Publ. Astron. Obs. Belgrade No. 65 (1999), 17 – 22

MODELING PLANETARY FORMATION

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Abstract. We present a simplified model of planetary formation through gravitational accretion. Using the model we have simulated the formation of planetary systems starting from as many as $N = 10^{10}$ initial particles, and have investigated the spontaneous appearance of two distinct types of condensate. The two types of condensate, light and heavy, are distinguished by the way they scale with changes of N. The positions, mass and spin of the heavy condensates (planets) are found to be in very good agreement with Solar system data.

1. INTRODUCTION

In recent years there has been a large increase in the amount of available data on extra-solar planetary systems (Marcy and Butler 1998). Even more dramatic increase is expected in the near future (Lattanzi, *et al.* 1997). In view of these achievements it is not surprising that a long standing problem of the detailed understanding of planet formation (Isaackman and Sagan 1977) has received a lot of attention lately (e.g. Ida and Makino 1992a, 1992b, Kokubo and Ida 1995, 1996, 1998). One of the ultimate goals of such studies is to assess the likelihood of the formation of Earth-like planets. The exponential increase in computing power during the last two decades has made numerical simulation the most promising tool to achieve this goal. However, if one wants to perform a numerical *ab initio* simulation of the planetary system formation starting from a protoplanetary nebula, a large number of initial particles needs to be considered. Since the range of planetary masses in the Solar system spans four orders of magnitude, the smallest possible number of initial particles needed to resolve them with reasonable accuracy is at the very least $N = 10^6$. Despite recent exceptional advances, state of the art simulations of gravitating systems are still not able to deal with this many initial particles.To make such large scale simulations feasible, we have devised a simplified model of planetary accretion. The simplifications used are rather natural, but at the same time quite radical, both regarding the geometry of particle motion, as well as its dynamics.The inherent simplicity of the model we present has made the condensation process not only feasible, but also transparent, which allowed certain analytical results to be obtained. The relatively small numerical cost involved in a single simulation enabled us to study a wide variety of possible situations, and to gain the predictive power needed to analyze data on extra-solar planetary systems, as well as multiple star systems.

2. THE MODEL

We begin by introducing a planetary condensation model that starts from a given planar distribution of N initial particles, all of the same mass and with no spin. The particles have a uniform angular distribution, while the radial distribution is given by $\rho(r)$, which determines the initial conditions.

The N-body dynamics is simplified by dividing it into two pieces—free propagation and instantaneous interactions. Between interactions all particles move in circular trajectories according to Kepler's laws. The only interaction allowed is the merging of two particles into one. The merging happens if the two particles satisfy an interaction criterion given bellow. The result of the merging of two bodies with masses m_1 and m_2, at positions $\vec{r_1}$ and $\vec{r_2}$, and with spins S_1 and S_2, is a new body with mass m_1+m_2, position $\vec{R}$, and spin $S = S_1 + S_2 + L_1 + L_2 - L$. L_1, L_2 and L are respectively the orbital angular momenta of the first, second and final particle. The point of joining follows from energy conservation (once we neglect heating due to accretion, energies due to the particles spin, as well as the potential energies between pairs of condensing particles). We find

$$\frac{m_1 + m_2}{R} = \frac{m_1}{r_1} + \frac{m_2}{r_2} \, . \tag{1}$$

The interaction criterion chosen is quite natural. We impose $F\Delta t \geq |\Delta\vec{p}|$, where F is a mean value of the gravitational force between the bodies during the collision and $\Delta t \sim |\Delta\vec{r}|/|\Delta\vec{v}|$ is a characteristic time of the collision. For the above criterion to be satisfied the two initial particles must be close in space. As a consequence, the angle between them must be small. In this paper we set $\theta = 0$, and disregard the $O(\theta^2)$ corrections. Using this, the criterion becomes

$$\frac{1}{m_1 m_2}\left|\frac{m_1+m_2}{\sqrt{R}} - \frac{m_1}{\sqrt{r_1}} - \frac{m_2}{\sqrt{r_2}}\right|\left|\frac{1}{\sqrt{r_1}} - \frac{1}{\sqrt{r_2}}\right| |r_1 - r_2| \leq K \, . \tag{2}$$

