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2002 J. Opt. B: Quantum Semiclass. Opt. 4 S358

(http://iopscience.iop.org/1464-4266/4/4/320)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS
J. Opt. B: Quantum Semiclass. Opt. **4** (2002) S358–S365 PII: S1464-4266(02)37881-9

Is the classical law of the addition of probabilities violated in quantum interference?

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Received 4 December 2001
Published 29 July 2002
Online at stacks.iop.org/JOptB/4/S358

Abstract
We analyse and compare the positive and negative arguments on whether quantum interference violates the classical law of the addition of probabilities. The analysis takes into account the results of recent interference experiments in neutron, electron and atom optics. Nonclassical behaviour of atoms was found in atomic experiments where the measurements included their time of arrival and space distribution. We determine probabilities of elementary events associated with the nonclassical behaviour of particles in interferometers. We show that the emergence of the interference pattern in the process of accumulation of such elementary events is consistent with the classical law of the addition of probabilities.

Keywords: Quantum interference, transverse momentum distribution, probability laws, atom optics, neutron optics, compatibility versus complementarity

1. Introduction

In the article *Quantum theory and the foundations of probability* Koopman [1] wrote:

> ‘Ever since the advent of modern quantum mechanics in the late 1920s, the idea has been prevalent that the classical laws of probability cease, in some sense, to be valid in the new theory. More or less explicit statements to this effect have been made in large number and by many of the most eminent workers in the new physics [1a]. Some authors have even gone further and stated that the formal structure of logic must be altered to conform to the terms of reference of quantum physics [1b].
>
> Such a thesis is surprising, to say at least, to anyone holding more or less conventional views regarding the position of logic, probability and experimental science: many of us have been apt—perhaps too naively—to assume that experiments can lead to conclusions only when worked up by means of logic and probability, whose laws seem to be on a different level from those of physical science.’

The primary object of Koopman’s paper is to show that (1) the thesis in question is entirely without validity and is the product of a confused view of the laws of probability, and (2) the situation can be straightened out at a very elementary level. All that is needed is to make clear and explicit the concept of *event*.

In section 2 we define events, taking as the starting point Koopman’s argumentation and conclusions, and in section 3 their probabilities which are appropriate for an interference phenomena. In section 4 we add the wavefunction of the transverse motion in the coordinate and the momentum representation, and in section 5 we show that the interference phenomena is a consequence of the fact that the particle behaviour behind the grating depends on the grating’s parameters. In section 6 we determine the probabilities of events associated with this behaviour and show that classical laws of probabilities are not necessarily violated in quantum interference. The conclusion is given in section 7.

2. Koopman's analysis of the standard arguments purporting to show that quantum physics is inconsistent with classical probability

The standard argument starts with the Young interference experiment where light emitted from the source S and collimated by passing through the slit C can reach any point x on a screen by passing through one or the other or both of the slits A, B. When A or B is closed, there are no interference fringes on the screen. However, such fringes are present when they are both open, meaning that the intensity of light at a point x is not the sum of intensities passing through A and B.

Quantum physics accounts for interference in terms of statistics of photons. When a corpuscule of light leaves C, it may reach x in any of the three cases: (i) with A open and B closed; (ii) with A closed and B open; (iii) with both open. And the intensity of light at x in each of these situations is proportional to the corresponding probability of the photon's reaching x. If the probabilities are denoted by p_A, p_B, $p_{A,B}$, respectively, we obviously do not have $p_{A,B} = p_A + p_B$, because of the interference. This is an alleged experimental violation of the law of total probability, according to the following reasoning: 'the event of the photon's reaching x when A and B are both open must occur in one of the following mutually exclusive ways: either by its passing through A (probability p_A) or through B (probability p_B); therefore probability should be $p_A + p_B$ and not the observed value $p_{A,B}$.'

The fallacy contained in the above argument, explains Koopman [1], lies in the fact that p_A is not the probability of the photon's reaching x passing through A. 'The only correct characterization of p_A in the framework of quantum mechanics is as the probability of photon's reaching x 'by the agency of A alone being open'. Similarly $p_{A,B}$ must be described as the probability of reaching x by the agency of both A and B being open. Thus p_A and p_B are not the probabilities of two different outcomes of the same trial but of the same outcomes of different types of trials: the principle of total probability does not even enter.'

3. Appropriate probabilities for interference phenomena

In fact, Koopman's argumentation leads to the main theoretical problem of quantum interference, that is the determination of the following probabilities, introduced by Selleri and Tarrozi [2]: P_A—probability of a quantum particle (quanton) reaching x after passing through A by the agency of both A and B being open; P_B—the probability of a quantum particle (quanton) reaching x after passing through B by the agency of both A and B being open.

From Koopman's explanation it follows that Feynman [1, 3] implicitly identified the probabilities P_A and P_B with the probabilities p_A and p_B, respectively: $P_A = p_A$; $P_B = p_B$. From this identification, which has not been supported by experiment, the conclusion about the violation of classical probability laws in quantum interference has been derived.

Implicitly, Bohr's discussion with Einstein [4] was about the relation between these probabilities. Apparently, Bohr had difficulty accepting the validity of the inequalities

$$P_A \neq p_A, \qquad P_B \neq p_B. \tag{1}$$

In order to avoid accepting the latter inequalities, Bohr denied the existence of probabilities P_A and P_B, by denying the existence of the particle's paths. The latter conclusion we draw from Bohr's analysis of the two-slit experiment [4]:

> 'A closer examination showed, however, that the suggested control of the momentum transfer would involve latitude in the knowledge of the position of the diaphragm, which would exclude the appearance of the interference phenomena in question....
>
> This point is of great logical consequence, since it is only the circumstance that we are presented with a choice of *either* tracing the path of a particle or observing interference effects, which allows us to escape from the paradoxical necessity of concluding that the behaviour of an electron or a photon should depend on the presence of a slit in the diaphragm through which it could be proved not to pass.'

