Non-perturbative approach to the quantum Hall bilayer

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We find in a systematic way universality classes of homogeneous ground state wave functions that describe superfluid disordering and possible phases in the quantum Hall bilayer at filling factor one. New quasiparticles with vorticity that emerge in this description are neutral fermions that constitute highly correlated states in the superfluid phase characteristic for smaller distances between layers. They emerge as unbound and free at a certain distance between layers in a quantum phase transition that resembles Berezinskii-Kosterlitz-Thouless (BKT) unbinding at T = 0. Their unbinding can also occur through finite temperature phase transition into the same incoherent phase as found in [A.R. Champagne et al., arXiv:0709.0718] This neutral fermion physics is stabilized by impurities in experiments which bind charged merons - the elementary vorticity quasiparticles of a translatory invariant system. In a translatory invariant system superfluid disordering via meron-antimeron loop condensation leads to a topological phase associated with the toric code model.

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Introduction The quantum Hall bilayer (QHB) at $\nu = 1$ consists of two layers of two two-dimensional electron gases that are brought close to one another in the quantum Hall regime of strong magnetic fields. When the distance between the layers is much smaller than the average distance between electrons inside each layer inter and intra Coulomb interactions are about the same. Then the expected $\nu = 1$ state is the state of a single layer filled lowest Landau level (LLL) generalized to two species. There is obvious degeneracy in dividing electrons into two groups which leads to the phenomenon of spontaneous symmetry breaking [1] and the existence of a Goldstone mode [2]. The expected superfluid behavior was verified also by very large zero bias voltage peak in tunneling conductance [3], but no clear evidence was found for finite temperature BKT transition [4] in transport experiments [5].

Therefore there is a need to systematically address the question of superfluid disordering in the QHB. In particular there is a need to understand the role of quantum disordering in this system that becomes important as the distance between the layers is increased. In the most of the previous work the starting point for the discussion of the physics of the bilayer was the ground state (GS) for the very small distance between the layers as a mean field solution to which none or some corrections were developed [4, 6]. We will take a non-perturbative approach inspired by the Laughlin solution of the $\nu = 1/3$ problem in which we will uniquely determine possible wave functions (WFs) for the GSs of the bilayer at an arbitrary distance.

There are two basic paradigms of superfluid disordering that we know: (1) BKT (2D XY model) for which the transition proceeds via unbinding of dipoles of vortex-antivortex pairs, and (2) λ transition type (3D XY model) for which the transition is characterized by a condensation of vortex-antivortex loops. In what follows through the WF analysis we will show that these two models correspond to two possible kinds of superfluid disordering in the QHB (1) one in the presence of impurities and (2) the other for translatory invariant system, respectively. The analysis of corresponding WFs will show the importance of neutral fermions. Each neutral fermion is a composite of two same vorticity but opposite charge elementary vortex quasiparticles - merons [4]. In the presence of impurities merons are locked, and associated transitions proceed via unbinding of pairs of opposite vorticity neutral fermions. The recently found finite temperature transition [7] and the quantum phase transition with respect to changing distance [7, 8] correspond to this unbinding. On the other hand, the analysis will show that in a translatory invariant system meron excitations via their loop condensation will produce an incompressible liquid state for the neutral sector. Its low-energy excitations can be described by a BF Chern-Simons theory [9], the same theory that desribes the excitations of the toric code model.

Universality classes of ground states A great deal is known from the experimental and theoretical point of view of the QHB in the two extremes when the distance between layers, d, is (1) much smaller or (2) much larger than the magnetic length, $l_B = (\hbar/eB)^{1/2}$, B - the magnetic field, the characteristic distance between the electrons inside any of the layers. When $d << l_B$, i.e. inter and intra Coulomb interactions are about the same, the good starting point and description is so-called (111) state [10],

$$\Psi_{111}(z_\uparrow, z_\downarrow) = \prod_{i<j}(z_{i\uparrow} - z_{j\uparrow}) \prod_{k<l}(z_{k\downarrow} - z_{l\downarrow}) \prod_{p,q}(z_{p\uparrow} - z_{q\downarrow}) \tag{1}$$

where $z_{i\uparrow}$ and $z_{i\downarrow}$ are two-dimensional complex coordinates of electrons in upper and lower layer respectively and we omitted the Gaussian factors. This is suggestive of the exciton binding [11]; any electron coordinate is also zero of the WF for any other electron coordinate - the