As we see, the interaction is given in terms of a single parameter K. From the derivation we have $K \sim 1/M$, where M is the mass of the star. Note that Newton's gravitational constant G has cancelled by the use of Kepler's law. The above criterion is homogeneous with respect to changes of mass scale as well as of distance scale. We will find it convenient to fix the mass scale by setting the mass of the protoplanetary material M_P equal to one. By doing this, the mass of the initial particles becomes $1/N$, while K becomes dimensionless. Distance scales are fixed by our choice of initial mass density. In this paper we work with the triangular initial mass density

$$\rho(r) = \begin{cases} \frac{1}{5}r & \text{if } r \leq 1 \\ \frac{1}{45}(10-r) & \text{if } 1 < r \leq 10 \, . \end{cases} \tag{3}$$

This is a rather simple density peaked at $r = 1$. The detailed analysis of how the results of accretion depend on the choice of initial density can be found in Balaž, *et al.* (1999a, 1999b).

It is convenient to work with reduced angular momentum quantities $\ell = L/\sqrt{MG}$, as well as $s = S/\sqrt{MG}$. In these units the orbital angular momentum of a body of mass m at a distance r from the star is equal to $\ell = m\sqrt{r}$. As a result of joining the spin of the new body becomes $s = s_1 + s_2 + m_1\sqrt{r_1} + m_2\sqrt{r_2} - (m_1 + m_2)\sqrt{R}$. It is easy to see that in this simple model spins are always positive, in accord with most of the planets in the Solar system.

3. BASIC RESULTS

The first physical quantity that we look at is $\Omega \equiv n/N$, the ratio of the number of final condensates and initial particles. Ω is a monotonous function of K that decreases from 1 (small K) to 0 (large K) and distinguishes between two phases, dominated by light and heavy condensates respectively. The behavior of the system is most complicated in the intermediate regime, in which Ω differs significantly from 0 or 1, i.e. when there is a significant mixture of both light and heavy condensates. It is rather easy to fit the obtained data to a simple law. We find

$$\Omega = \frac{1}{1 + AN^{\alpha}K^{\beta}} , \tag{4}$$

where $\alpha = 0.737(6)$, $\beta = 0.251(2)$, while $A = 2.10(5)$. Since $n_{light} \gg n_{heavy}$, we see that Ω is just the relative number of light condensates. Ω is a global property of light condensates. A more detailed understanding of their structure is achieved by studying their distributions in mass and distance from the star. The mass distribution, i.e. the fraction of condensates at mass m, can be fitted by a simple power law

$$\Delta(m) = \begin{cases} 0 & \text{if } m < 1/N \\ \tau N^{-\tau} m^{-\tau-1} & \text{if } 1/N \leq m < m^* \end{cases} , \tag{5}$$

where $\tau = 1.2(2)$. The mass scale m^* is the dividing line distinguishing between light and heavy condensates. More details about this and other properties of light condensates are given in Balaž, *et al.* (1999b). The radial distribution of light condensates, i.e. the fraction of condensates found at distance r from the star, also displays a power law behaviour $\Lambda(r) \propto r^{-\zeta}$. Unlike the mass distribution, the radial distribution changes after condensation due to various effects like the solar wind and bombardement by extra-solar dust particles. For this reason, Λ can not easily be compared to present day radial distributions of dust.

Condensates with masses greater than m^* are designated as heavy. Planets belong to this category. We have studied their masses, locations and spins. Unlike the light condensates, the properties of the heavy condensates do not scale with N. The K dependence of the masses of the four heaviest condensates is shown in Fig. 1. The choice of $K = 0.1$ follows from equating m_2/m_1 with the ratio of the masses of Saturn and Jupiter as is shown in Fig. 2. By fixing K we have completely determined our model. The predicted masses of the other giants also agree well with Solar system data.

