to the edge of free-electron gas (with an appropriate boundary condition).

### ACKNOWLEDGMENTS

We gratefully acknowledge useful discussions with A. Auerbach, J. Feinberg, J. H. Han, A. MacDonald, R. Rajaraman, S. Sondhi, A. Stern, and X.-G. Wen, and especially with N. Read. This work was partially supported by Grant No. 96-00294 from the United States–Israel Binational Science Foundation (BSF), Jerusalem, Israel, and the Technion–Haifa University Collaborative Research Foundation. M.M. also acknowledges support from the Fund for Promotion of Research at Technion, and the Israeli Academy of Sciences. E.S. would like to thank the hospitality and support of the Institute of Theoretical Physics in UCSB, Santa Barbara, where part of this work was carried out.

### APPENDIX A

We consider the integral equation (34) for the case of a general vertex

$$c \left[ \frac{1}{-i(q_i - q_f)} + a\pi\delta(q_i - q_f) \right]. \quad (\text{A1})$$

Then, the integral equation can be rewritten as

$$V(q_i, q_f) \left[ 1 - \frac{\pi acb}{q^2 + q_i^2} \right] = c \left[ \frac{1}{-i(q_i - q_f)} + \pi a\delta(q_i - q_f) \right. \\ \left. + b \int dk \frac{V(k, q_f)}{i(k - q_i)(k^2 + q^2)} \right], \quad (\text{A2})$$

where  $b = -4\pi m$ . If we try simply to iterate the equation in the limit when  $q_i \rightarrow q_f$ , we find that each iteration produces a solution of the form

$$V(q_i, q_f) = \frac{\alpha(q_i, q_f; q)}{-i(q_i - q_f)} + \beta(q_i; q)\delta(q_i - q_f) + f(q_i, q_f; q), \quad (\text{A3})$$

where  $\alpha$  and  $\beta$  are fixed, i.e., do not change after some iterations, and  $f(q_i, q_f; q)$  keeps changing but does not have any (new) singularity as  $q_i \rightarrow q_f$ . It can be assumed, from the iteration analysis, that  $f(q_i, q_f; q)$  is analytic in all of its variables in the long-distance limit.

If we assume that the solution is of the form (A3), and that  $\alpha$  is an analytic function of its variables, the integration on the right-hand side can be done and yields the expression

$$c \left\{ \frac{1}{-i(q_i - q_f)} \left[ 1 + \frac{\beta b}{q^2 + q_i^2} \right] + \pi a\delta(q_i - q_f) \right. \\ \left. + b\pi \left[ \frac{f(q_i, q_f)}{q_i^2 + q^2} + \frac{f(iq, q_f)}{iq(iq - q_i)} \right] \right. \\ \left. + b\pi \frac{1}{-i(q_i - q_f)} \left[ \frac{\alpha(q_i)}{q_i^2 + q^2} - \frac{\alpha(q_f)}{q_f^2 + q^2} \right] \right. \\ \left. + b\pi \frac{\alpha(iq)}{q(iq - q_i)(iq - q_f)} \right\}, \quad (\text{A4})$$

where  $\alpha(k) \equiv \alpha(k, q_f; q)$ . In order to equate the  $\delta$  functions on both sides (at  $q_i = q_f$ ),  $\beta$  must be

$$\beta = \frac{\pi ac}{\left[ 1 - \frac{\pi acb}{q^2 + q_i^2} \right]}. \quad (\text{A5})$$

Then we equate the coefficients with  $1/-i(q_i - q_f)$  at the point  $q_i = q_f$  to get

$$\alpha(q_i, q_f; q) \Big|_{q_i=q_f} = \frac{c(q^2 + q_i^2)^2}{(q^2 + q_i^2 - \pi abc)^2}. \quad (\text{A6})$$

To be consistent with the iteration result (and also with symmetry arguments),

$$\alpha(q_i, q_f; q) = c \frac{q^2 + q_i^2}{(q^2 + q_i^2 - \pi abc)} \frac{q^2 + q_f^2}{(q^2 + q_f^2 - \pi abc)}, \quad (\text{A7})$$

although it is not consistent with our assumption that  $\alpha$  is an analytic function at the beginning of the substitution of Eq. (A3). Still, it does satisfy the assumption in its long-distance version,

$$\alpha(q_i, q_f; q) \approx \frac{c}{(\pi abc)^2} (q^2 + q_f^2)(q^2 + q_i^2), \quad (\text{A8})$$

and the same approximation must be employed in the previous equations.