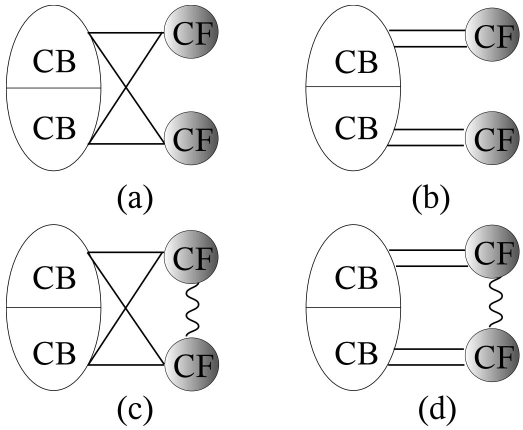

FIG. 1: Possible universality classes of wave functions

correlation hole is just opposite to electron. This exciton description can be a viewpoint of the phenomenon of superfluidity found in these systems [2, 3] and is closely connected to the concept of composite bosons (CBs) [12] that can be used as natural quantum Hall quasiparticles in this system. When $d >> l_B$ we have the case of the decoupled layers and the GS is a product of single layer filling factor 1/2 WFs; each describes a Fermi-liquid-like state [13],

$$\Psi_{1/2}(w) = \mathcal{P}\{\mathcal{F}_s(w,\overline{w}) \prod_{i<j}(w_{i\uparrow} - w_{j\uparrow})^2\} \tag{2}$$

where $\mathcal{F}_s$ is the Slater determinant of free waves of noninteracting particles in zero magnetic field and $\mathcal{P}$ represents projection to LLL. Underlying quasiparticles are composite fermions (CFs), the usual quasiparticles of the single layer quantum Hall physics.

To answer the question of intermediate distances we may try to, classically speaking, divide electrons into two groups, one in which electrons correlate as CBs and the other as CFs [14]. (The ratio between the numbers of CBs and CFs would be determined by the distance between layers.) The WF constructed in this way would need an overall antisymmetrization in the end, but also intercorrelations among the groups as each electron of the system sees the same number of flux quanta through the system (equal to the number of electrons). This requires that the highest power of any electron coordinate is the same as the number of electrons in the thermodynamic limit. If we denote by a line the Laughlin-Jastrow factor $\prod_{A,B}(z_A - z_B)$ between two groups of electrons, A and B ($A, B = CB, CF$), the possibilities for the QHB GSWFs can be summarized as in Fig. 1. If we ignore the possibility of pairing between CFs denoted by wriggly lines in Fig.1(c,d) we have two basic families of the GSWFs depicted in Fig.1(a,b). The intercorrelations in the first family in Fig.1(a) are in the spirit of Ψ_{111} correlations, and those in the second family in Fig.1(b) are in the spirit of the decoupled state, $\Psi_{1/2} \times \Psi_{1/2}$, where we correlate exclusively inside each layer. We can imagine a mixture of both intercorrelations in a single wave function but these mixed states, by their basic response [15], fall into one of the universality classes depicted in Fig. 1. Phenomenological (that ignore the overall antisymmetrization) Chern-Simons field theories can be easily constructed for the wave functions in Fig.1 and their basic response extracted [15]. We get that the states in Fig.1(a) and Fig.1(c) are superfluids, and the states in Fig.1(b) and (d) are disordered superfluids, compressible and incompressible, respectively.

The two basic possibilities of connecting two extremes as depicted in Fig. 1, i.e. without and with pairing of CFs, must correspond to the two possible ways or paradigms that we know of disordering a superfluid. We will substantiate this claim further by examining the two superfluid constructions (Fig. 1(a) and (c)) in more detail. Also we will find that the form of the long distance pairing among CFs is fixed.

Neutral fermions and BKT disordering Let's write out the unprojected in the LLL version of the construction in Fig. 1(a):

$$\begin{aligned}\Psi_1 = \mathcal{A}\ \{\ & \Psi_{111}(z_\uparrow, z_\downarrow)\Psi_{1/2}(w_\uparrow)\Psi_{1/2}(w_\downarrow) \\ & \times \prod_{i,j}(z_{i\uparrow} - w_{j\uparrow}) \prod_{k,l}(z_{k\uparrow} - w_{l\downarrow}) \\ & \times \prod_{p,q}(z_{i\downarrow} - w_{q\uparrow}) \prod_{m,n}(z_{m\downarrow} - w_{n\downarrow})\},\end{aligned} \tag{3}$$