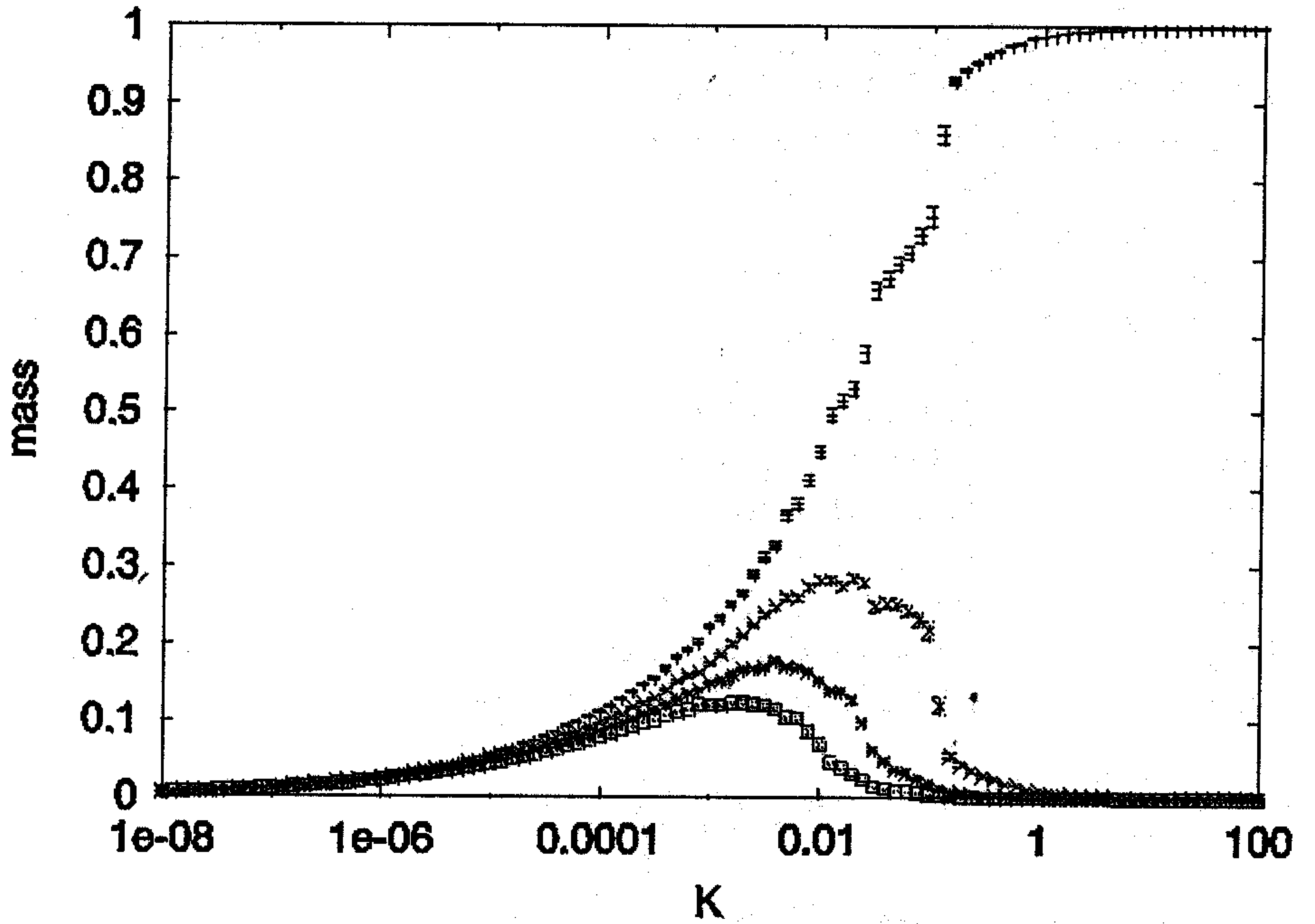

Fig. 1. The mass of the four heaviest condensates as functions of K.