Therefore, for Bohr the dependence of the particle's behaviour on its environment (boundary conditions on the grating) was paradoxical. In order to avoid this 'paradox' Bohr formulated the principle of complementarity.

In the present time we are forced to reconsider this conclusion of Bohr's, taking into account the results of quantum interference experiments performed with neutrons (reviewed by Rauch and Werner [5]), with electrons [6], with photons [7], with atoms [8, 9], and with large molecules [10]. In our opinion, the results of those experiments force us to conclude that the behaviour of a photon, neutron, electron, atom, molecule behind the grating depends on the number of open slits, their widths and mutual distances. Consequently, the behaviour of an electron or a photon DOES DEPEND on the presence of a slit in the diaphragm through which it had not passed.

4. The wavefunction of the transverse motion in the coordinate and in the momentum representations

A particle's behaviour and motion behind the grating is determined by its wavefunction, which is a solution of the Schrödinger equation. If the source is far from the grating the solution in front of the grating is a plane wave with the initial momentum $p = \hbar k$ along the longitudinal direction y. Behind the grating the solution was most often written in the Fresnel–Kirchhoff form [9–11]. By invoking the approximation which is equivalent to the paraxial approximation in optics, Tomonaga [12] wrote the solution in the form of a product of the longitudinal and transverse parts, the former being a plane wave. The transverse part was written in the form of a superposition of Gaussians (which spread in time) by Tomonaga [12], Zurek [13], Bonifacio and Olivares [14], and others. Božić *et al* [15] showed that the following form of the solution:

$$\Psi(x, y, t) = \frac{1}{\sqrt{2\pi}} e^{-i\omega t} e^{iky} \int_{-\infty}^{+\infty} dk_x\, c(k_x) e^{ik_x x} e^{-ik_x^2 y/2k}, \qquad y \geqslant 0 \tag{2}$$

is equivalent to the Fresnel–Kirchhoff form. The function $c(k_x)$ in equation (2) is determined by the boundary values of the wavefunction $\Psi(x, y, t)$ at $y = 0$ and $t = 0$,

$$c(k_x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}x\, \Psi(x, 0, 0)\mathrm{e}^{-\mathrm{i}k_x x}. \tag{3}$$

It is assumed that $c(k_x)$ tends to zero when $p_x/p = k_x/k$ is not much smaller than one. This assumption is equivalent to the paraxial approximation in optics.

The solution is the product of the plane wave propagating along the y-axis, and of an integral which depends on x and y. Now we invoke wave–particle duality, as understood by de Broglie [16], and start to consider a particle surrounded by the wave described by the wavefunction (2). Its form suggests that a particle arriving at the slits continues to move with longitudinal momentum $p = mv$ along the y-axis. But, there is a probability density $c(k_x)$ that the particle acquires a value $p_x = \hbar k_x$ of the transverse momentum. So, its motion (evolution) along the x-axis is nonstationary and it is described by a nonstationary solution of the Schrödinger equation. From equation (2) one derives the wavefunction of the transverse motion by substituting the value $y = vt$ of the particle's y-coordinate at time t in the integral. So,

$$\begin{aligned}\psi_{tr}(x, t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}k_x\, c(k_x)\mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{-\mathrm{i}k_x^2 y/2k} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}k_x\, c(k_x)\mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{-\mathrm{i}\omega_x t}, \qquad y = vt \geqslant 0\end{aligned} \tag{4}$$

where $\omega_x \equiv p_x^2/2m\hbar$ and

$$c(k_x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}x\, \psi_{tr}(x, 0)\mathrm{e}^{-\mathrm{i}k_x x}. \tag{5}$$

The function $c(k_x)$ is proportional to the probability amplitude of the particle's transverse momentum $c'(p_x)$.

By substituting the expression (5) into (4) one derives

$$\begin{aligned}&\psi_{tr}(x, t = ym/\hbar k) \\ &= \frac{\sqrt{k}}{\sqrt{2\pi y}}\mathrm{e}^{-\mathrm{i}\pi/4} \int_{-\infty}^{+\infty} \psi_{tr}(x', 0)\mathrm{e}^{\mathrm{i}k(x-x')^2/2y}\, \mathrm{d}x'.\end{aligned} \tag{6}$$

The latter expression of $\psi_{tr}(x, t)$ is particularly useful when $\psi_{tr}(x, 0)$ consists of pieces where it takes zero value, like the example shown in figure 1. This is just the boundary condition corresponding to the n-slit grating.

For large values of y, equation (6) is approximated by

$$\begin{aligned}&\psi_{tr}(x, t = ym/\hbar k) \\ &= \frac{\sqrt{k}}{\sqrt{2\pi y}}\mathrm{e}^{-\mathrm{i}\pi/4}\mathrm{e}^{\mathrm{i}kx^2/2y} \int_{-\infty}^{+\infty} \mathrm{d}x' \psi_{tr}(x', 0)\mathrm{e}^{-\mathrm{i}kxx'/y}.\end{aligned} \tag{7}$$

By comparing (5) and (7) one finds

$$\psi_{tr}(x, t = ym/\hbar k) = \frac{\sqrt{k}}{\sqrt{y}}\mathrm{e}^{-\mathrm{i}\pi/4}\mathrm{e}^{\mathrm{i}kx^2/2y}c(kx/y). \tag{8}$$

Far from the slits, the transverse wavefunction in the coordinate representation is proportional to the wavefunction of the transverse motion in the momentum representation, where kx/y plays the role of k_x. This is an indirect proof of the equivalence of two forms of the solution of Schrödinger's equation; the form presented by equation (2) and the Fresnel–Kirchhoff form. The latter form leads (up to a constant) to the relation (8), as shown in [17].