where z_σ's and w_σ's denote coordinates of electrons belonging to the layer with index σ and $\mathcal{A}$ stands for an overall antisymmetrization. By using the expressions for the densities of electrons in each layer, $\rho^\sigma(\eta) = \sum_i \delta^2(\eta - z_i^\sigma)$, here z_σ's denote all electrons of the layer σ, we can rewrite the wave function in the following way,

$$\begin{aligned}\Psi_1 = \int d^2\eta_{1\uparrow} \cdots \int d^2\eta_{n\downarrow} \quad & \frac{\prod_{k<l}(\eta_{k\uparrow} - \eta_{l\uparrow}) \prod_{p<q}(\eta_{p\downarrow} - \eta_{q\downarrow})}{\prod_{i,j}(\eta_{i\uparrow} - \eta_{j\downarrow})} \\ & \mathcal{F}_s(\eta_\uparrow) \times \mathcal{F}_s(\eta_\downarrow) \times \\ & \rho^\uparrow(\eta_{1\uparrow}) \cdots \rho^\downarrow(\eta_{n\downarrow})\Psi_{111}(z_\uparrow, z_\downarrow),\end{aligned} \tag{4}$$

where n is the total number of electrons that correlate as CFs. The expression in Eq.(4) reminds us of a dual picture in terms of some quasiparticles with η coordinates as in [16]. To find those quasiparticles we will rewrite Eq.(4) as

$$\begin{aligned}\Psi_1 = \int d^2\eta_{1\uparrow} \cdots \int d^2\eta_{n\downarrow} \quad & \frac{\prod_{k<l}|\eta_{k\uparrow} - \eta_{l\uparrow}| \prod_{p<q}|\eta_{p\downarrow} - \eta_{q\downarrow}|}{\prod_{i,j}|\eta_{i\uparrow} - \eta_{j\downarrow}|} \\ & \mathcal{F}_s(\eta_\uparrow) \times \mathcal{F}_s(\eta_\downarrow) \times \\ & \{\exp\{i\phi(\eta_{1\uparrow} \cdots \eta_{n\downarrow})\} \times \\ & \rho^\uparrow(\eta_{1\uparrow}) \cdots \rho^\downarrow(\eta_{n\downarrow})\Psi_{111}(z_\uparrow, z_\downarrow)\}\end{aligned} \tag{5}$$

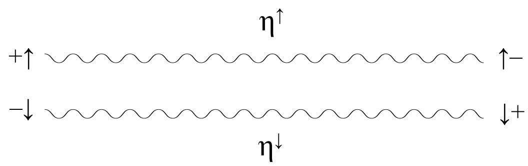

FIG. 2: The quadrupolar configurations of merons that make neutral fermion pair

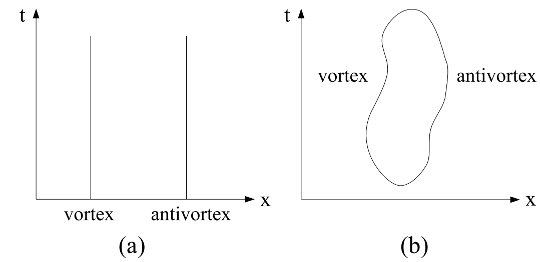

FIG. 3: The time evolution of a vortex-antivortex pair (a) without and (b) with quantum fluctuations

where $\exp\{i\phi(\eta)\}$ factor denotes the phase part of the Laughlin-Jastrow factors in front of the Fermi seas in Eq.(4). If we take and define that the phase factor always vanishes when any of two η's from the same layer coincide, we can prove, using plasma properties of the charge sector of Ψ_{111} [16, 17], that the expression in the curly brackets,

$$|\eta_{1\uparrow}\cdots\eta_{n\downarrow}>= \exp\{i\phi(\eta_{1\uparrow}\cdots\eta_{n\downarrow})\}\rho^{\uparrow}(\eta_{1\uparrow})\cdots\rho^{\downarrow}(\eta_{n\downarrow})|\Psi_{111}> \quad (6)$$

makes a Fock basis in the coordinate space for some neutral fermion quasiparticles. (Neutral because in the construction of the state there is no net magnetic flux through the system.) More precisely,