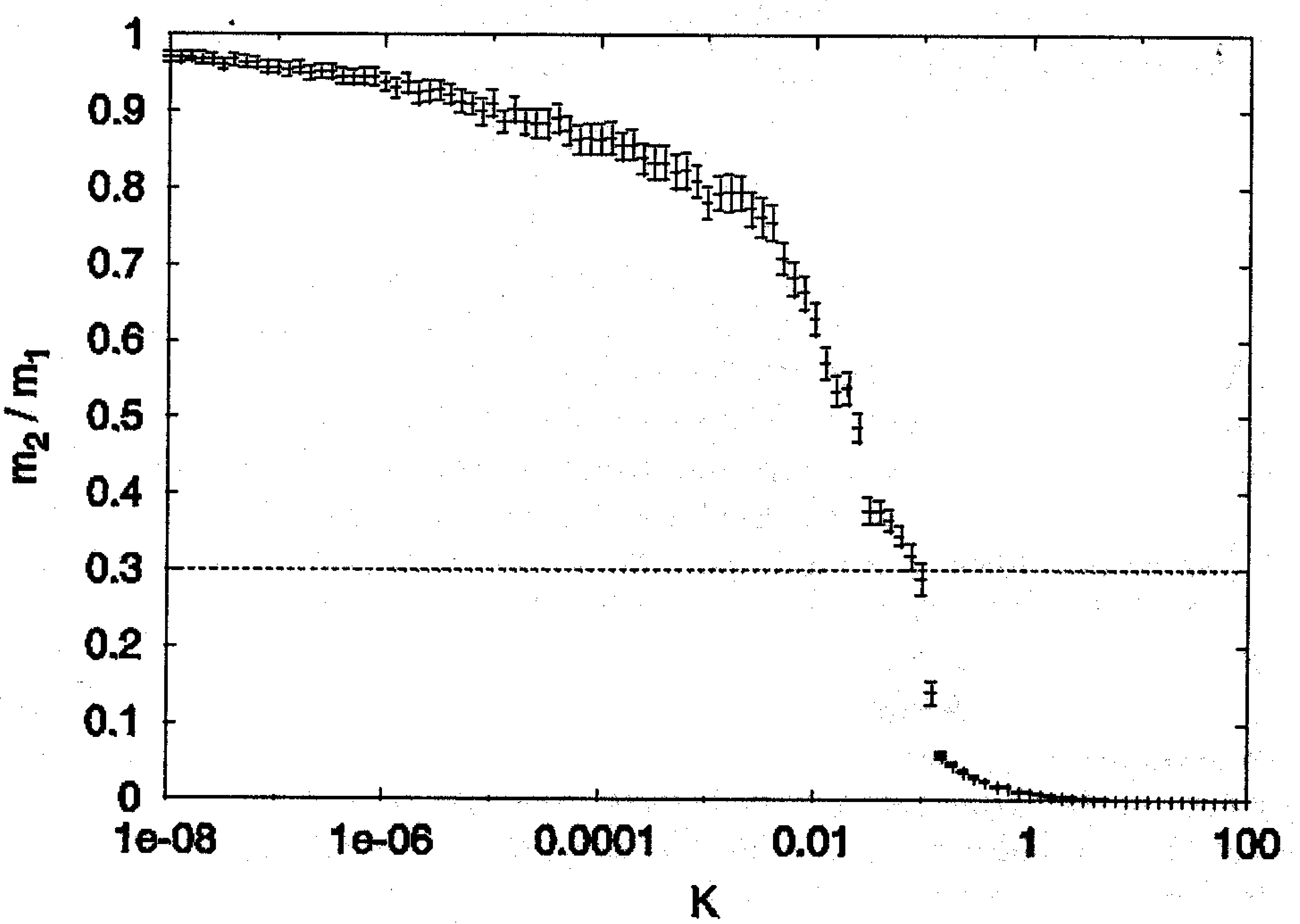

Fig. 2. m_2/m_1 as a function of K. The horizontal line is the ratio of the masses of Saturn and Jupiter.

The situation with spin is even more interesting. We find that

$$s \propto K^{\epsilon} m^{\omega} , \tag{6}$$

where $\omega = 1.75(3)$, and $\epsilon = 0.40(2)$. This is shown in Fig. 3. Except for the very lightest particles whose mass is near the $1/N$ cut-off all the points lie on the law given in equation (6).