To complete the solution we must find  $f(q_i, q_f; q)$  from the remaining equation

$$\left[ 1 - \pi(a+1)bc \frac{1}{q^2 + q_i^2} \right] f(q_i, q_f) = \pi cb \left[ \frac{f(iq, q_f)}{iq(iq - q_i)} \right]. \quad (\text{A9})$$

[We used  $\alpha(iq) = 0$ , which is consistent with the fact that the poles at  $k = \pm iq$  in  $\alpha(k)/(k^2 + q^2)$  at the beginning of the calculation were spurious.] In the long-distance (or small-momentum) approximation, i.e., when

$$(a+1) \frac{f(q_i, q_f; q)}{(iq + q_i)} \approx \frac{f(iq, q_f; q)}{iq}, \quad (\text{A10})$$

a nontrivial (nonzero) solution exists only when  $a = 1$  (Laughlin case). It is

$$f(q_i, q_f; q) = q_i + iq \quad (\text{A11})$$

in the limit when  $q_i \rightarrow q_f$  (irrespective from the value of  $c$ ).

By power counting or by explicit calculation, we can find out that the leading contribution in the Laughlin case comes from the latter part of the solution. An introduction of an infrared cutoff is necessary, but the dependence on it disappears when the  $\delta$ -function part of the solution is included. When substituted in Eq. (35), this yields

$$-m \ln(\xi\Lambda) \quad (\text{A12})$$

as the electrostatic energy of the impurity and its image counterpart in the long-distance approximation, where  $\Lambda$  is an ultraviolet cutoff (corresponding, e.g., to the inverse screening length of the plasma). There is no dependence on the infrared cutoff, because we are considering a half-plane (semi-infinite) system and are recovering the well-known result for that case.

### APPENDIX B: “IMAGE CHARGE” INTERACTION ENERGY IN A MODIFIED PLASMA

We consider a pointlike impurity of charge  $m$ , placed at a distance  $\xi$  from the edge of a two-dimensional modified plasma that occupies the half-plane  $x \leq 0$ . The plasma is characterized by a wave-vector dependent dielectric constant of the form

$$\epsilon(q) = 1 + \frac{q_0}{q}, \quad \text{where } q_0 = \frac{3}{4} \frac{k_f}{\pi^2} \quad (\text{B1})$$

and  $q = |\mathbf{q}|$  [see Eqs. (19) and (20)]. The electrostatic potential generated by the charge distribution, that the external impurity induces in the plasma, is given by

$$V_{\text{ind}}(\mathbf{r}) = V_{\text{sc}}(\mathbf{r}) - V_{\text{ex}}(\mathbf{r}). \quad (\text{B2})$$

Here  $V_{\text{sc}}(\mathbf{r})$  is the screened potential of the impurity

$$V_{\text{sc}}(\mathbf{r}) = m \int d^2r' D(\mathbf{r}, \mathbf{r}') \ln|\mathbf{r} - \xi\hat{x}|,$$

where

$$D(\mathbf{r}, \mathbf{r}') = \theta(-x)\theta(-x') \int \frac{d^2q}{(2\pi)^2} e^{-i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \epsilon^{-1}(q), \quad (\text{B3})$$

and  $\epsilon(q)$  is given by Eq. (B1) (the theta functions restrict the screening to the half-plane occupied by the plasma). This yields (in  $q$  space)

$$V_{\text{ind}}(\mathbf{q}) = - \frac{m\pi q_0 e^{-|q_y|\xi}}{(q+q_0)|q_y|(|q_y|-iq_x)}. \quad (\text{B4})$$

The (two-dimensional) Poisson equation then relates this component of the potential to the induced charge

$$\rho_{\text{ind}}(\mathbf{q}) = \frac{q^2}{2\pi} V_{\text{ind}}(\mathbf{q}) = - \frac{mq_0 (|q_y| + iq_x) e^{-|q_y|\xi}}{2(q+q_0)|q_y|}. \quad (\text{B5})$$

The Fourier transform of Eq. (B5) yields

$$\rho_{\text{ind}}(\mathbf{r}) = -\theta(-x) \frac{m}{4\pi q_0} \mathcal{R}(r^2 - \xi^2 + 2i\xi y)^{-3/2}, \quad (\text{B6})$$

where  $\mathcal{R}$  denotes the real part and  $r^2 = x^2 + y^2$ . Note that this charge distribution decays algebraically towards the interior of the plasma, indicating its anomalously poor screening properties. The electrostatic energy associated with the interaction of the impurity and the induced charge is then found to be (to leading order in small  $\xi$ )

$$E_{\text{el}} \approx - \frac{m^2 \Lambda}{4q_0} \ln \frac{\Lambda}{q_c}. \quad (\text{B7})$$

The higher-order corrections decrease as a function of  $\xi$ . Multiplying by the inverse temperature  $\beta = 2/m$ , and with the appropriate definition of the ultraviolet cutoff  $\Lambda$ , this result coincides with Eq. (43), and hence is consistent with our diagrammatic approach.

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