$$< \eta_{1\uparrow}^{'}, \eta_{2\uparrow}^{'}\cdots\eta_{n\downarrow}^{'}|\eta_{1\uparrow}, \eta_{2\uparrow}\cdots\eta_{n\downarrow}> \delta^2(\eta_{1\uparrow}^{'}-\eta_{1\uparrow})\delta^2(\eta_{2\uparrow}^{'}-\eta_{2\uparrow})\cdots\delta^2(\eta_{n\downarrow}^{'}-\eta_{n\downarrow}) - \delta^2(\eta_{1\uparrow}^{'}-\eta_{2\uparrow})\delta^2(\eta_{2\uparrow}^{'}-\eta_{1\uparrow})\cdots\delta^2(\eta_{n\downarrow}^{'}-\eta_{n\downarrow}) + \cdots \quad (7)$$

On the other hand, merons are true elementary vorticity quasiparticles of the translatory invariant QHB system at least for small distances between layers as shown in Ref. [4] and carry both charge and vorticity. Therefore the neutral fermion basis can not be a complete basis for neutral excitations of the QHB in the translatory invariant case because the neutral fermions carry layer index and can not describe, for example, meron dipole configurations with no net vorticity and no layer denomination.

Just by looking at Eq.(5) we can read out the GSWF in the dual picture in terms of neutral fermions,

$$\Psi_{dual}(\eta) = \frac{\prod_{k<l}|\eta_{k\uparrow}-\eta_{l\uparrow}|\prod_{p<q}|\eta_{p\downarrow}-\eta_{q\downarrow}|}{\prod_{i,j}|\eta_{i\uparrow}-\eta_{j\downarrow}|}\mathcal{F}_s(\eta_\uparrow)\mathcal{F}_s(\eta_\downarrow) \quad (8)$$

This is a wave function of 2D Coulomb fermionic plasma [18] and it describes the superfluid state in Fig 1(a). It encodes dipole positioning of opposite vorticity (layer index) neutral fermions. In the superfluid phase, with respect to merons, a neutral fermion dipole should be in essence a superposition of quadrupolar combinations of merons - two dipoles which come in pairs but at arbitrary distance as illustrated in Fig. 2. In this way, as dipoles, neutral fermions constitute the lowest lying states of the QHB - (pseudo)spin waves [2, 12]. If neutral fermions may be considered as eigenstates they must lie very high in spectrum; like electrons in fractional quantum Hall states they constitute the physics of Ψ_1 but their wave function Eq.(8) describes a highly correlated state. It is our conjecture that the functions, Eq.(8), make a complete set for the description of the superfluid ground state evolution in the presence of impurities in experiments and should constitute a basis for explanation of quantum and finite temperature phase transitions [7] through a BKT neutral fermion unbinding.

Quantum fluctuations and loop condensation The two paradigms - models of superfluid disordering as applied to our 2+1 dimensional system mean that the time evolution is such that (1) meron -antimeron pairs are locked as in Fig. 3(a) or (2) created and annihilated at some later time and therefore making a loop as in Fig. 3 (b). The loops in time signify the presence of quantum fluctuations. The mean field approach to a 2D or 3D superfluid entails a spin wave contribution to the GSWF, in our case $\exp\{c\sum_k \rho_k^{\uparrow}\rho_{-k}^{\downarrow}\}\Psi_{111} = \Psi_{111}$ i.e. a trivial contribution. We find this by simple application of the definitions of the densities:$\rho^\sigma(x) = \sum_i \delta^2(x - z_i^\sigma)$ (no projection to the LLL implied). On the other hand, the Chern-Simons (CS) field theory approach based on the mean field Ψ_{111} solution [6, 19] in the RPA approximation of treating quantum corrections finds $\exp\{\sum_k \frac{f(d)}{k}\rho_k^{\uparrow}\rho_{-k}^{\downarrow}\}\Psi_{111}$, where $f(d)$ is a positive function of d (distance between the layers). In order to find out the spin wave part in our WFs we first take the $1 + 1$ neutral fermion construction as in Eq.(4):

$$\int d^2\eta_{1\uparrow}\int d^2\eta_{2\downarrow}\frac{1}{(\eta_{1\uparrow}-\eta_{2\downarrow})}\rho^{\uparrow}(\eta_{1\uparrow})\rho^{\downarrow}(\eta_{2\downarrow}) \quad (9)$$