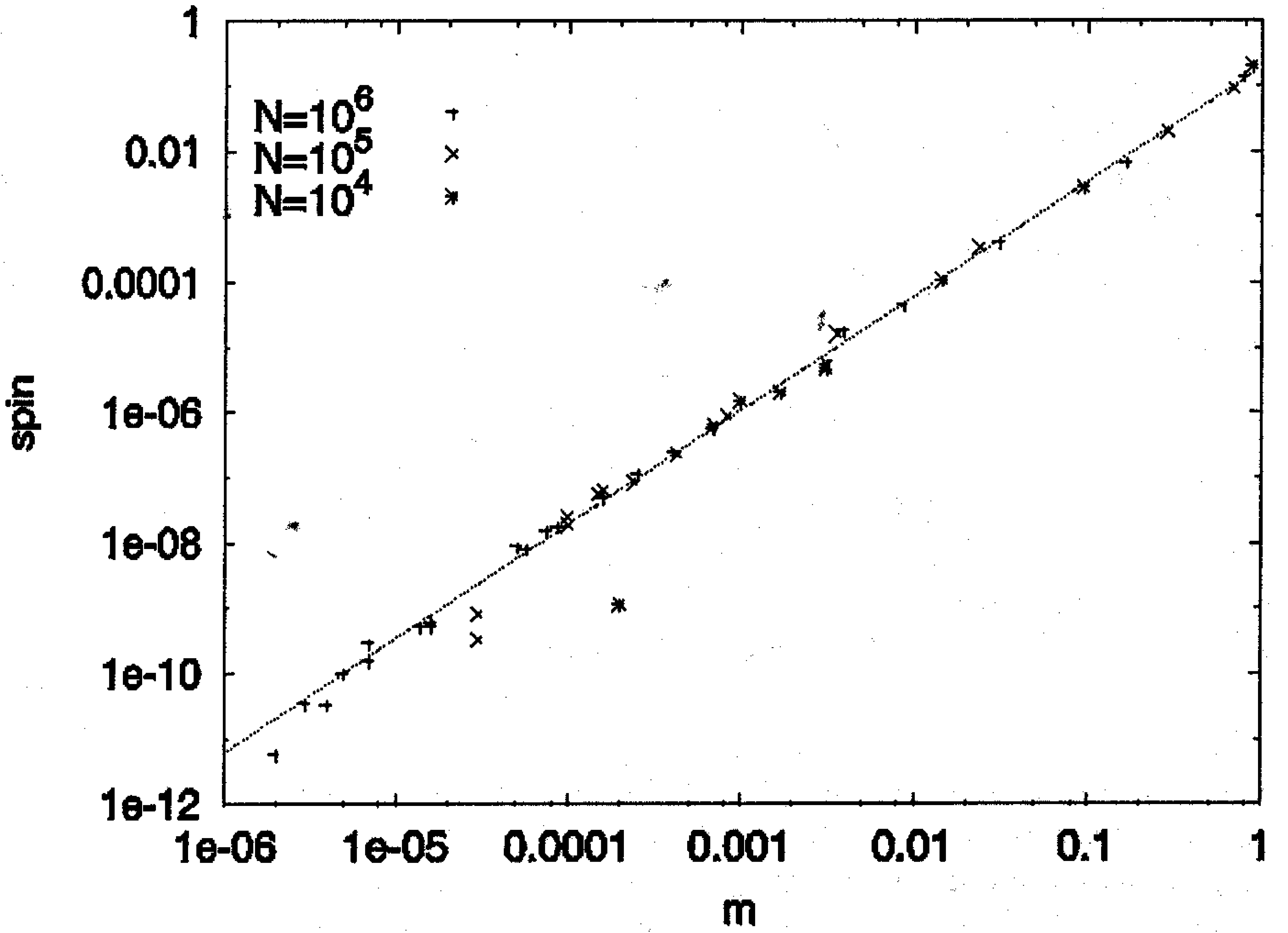

Fig. 3. Spin as a function of mass for $K = 0.1$ and $N = 10^4, 10^5, 10^6$.

The corresponding data for the planets in the Solar system is given in Fig. 4. We see that our simplified accretion model agrees quite well with phenomenology. The only two planets that do not satisfy the above spin-mass relation are Mercury and Venus. This is not surprising, as these are the two planets nearest to the sun, where additional tidal lock effects play an important role.

Of the above properties, the exponents governing Ω, Δ and Λ have been found to be independent of the choice of initial mass density. The same is true of spin. The masses of the heavy condensates do depend on ρ, but the general form of this dependence is quite similar for all ρ's. The properties of the condensates that depend *very strongly* on the initial mass density are the positions of the planets. The details of the analysis of dependence on initial conditions, as well as some analytical derivations in the $K \to 0$ and $K \to \infty$ limits, can be found in Balaž, *et al.* (1999a). We are currently working on identifying properties that do not change during the accretion process, and depend only on the initial conditions. Such quantities have been found