For the boundary conditions of figure 1 it is easy to perform the integration in equation (5). The resulting expressions for the one-slit (centred at $x_c = z$) function $c_1(k_x)$ and n-slit function $c_n(k_x)$ are

$$c_1^z(k_x) = \frac{\sqrt{2}}{\sqrt{\pi\delta}} \frac{\sin\frac{k_x\delta}{2}}{k_x}\mathrm{e}^{-\mathrm{i}k_x z} \tag{9}$$

$$\begin{aligned}c_n(k_x) &= \frac{\sqrt{2}}{\sqrt{\pi\delta}} \frac{\sin\frac{k_x\delta}{2}}{k_x} \sum_{j=1}^{n} \mathrm{e}^{-\mathrm{i}k_x z_j} \\ &= \frac{\sqrt{2}}{\sqrt{\pi n\delta}} \frac{\sin\frac{k_x\delta}{2}}{k_x} \frac{\sin(\frac{k_x dn}{2})}{\sin\frac{k_x d}{2}}, \qquad n > 1.\end{aligned} \tag{10}$$

The one-slit function is of particular interest since the n-slit function is the sum of one-slit functions. The one-slit function is a product of a real factor, which depends on the slit width, by the complex factor, which depends on the centre of the slit coordinate. The n-slit function $c_n(k_x)$ contains the former factor too. Its second factor, resulting from the summation of phase factors of all slits, depends on the slit separation d.

It is important to note that the function $c_n(k_x)$ is independent of the initial longitudinal momentum and of the coordinate y. Its modulus square, $|c_n(k_x)|^2$ is proportional to the probability density of the particle transverse momentum $p_x = \hbar k_x$. It is graphically represented in figure 2 for the gratings with $n = 1, 2, 4, 6$ and 8 slits. The characteristic quantity, the ratio of slit width to slit separation, is $\delta/d = 0.5$.

5. The interference patterns as a consequence of the dependence of the particle's behaviour behind the grating on the grating's parameters

The independence of $|c_n(k_x)|^2$ on the distance from the grating means that the particle's transverse momentum distribution is determined only by the width of the slits and their mutual distances. It is the same for all values of the initial longitudinal momentum, as far as the values of the transverse momenta are much smaller than the initial longitudinal momentum. From this fact we conclude that the particle's behaviour is determined by all slits illuminated by the initial particle wave, not only by the one through which the particle has passed.

It seems that this influence can be attributed to the wave which accompanies a particle and which evolves according to equation (2). The transverse motion of the particle is determined by the transverse momentum it acquired in passing through a slit and by the evolution of its wave. Its longitudinal motion is classical and satisfies the relation $y = (p/m)t$. The modulus square of the transverse wavefunction, presented in figure 3 ($n = 2$) and figure 4 ($n = 8$), determines the distribution along the x-axis for the sets of fixed distances y.

The latter figures explain the 'stationary' interference pattern as a phenomenon built up by the process of accumulation of individual events. The particles of the same initial momentum $p = mv$ sent through the slits arrive at various points x on the screen at the distance y because they have acquired various transverse velocities passing through the slits. The probability of particle arrival at the point (x, y) at time $t = y/v$ is equal to $|\psi_{tr}(x, y = tv)|^2$. But, for the final accumulated pattern

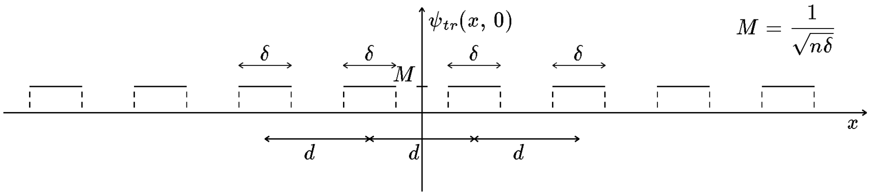

Figure 1. Boundary conditions for the wavefunction $\psi_{tr}(x, t)$ at $t = 0$ $(y = 0)$.

