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PLANETARY FORMATION ALGORITHM

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Abstract. We present a detailed exposition of the algorithm used in a new model of planetary formation through gravitational accretion. The algorithm has allowed us to work with up to $N = 10^{10}$ initial particles, to average over large numbers of individual runs, as well as to investigate the dependence of the outcome of the condensation process on initial conditions. This has been possible due to the fact that the computing time of the algorithm effectively scales as $O(N)$.

1. INTRODUCTION

In recent years there has been an added research effort in building accretion models and analyzing their predictions. The reasons for this renewed activity are twofold. On the one hand, new observational results have appeared regarding extra-solar planetary systems (e.g. Marcy and Butler 1998). On the other hand, the explosive increase in computing power has enabled us to look at N-body simulations with a much larger number of initial particles than before. State of the art direct simulations of N gravitating bodies, implemented on dedicated super-computers, can now work with up to $N \sim 10^4$ bodies (Ida and Makino 1992a, 1992b, Kokubo and Ida 1995, 1996, 1998), which is insufficient to resolve the masses of all the planets in the Solar system.

For this reason we have recently constructed a simplified condensation model described in Balaž, *et al.* (1999a, 1999b). In this paper we will focus on the computer implementation of this model. We will also give a brief description of an important property of the model that does not depend on the details of the condensation process but only on the initial conditions.

2. THE ALGORITHM

The straight forward way to simulate planetary accretion model (Balaž, *et al.* 1999a, 1999b) would be to generate the positions of all N initial particles according to initial mass distribution $\rho(r)$ and then to randomly pick the pairs and merge them if the interaction criterion is satisfied. The random number generator used in this work is RAN3 described in Press, *et al.* (1995). This process would continue until no further merging was possible, i.e. no pair of bodies satisfied the criterion. Such a strategy (definition algorithm) has been investigated and the memory requirements needed to simulate N body accretion scale as $O(N)$, whereas computing time has been found to be of order $O(N^{2.2})$. This makes it impractical for the study of sufficiently large systems.

Fortunately, there is a more efficient way to simulate our model. It is not necessary to specify all the N bodies at the very beginning, due to the fact that we are dealing with a two body interaction criterion. For example, in the very first merger the position of the $N-2$ spectators are irrelevant, and need not be generated at that time. After the question of this merging is resolved another particle is added and so on. At each step the possible merging of the newly added particle with the ones already present is investigated. This operation is local, i.e. if the newly introduced particle merges that can only happen with one of the two particles with the nearest radial distance, and for this reason it is useful to keep particles sorted according to increasing r throughout the simulation. Obviously, the positions and masses of particles change after merging, making further merging possible. After all the merging possibilities are exhausted a new particle is added to the system, and the procedure is recursively repeated until all N initial particles are considered. Although we can't prove the strict equivalence of these two simulation schemes numerical evidence shows that the results are equivalent within statistical errors.