This can not belong to the spin wave contribution because it is antisymmetric under ↑↓ exchange, and our conclusion must be that in the neutral fermion constructions (Fig.1(a)) the spin wave contribution is trivial - a mean field one as introduced above. In the case of the pairing constructions (Fig.1(c)) the $1 + 1$ CF part can be

$$\int d^2\eta_{1\uparrow}\int d^2\eta_{2\downarrow}\frac{1}{|\eta_{1\uparrow}-\eta_{2\downarrow}|^2}\rho^{\uparrow}(\eta_{1\uparrow})\rho^{\downarrow}(\eta_{2\downarrow}) \quad (10)$$

where we took $g(z) = \frac{1}{z^*}$ for the pairing function and the expression is symmetric under $\uparrow\downarrow$ exchange. This in the long-distance limit reduces to $\{\sum(-)\ln(kl_B)\rho_k^{\uparrow}\rho_{-k}^{\downarrow}\}\Psi_{111}$ with l_B as the small distance cutoff. Extracting the spin wave contributions from other $n+n$ neutral fermion constructions [17] we can get an overall contribution of a form, $\exp\{\sum_k \tilde{f}(d)(-)\ln(kl_B)\rho_k^{\uparrow}\rho_{-k}^{\downarrow}\}\Psi_{111}$. So indeed we get corrections from quantum fluctuations like in [6, 19] although a slightly different expression [21]. Therefore, from this analysis, we can conclude that a perturbative expansion in the $n+n$ CF constructions of Fig.1(c) with $g(z) = \frac{1}{z^*}$ is well justified and parallels previous approaches in a translatory invariant system [6, 19] that are based on corrections to Ψ_{111} state.

There is also another justification for the choice $g(z) = \frac{1}{z^*}$ for our WF. While calculating the spin wave contribution [17] using the $n+n$ neutral fermion constructions, it was necessary to use the following identity,

$$\mathcal{D}et\{\frac{1}{\eta_{i\uparrow}-\eta_{j\downarrow}}\} \times \mathcal{D}et\{\frac{1}{\eta_{i\uparrow}^*-\eta_{j\downarrow}^*}\} = \frac{\prod_{k<l}|\eta_{k\uparrow}-\eta_{l\uparrow}|^2\prod_{p<q}|\eta_{p\downarrow}-\eta_{q\downarrow}|^2}{\prod_{i,j}|\eta_{i\uparrow}-\eta_{j\downarrow}|^2}, \qquad (11)$$

which follows from the bosonization theory [22] in which two Majorana field correlators in the holomorphic and antiholomorfic sector on the l.h.s. are equal to the correlator on the r.h.s. of bosonic vertex operators. If we leave the dual picture and examine the final form of the state of Fig.1(d) when there are no CBs, we are lead to its following forms,

$$\begin{aligned}\Psi_2 &= \mathcal{D}et\{\frac{1}{z_{i\uparrow}^*-z_{j\downarrow}^*}\}\prod_{i<j}(z_{i\uparrow}-z_{j\uparrow})^2\prod_{k<l}(z_{k\downarrow}-z_{l\downarrow})^2 \\ &= \mathcal{D}et\{\frac{1}{z_{i\uparrow}^*-z_{j\downarrow}^*}\}\mathcal{D}et\{\frac{1}{z_{k\uparrow}-z_{l\downarrow}}\}\Psi_{111}, \qquad (12)\end{aligned}$$

where to get the last line we used the Cauchy determinant identity. The neutral part of Ψ_2 (not carrying a net flux through the system as Ψ_{111} does) that consists of the two determinants is, as we have written in Eq.(11) above, nothing but a correlator of a single (nonchiral) bosonic CFT. According to [23] CFT theory correlators not only describe quantum Hall system WFs but also can be used to find out about and connect to its edge and bulk theories. The bulk theory in this case, for the neutral part, is the P, T invariant BF Chern-Simons theory [9, 17] and its edge theory is associated with the bosonic CFT. So at the end of the QHB evolution with distance in a clean system we may have a topological phase. The superfluid disordering via meron-antimeron loop condensation can produce such a phase whose ground state on the other hand can be viewed as a condensate of loops in space and time.

The complete bulk theory when we consider also the charge degrees of freedom contains also $U(1)_1$ CS theory. The degeneracy of the system GSs on the torus must be 4 [20, 24]. Therefore sufficiently clean QHB systems may be used as generators for states described by doubled CS field theories [24] as the Abelian BF theory - a non-trivial example. The states are generated through superfluid disordering via loop condensation. They are important because their non-Abelian varieties may be used for universal topological quantum computing [25].

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