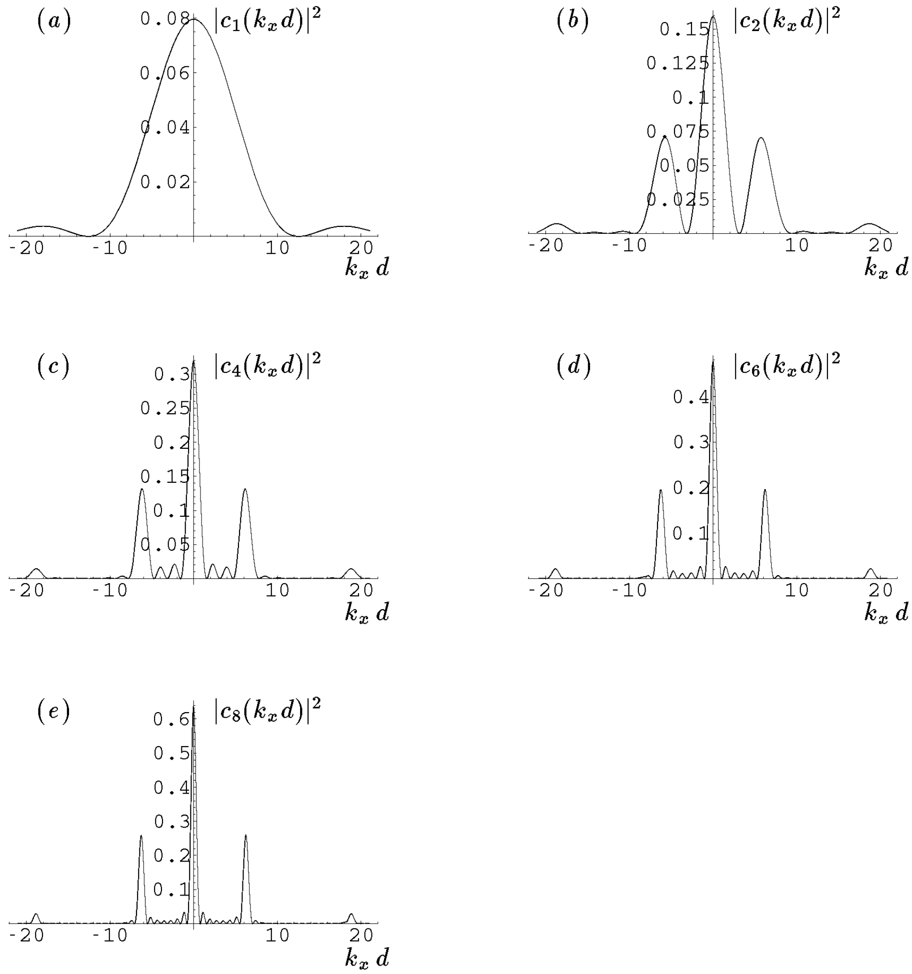

Figure 2. The particle's transverse momentum distribution $|c_n(k_x d)|^2$ behind the grating with (*a*) one slit, (*b*) two slits, (*c*) four slits, (*d*) six slits and (*e*) eight slits; $\delta/d = 0.5$.

('stationary' pattern), time of arrival is unimportant. Consequently, the distribution over x of accumulated particles, for a fixed value of y, is given by $|\psi_{tr}(x, y = t\hbar k/m)|^2$. This is why the same results are obtained by sending many particles at once with the same initial velocity.

By measuring the particle's position and time of arrival Kurtsiefer *et al* [9] measured the time dependence of the modulus square of the transverse wavefunction. The beam of atoms used in the experiment was characterized by a wide distribution of the initial longitudinal velocities. This method of measurement is based on the relation $t = y/v$ and the independence of the function $|\psi_{tr}(x, t = y/v)|^2$ from the initial velocity v. The experimental results of Kurtsiefer *et al* are in good agreement with $|\psi_{tr}(x, t)|^2$ evaluated using the Fresnel–Kirchhoff integral. Since the modulus square of the transverse wavefunction (4) is equivalent to the one evaluated from the Fresnel–Kirchhoff integral (as was shown by Božić *et al* [15]), the distributions evaluated here are in agreement with the experimental results.

By comparing the momentum distribution (figure 2) and the space distribution (figures 3 and 4) one sees that in the Fraunhofer region, the values of x corresponding to the maximum of the space distribution are consistent with the values corresponding to the maximum of the transverse momentum distribution. The following relations are valid:

$$x_{max} = (p_{x,max}/m)t = (p_{x,max}/m)(ym/p) = k_{x,max}y/k. \tag{11}$$

This property is the direct consequence of the relation (8) between the transverse momentum wavefunctions in the coordinate and in the momentum representations.

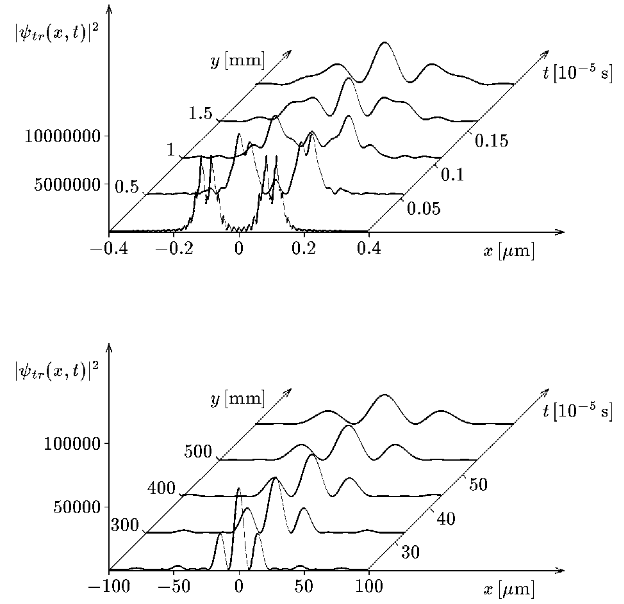

Figure 3. The function $|\psi_{tr}(x, t)|^2$ behind the $n = 2$ grating ($\delta = 0.1$ μm, $d = 0.2$ μm) close to the slits (above) and far from the slits (below). The initial longitudinal wavevector is $k = (\pi/8) \times 10^{12}$ m^{-1}, the particle mass is $m = 3.8189 \times 10^{-26}$ kg, the initial particle velocity $v = 1084$ m s^{-1}.