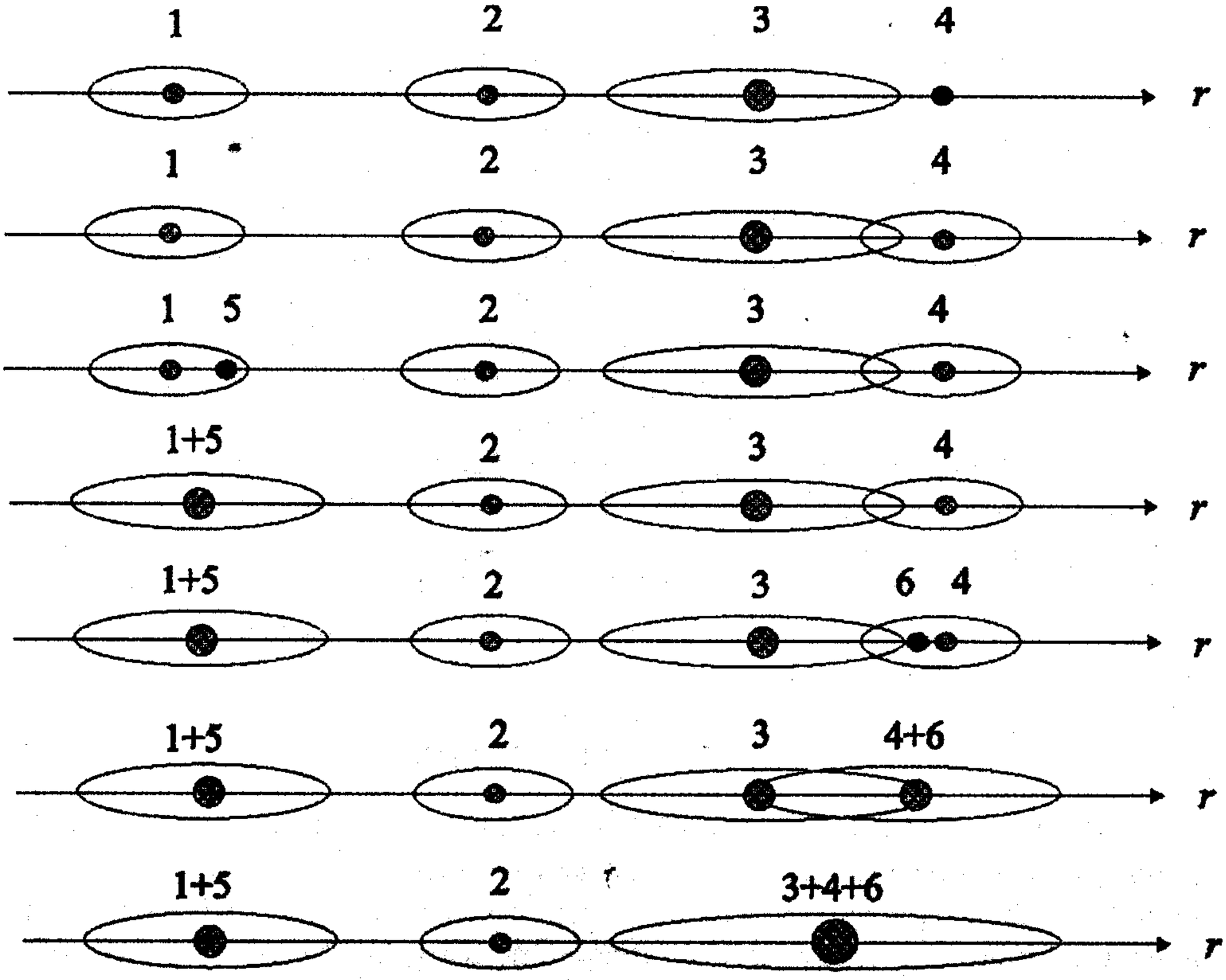

Fig. 1. Schematic presentation of the order of introduction of initial particles used in the employed algorithm.

Fig. 1 gives a schematic presentation of how initial particles are introduced. Black dots represent newly introduced particles, gray dots represent existing particles, while ellipses correspond to their regions of attraction for the capture of initial particles. At the top line we introduce a new particle that does not merge with the rest of the particles. The resulting situation is shown in line two. Lines three and four depict a typical merging. Lines five to seven show the introduction of a new particle that leads to a two step merging cascade. In line two we see that particles 3 and 4, while not interacting, have overlapping regions of attraction for the capture of initial particles. In cases like this we need to specify whether the merging proceeds to the left or right. We have investigated both the cases when all such merging is to the left and to the right. The difference is quite small and may be absorbed into a change of K. Throughout this paper we resolve the case of overlapping regions of attraction by always merging to the left, i.e. to smaller values of r.