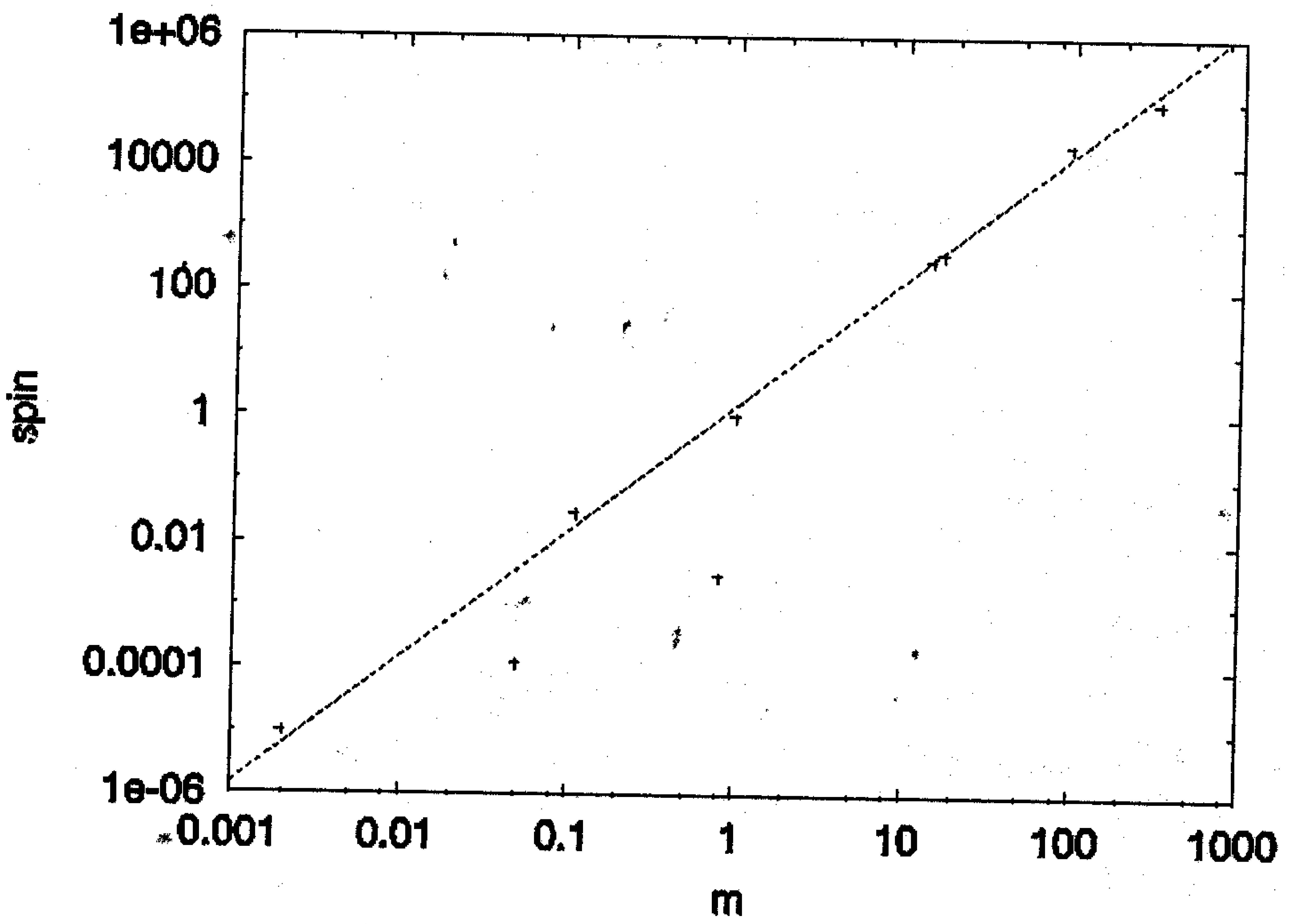

Fig. 4. In the Solar system the planet's spin fits a $s \propto m^2$ law, that is, the exponent is $\omega = 2$.

(see Balaž, *et al.* 1999c), and they allow the determination of the initial conditions that produce the correct radial distribution of planets.

The numerical simulations presented in this contribution have been performed on an SGI Origin 2000 super-computer. We would like to acknowledge the help of the staff of the IPCF (Institute of Physics Computing Facilities). This research was supported in part by the Serbian Ministry of Science and Technology under research projects 01M01 and 01E15.

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