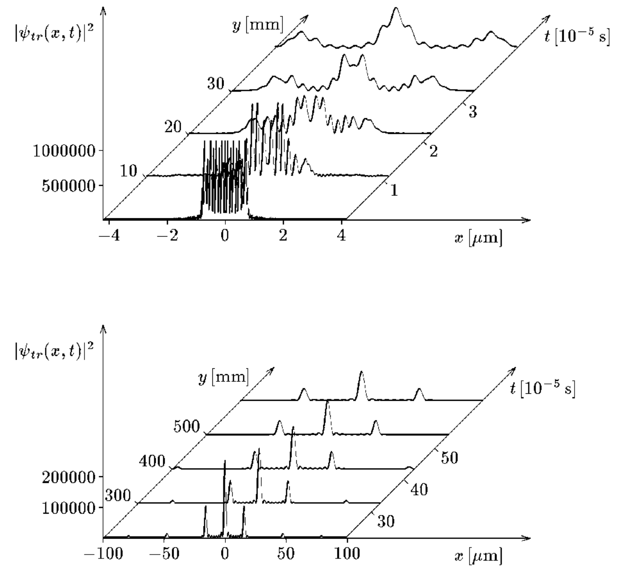

Figure 4. The function $|\psi_{tr}(x, t)|^2$ behind the $n = 8$ grating ($\delta = 0.1$ μm, $d = 0.2$ μm) close to the slits (above) and far from the slits (below). The initial longitudinal wavevector is $k = (\pi/8) \times 10^{12}$ m^{-1}, the particle mass is $m = 3.8189 \times 10^{-26}$ kg, the initial particle velocity $v = 1084$ m s^{-1}.

6. The de Broglian probabilities in the n-slit interferometer

The evident dependence of the momentum distribution on the grating parameters and the close relation between the momentum distribution and the space distribution clearly show, in our opinion, that the behaviour of a photon, neutron, electron, atom, molecule behind the grating depends on the number of open slits, their widths, and mutual distances.

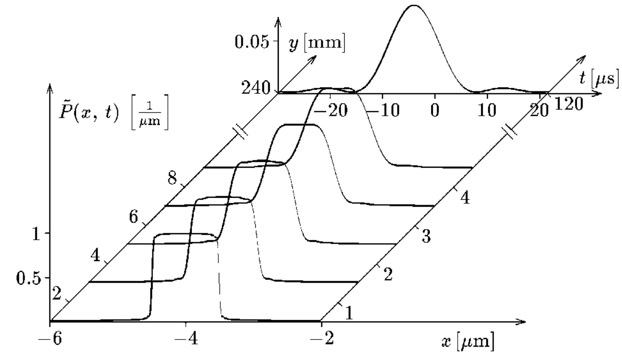

Figure 5. The probability density $\tilde{P}(x, t)$ of a particle's arrival at the point x at time $t\,(y = vt)$, behind the $n = 1$ grating ($\delta = 1$ μm), evaluated from (14) employing the following parameters: $m = 6.6432 \times 10^{-27}$ kg, $k = 4\pi \times 10^{10}$ m^{-1}, and $v = 1995.58$ m s^{-1}.

This dependence can be attributed to the wave, which is the superposition of all waves emerging from every slit illuminated by the initial particle wave. This wave surrounds the particle and propagates with it.

The probability of the particle's arrival at a certain point (x, y) at time t is $P(x, y, t) = |\Psi(x, y, t)|^2$. If the law of the addition of probabilities is not violated, this probability should be equal to the sum of probabilities $P_i(x, y, t)$, where $P_i(x, y, t)$ is the probability of the quantum particle (quanton) reaching (x, y) at time t after passing through slit i by the agency of all n slits being open. Thus,

$$P(x, y, t) = |\Psi(x, y, t)|^2 = |\psi_{tr}(x, t)|^2 = \sum_{i=1}^{n} P_i(x, y, t). \tag{12}$$

The probabilities $P_i(x, y, t)$ are called de Broglian probabilities [20].

The direct experimental determination of probabilities $P_i(x, y, t)$ would be equivalent to the so-called which-path experiment. But, experiments aimed to determine the path with certainty have resulted in the destruction of the interference pattern [18, 19]. Because of that, it was not possible to measure the probabilities $P_i(x, y, t)$ directly. Moreover, it was concluded from this fact that the interference patterns and particle trajectories were incompatible [4, 18, 19]. This is why there are no comprehensive theoretical studies of the determination of the probabilities $P_i(x, y, t)$.

In our opinion, the incompatibility of the interference and the particle trajectories does not follow from the results of which-way experiments. We consider that these results point to the necessity to determine theoretically the probabilities $P_i(x, y, t)$. The first step would be theoretical investigation of all possible classes of unknown positive definite probabilities $P_i(x, y, t)$, $i = 1, 2, \ldots, n$, defined by (12). For this purpose it is necessary to introduce additional plausible assumptions, as was done in [20–22]. The final goal is to determine which class corresponds to the real experiments and interference phenomena.

In this paper we determine a class of $P_i(x, y, t)$, taking into account the above comparison of the transverse momentum distribution with the space distribution in the Fraunhofer region. Based on the results of this comparison, we assume