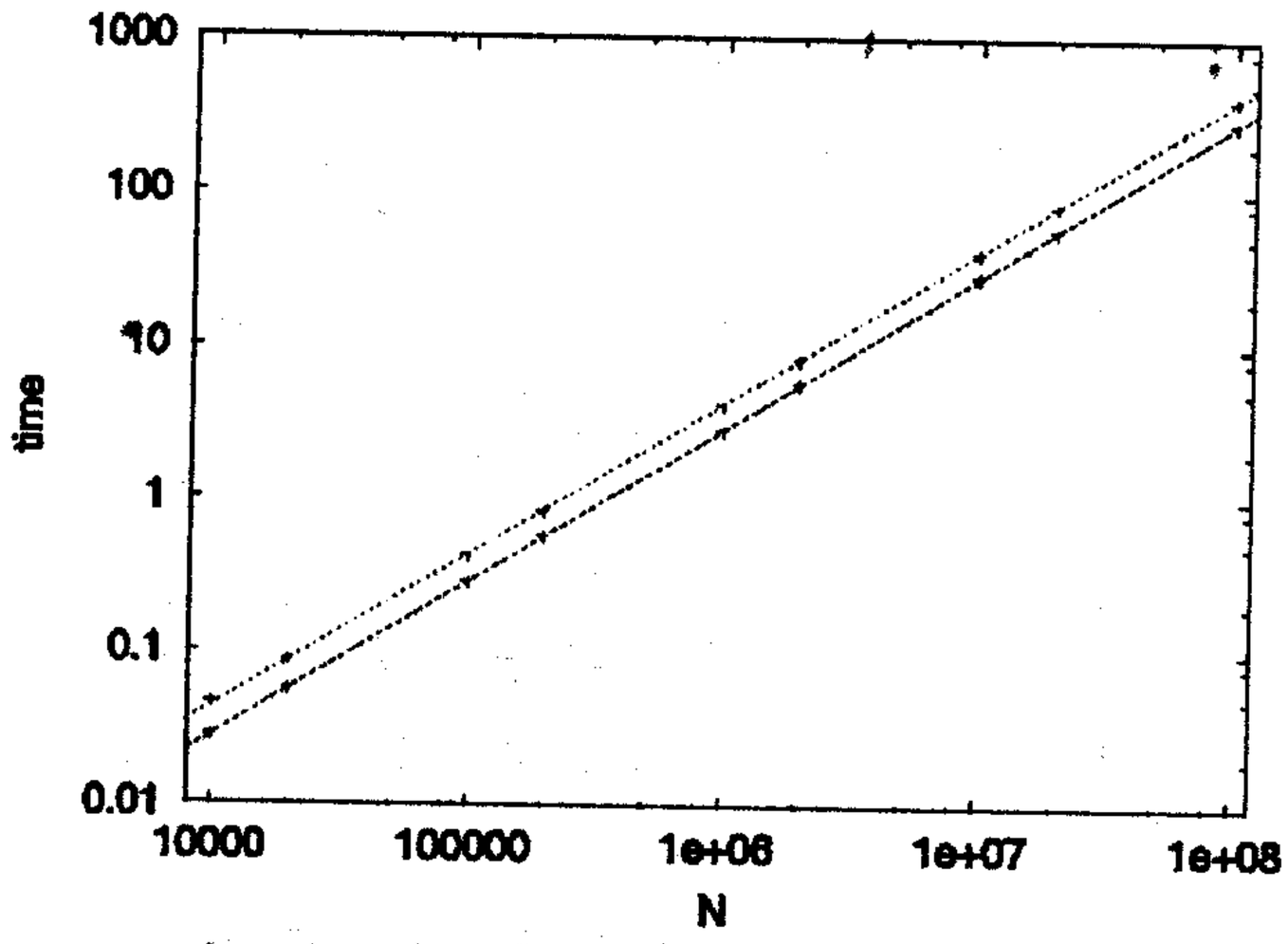

Fig. 2. The computing time (per R10000 processor) for a single run as a function of N for $K = 10^{-6}$ (top) and $K = 10$ (bottom line).

The memory required for this algorithm is of order $O(n)$, where n (typically $n \ll N$) is the final number of condensates. On the other hand, we expect the computing time to be $t \propto N(a + b \log N)$. The overall factor of N comes from the loop over N initial particles. The term in brackets represents the time for the calculations inside the loop, i.e. for a single particle. The $b \log N$ term comes from keeping particles sorted according to increasing r throughout the simulation, while the constant term a represents the number of other operations inside the loop, regardless of sorting. Obviously, for $N \gg 10^{a/b}$ the term containing the logarithm will dominate and the algorithm will be $O(N \log N)$. However, since in our code $a/b \sim 50$ that regime is never reached in practice and, for the considered numbers $N = 10^3 - 10^{10}$, the algorithm is $O(N)$, as can be seen from Fig. 2.

At the end let us display an important property of the model – the fact that the accretion process preserves the cumulative distribution $c(r) = \int_0^r \rho(x)dx$. Two examples of this, for $K = 10^{-6}$ and $K = 10^{-3}$, are shown in Fig. 3. Smaller values

of K correspond to a larger number of condensates. The curve for $K = 10^{-6}$ thus represents an excellent approximation to the initial cumulative distribution $c(r)$. The $K = 10^{-3}$ run has a much smaller number of condensates, corresponding to a smaller number of steps in the figure. Despite that, it shows an obvious relationship with $c(r)$. The above hypothesis that the cumulative distribution is preserved is validated by using the Kolmogorov–Smirnov test (Balaž, *et al.* 1999a). The fact that the cumulative distribution is unchanged by accretion can be used to determine the initial conditions that have led to the Solar system. In this way one can obtain good agreement even with those properties of the condensates that strongly depend on initial conditions (like the positions of heavy condensates). Investigations in this direction are currently in progress.

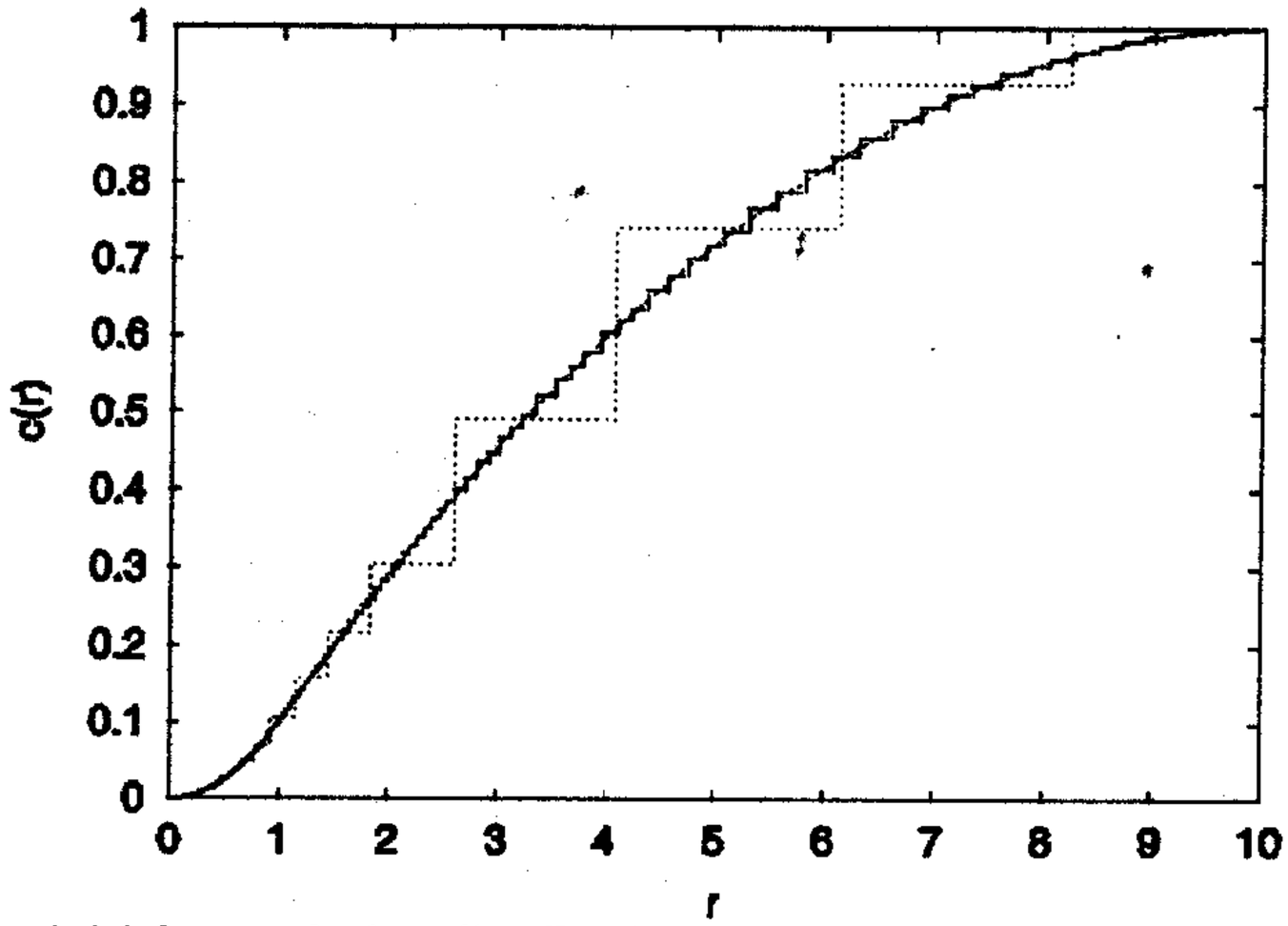

Fig. 3. The initial cumulative distribution compared with the outcomes of condensation for two runs at $K = 10^{-6}$ and $K = 10^{-3}$.

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