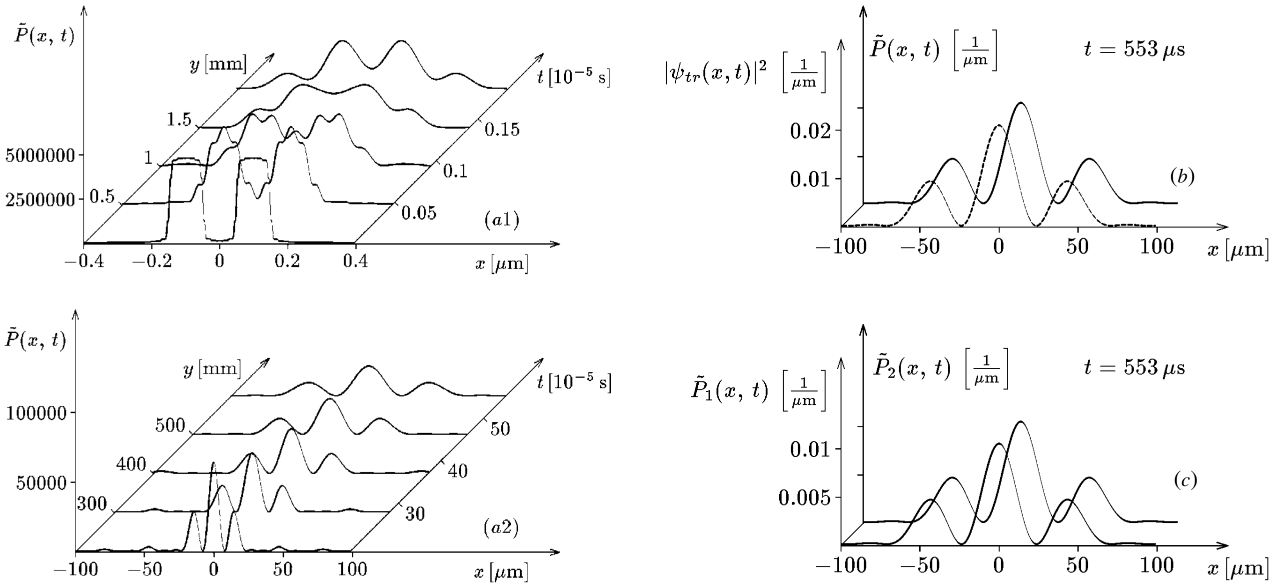

Figure 6. All graphs are calculated for the two-slit grating with $\delta = 0.1$ μm, $d = 0.2$ μm, $k = (\pi/8) \times 10^{12}$ m^{-1}, $m = 3.8189 \times 10^{-26}$ kg, $v = 1084$ m s^{-1}. (*a1*) and (*a2*) The probability density $\tilde{P}(x, t)$ of a particle's arrival at point x at time $t(y = vt)$ behind the grating evaluated from (14). (*b*) The distribution functions $|\psi_{tr}(x, t)|^2$ and $\tilde{P}(x, t)$ for one value of $t(y)$ far from the slits. (*c*) The probabilities $\tilde{P}_i(x, t)$ evaluated from (15).

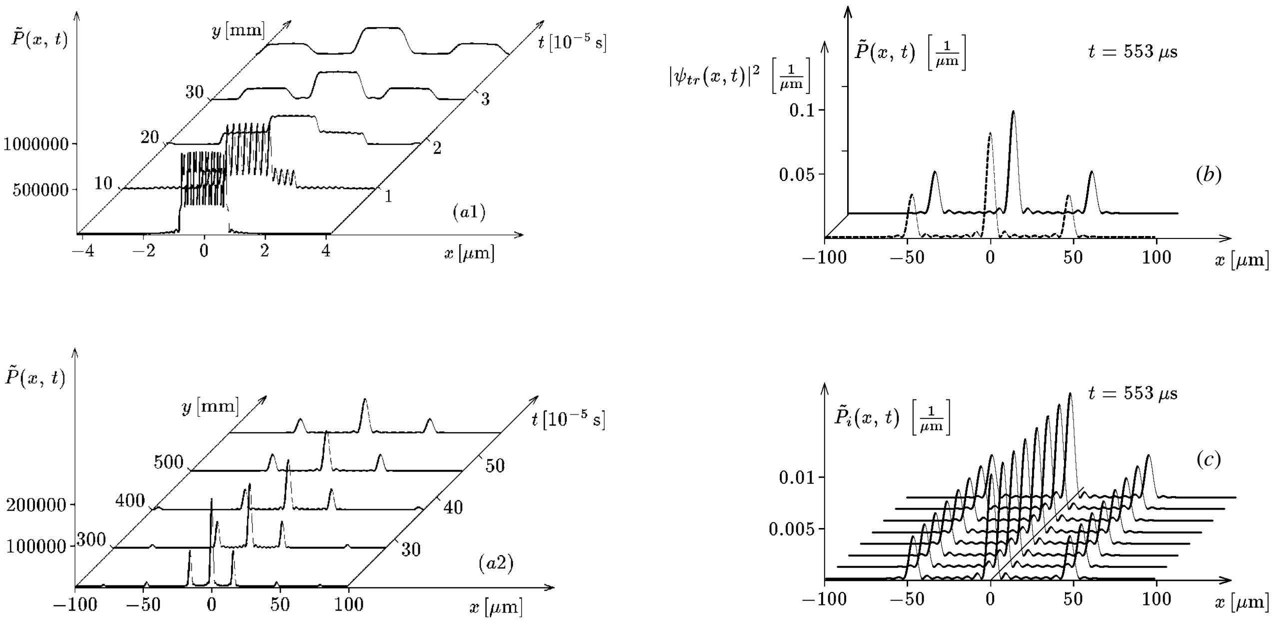

Figure 7. All graphs are calculated for an eight-slit grating with $\delta = 0.1$ μm, $d = 0.2$ μm, $k = (\pi/8) \times 10^{12}$ m^{-1}, $m = 3.8189 \times 10^{-26}$ kg, $v = 1084$ m s^{-1}. (*a1*) and (*a2*). The probability density $\tilde{P}(x, t)$ of a particle's arrival at point x at time $t(y = vt)$ behind the grating evaluated from (14). (*b*) The distribution functions $|\psi_{tr}(x, t)|^2$ and $\tilde{P}(x, t)$ for one value of $t(y)$ far from the slits. (*c*) The probabilities $\tilde{P}_i(x, t)$ evaluated from (15).

that a particle with transverse momentum $p_x = \hbar k_x$ which was at point $(x' = x - \hbar k_x t/m,\ y = 0)$ at time $t = 0$ arrives at point (x, y) at time t. Of course, we have to integrate over all possible k_x and x'. Thus, we assume that $|\Psi(x, y, t)|^2 = |\psi_{tr}(x, t)|^2$ can be approximated by the expression

$$\int_{-\infty}^{+\infty} \mathrm{d}k_x \int_{-\infty}^{+\infty} \mathrm{d}x' |c_n(k_x)|^2 |\psi_{tr}(x', 0)|^2 \delta(x - x' - \hbar k_x t/m) \equiv \tilde{P}(x, t). \qquad (13)$$

After substitution of the boundary values of $|\psi_{tr}(x, 0)|^2$ of figure 1 into equation (13), integration over x' leads to

$$\tilde{P}(x, t) = \frac{1}{n\delta} \sum_{i=1}^{n} \int_{(m/\hbar t)(x - x_r^i)}^{(m/\hbar t)(x - x_l^i)} |c_n(k_x)|^2 \mathrm{d}k_x = \sum_{i=1}^{n} \tilde{P}_i(x, t) \qquad (14)$$

where x_l^i and x_r^i are the coordinates of the left and right edges of the ith slit. In addition, by comparing equations (14) and (12) we conclude that the probabilities $P_i(x, y, t)$ of elementary events, in the region far from the slits, are given by

$$\tilde{P}_i(x, t) = \frac{1}{n\delta} \int_{(m/\hbar t)(x - x_r^i)}^{(m/\hbar t)(x - x_l^i)} |c_n(k_x)|^2 \, \mathrm{d}k_x. \tag{15}$$

This is the expression for the probability of a particle reaching (x, y) at time t after passing through the slit i of the n-slit grating, derived by assuming that particle and wave properties are compatible. Relation (8) is implicitly used in this derivation, as well as the law of the addition of probabilities of elementary events (a particle's arrivals along the allowed trajectories).

In figures 5–8 we present graphically various combinations of probabilities $\tilde{P}(x, t)$, $\tilde{P}_i(x, t)$ and $P(x, y, t) = |\psi_{tr}(x, t)|^2$ for one-slit, two-slit and eight-slit gratings. In order to calculate the graphs presented in figures 5 and 8 we used the following numerical values: grating parameters $\delta = 1$ μm and $d = 8$ μm; particle mass, $m = 6.6432 \times 10^{-27}$ kg; initial longitudinal wavevector, $k = 4\pi \times 10^{10}$ m^{-1}; initial longitudinal velocity, $v = 1995.58$ m s^{-1}. These parameters are the same as in the experiment of Kurtsiefer *et al* [9]. The graphs presented in figures 6 and 7 are calculated with the following numerical values: $\delta = 0.1$ μm, $d = 0.2$ μm, $k = (\pi/8) \times 10^{12}$ m^{-1}, $m = 3.8189 \times 10^{-26}$ kg, $v = 1084$ m s^{-1}, which are the parameters in the experiment of Keith *et al* [8].

By comparing the $\tilde{P}(x, t)$ graphs in figures 5, 6(*a*), 7(*a*), and 8(*a*) with the corresponding graphs of $P(x, y, t) = |\psi_{tr}(x, t)|^2$ in figures 3 and 4 in this paper and in figures 2 and 3 in [15], one sees that near the slits (Fresnel regime) the distributions $\tilde{P}(x, t)$ and $|\psi_{tr}(x, t)|^2$ qualitatively look similar but they differ numerically. Far from the slits (Fraunhofer region) the distributions $\tilde{P}(x, t)$ and $|\psi_{tr}(x, t)|^2$ are almost identical.

In order to show more clearly the latter agreement, the functions $\tilde{P}(x, t)$ and $|\psi_{tr}(x, t)|^2$ are presented on the same graphs in figures 6(*b*), 7(*b*), and 8(*b*), in the region where equation (14) approximates very well the modulus square of equation (4). By taking into account that $\tilde{P}(x, t)$ is obtained by summing the probabilities of particle arrival at the point (x, y) at time t along various possible trajectories, we conclude that far from the slits the function $\tilde{P}_i(x, t)$ might present the probability of a particle reaching (x, y) at time t after passing through the slit i of the n-slit grating. Near the slits it does not present that probability, because the quanton trajectories near the slits are more complicated than further from the slits. The straight lines could only approximate the quanton trajectories far from the slits. The graphs of distributions $\tilde{P}_i(x, t)$ are presented in figures 6(*c*), 7(*c*), and 8(*c*).

7. Conclusion

The wavefunction of a particle in the n-slit interferometer is determined, in the coordinate and in the momentum representations. The evident dependence of the momentum distribution on the grating parameters and the close relation between the momentum distribution and the space distribution clearly show, in our opinion, that the behaviour of a photon, neutron, electron, atom, molecule behind the grating depends on the number of open slits, their widths, and mutual distances.

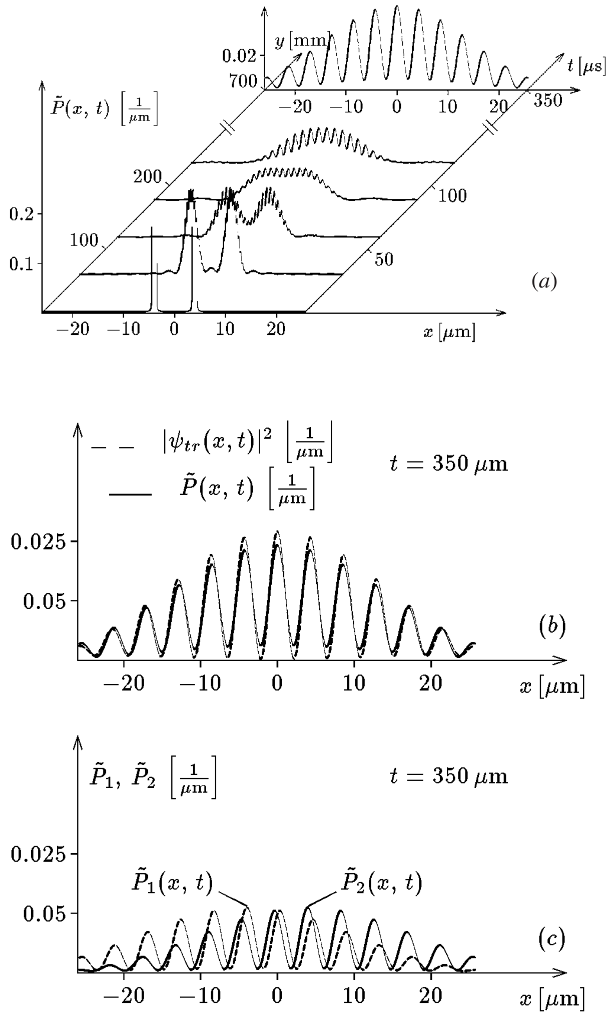

Figure 8. All graphs are calculated for the two-slit grating with $\delta = 1$ μm, $d = 8$ μm, $m = 6.6432 \times 10^{-27}$ kg, $k = 4\pi \times 10^{10}$ m^{-1}, and $v = 1995.58$ m s^{-1}. (*a*) The probability density $\tilde{P}(x, t)$ of a particle's arrival at point x at time $t(y = vt)$ behind the grating evaluated from (14). (*b*) The distribution functions $|\psi_{tr}(x, t)|^2$ and $\tilde{P}(x, t)$ for one value of $t(y)$ far from the grating. (*c*) Probabilities $\tilde{P}_i(x, t)$ evaluated from (15).

We determined the probabilities of events associated with this behaviour and explained how the interference pattern emerges in the process of accumulation of such individual events (particles arrive at the screen along the allowed trajectories). These results invalidate the assertions that classical laws of probability are violated in quantum interference.

References

[1] Koopman B O 1955 Quantum theory and the foundations of probability *Proc. Symp. Applied Mathematics* vol 7, ed L A MacColl (New York: McGraw-Hill) p 97

[1a] References would be too numerous to give here, although we may cite as typical an expository note by
Feynman R P 1951 The concept of probability in quantum mechanics *Proc. 2nd Berkeley Symp. on Mathematical Statistics and Probability* (Berkeley, CA: University of California Press)

[1b] Birkoff G von Neumann J 1936 The logic of quantum mechanics *Ann. Math.* **37** 823–43

[2] Selleri F and Tarrozi G 1978 *Nuovo Cimento* A **43** 31
[3] Feynman R P, Leighton R B and Sands M 1965 *The Feynman Lectures on Physics* vol 3 (Reading, MA: Addison-Wesley) p I-1
[4] Bohr N 1949 Discussion with Einstein on epistemological problems in atomic physics *Albert Einstein: Philosopher–Scientist* ed P A Shilpp (Evanston, IL: The Library of Living Philosophers) pp 200–41
[5] Rauch H and Werner S A 2000 *Neutron Interferometry, Lessons in Experimental Quantum Mechanics* (Oxford: Clarendon)
[6] Tonomura A, Endo J, Matsuda J, Kawasaki T and Ezawa H 1989 *Am. J. Phys.* **57** 117
[7] Grangier P, Roger G and Aspect A 1986 *Europhys. Lett.* **1** 173
[8] Keith D W, Ekstrom C R, Turchette Q A and Pritchard D E 1991 *Phys. Rev. Lett.* **66** 2693
[9] Kurtsiefer Ch, Pfau T and Mlynek J 1997 *Nature* **386** 150
[10] Arndt J, Nairz J, Andreae J V, Keller C, van der Zouw G and Zeilinger A 1999 *Nature* **401** 681
[11] Born M and Wolf E 1965 *Principles of Optics* (Oxford: Pergamon)
[12] Tomonaga S I 1962 *Quantum Mechanics* vol 2 (Amsterdam: North-Holland)
[13] Zurek W H 1991 *Phys. Today* 36
[14] Bonifacio R and Olivares S 2001 Young's experiment, Schrödinger's spread and spontaneous intrinsic decoherence *Proc. 3rd Workshop on Mysteries, Puzzles and Paradoxes in Quantum Mechanics* ed R Bonifacio *et al Z. Naturf.* a **56** 41
[15] Božić M, Arsenović D and Vušković L 2001 Transverse momentum distribution of atoms in an interferometer *Proc. 3rd Workshop on Mysteries, Puzzles and Paradoxes in Quantum Mechanics* ed R Bonifacio *et al Z. Naturf.* a **56** 173
[16] de Broglie L 1963 *Etude Critique des Bases de l'Interpretation Actuelle de la Mecanique Ondulatoire* (Paris: Gauthier-Villars) (Engl. transl. 1964 (Amsterdam: Elsevier))
[17] Hecht E 1987 *Optics* (Reading, MA: Addison-Wesley)
[18] Hellmuth T, Walther H, Zajonc A and Schleich W 1987 *Phys. Rev.* A **35** 2532
[19] Buks E, Schuster R, Heiblum M, Mahalu D and Umansky V 1998 *Nature* **26** 871
[20] Božić M and Marić Z 1991 *Z. Phys. Lett.* A **158** 33
[21] Božić M, Marić Z and Vigier J P 1992 *Found. Phys.* **22** 1325
[22] Božić M and Marić Z 1998 *Found. Phys.* **28** 415
[23] Vušković L, Arsenović D and Božić M webpage http://xxx.lanl.gov/archive/quant-ph/0105129