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# SCALING EXPONENTS FOR ACCRETION

A. BALAŽ, A. BELIĆ and A. BOGOJEVIĆ
*Institute of Physics, P.O.B. 57, Belgrade 11001, Yugoslavia*

**Abstract.** A recently developed model of planetary accretion is used to study various properties of the produced condensates. The properties are given in terms of a set of scaling exponents. We have investigated the dependence of the results upon the choice of the initial mass distribution of the protoplanetary nebula. It is found that these scaling exponents do not depend on the initial conditions.

## 1. INTRODUCTION

The interest in the detailed modeling of the planetary accretion process has increased in recent years due to a wealth of new data on extrasolar planets (Marcy and Butler 1998). A large amount of effort has been invested to surmount the principal difficulty in such *ab-initio* simulations, the fact that huge number of bodies needs to to be considered in order to resolve planets whose masses span four orders of magnitude. To achieve this goal, we have recently developed an effective model of planetary accretion utilizing a set of physical assumptions that simplify the accretion process considerably (Balaž, *et al.* 1999a, 1999b). This approach is conceptually different from the now prevalent brute-force gravitational simulations (e.g. Ida and Makino 1992).

Working with a large number of particles $N$ (typically $10^6$-$10^{10}$) has enabled us to clearly distinguish two types of condensates that we identify as light and heavy. The distinction of these two types is based upon their scaling behavior when $N$ is changed.

The problem of not knowing the initial distribution of matter in the protoplanetary nebula is a serious one. One of the main motivations for developing our model was to investigate how sensitive are the outcomes of the accretion process to variations in the initial conditions. We have identified a set of properties of the final distributions of light and heavy condensates that do not depend on the details of the initial mass distribution. Conversely, it is possible to identify the properties that strongly depend upon the initial conditions. These properties can be fitted to the solar system data and used to extract the likely initial conditions as discussed in Balaž, *et al.* 1999c.

## 2. SCALING EXPONENTS

The model we use to simulate the planetary accretion process (Balaž, *et al.* 1999a, 1999b) depends on a parameter $K$, a number of initial bodies $N$, and on the initial mass distribution $\rho(r)$. In order to be able to make a quantitative comparison between the outcomes of accretion simulations for different choices of $K$, $N$, and $\rho(r)$, we need to identify a set of quantities that characterize the final distribution of condensates.

The first, and principal, such quantity is a ratio of final and initial number of bodies $\Omega = n/N$. A typical result for $\Omega$ is shown in Figure 1. It is seen that $\Omega$ monotonically decreases with $K$ from its maximal value of 1 at $K = 0$, to the minimal value of $1/N \sim 0$ at large $K$. These two extreme cases correspond respectively to no accretion at all, and to a collapse of all the material of the protoplanetry nebula into a single large body, i.e. to the formation of a binary system. Thus, the parameter $K$ is seen to regulate the amount of the accretion that takes place.

Since we model the planetary accretion using a stochastic process, the outcomes of various runs do not necessarily coincide. Averaging the results over several runs give their expectation values and error bars. The data shown in Figure 1 are produced by averaging over 100 runs, and error bars can barely be seen.

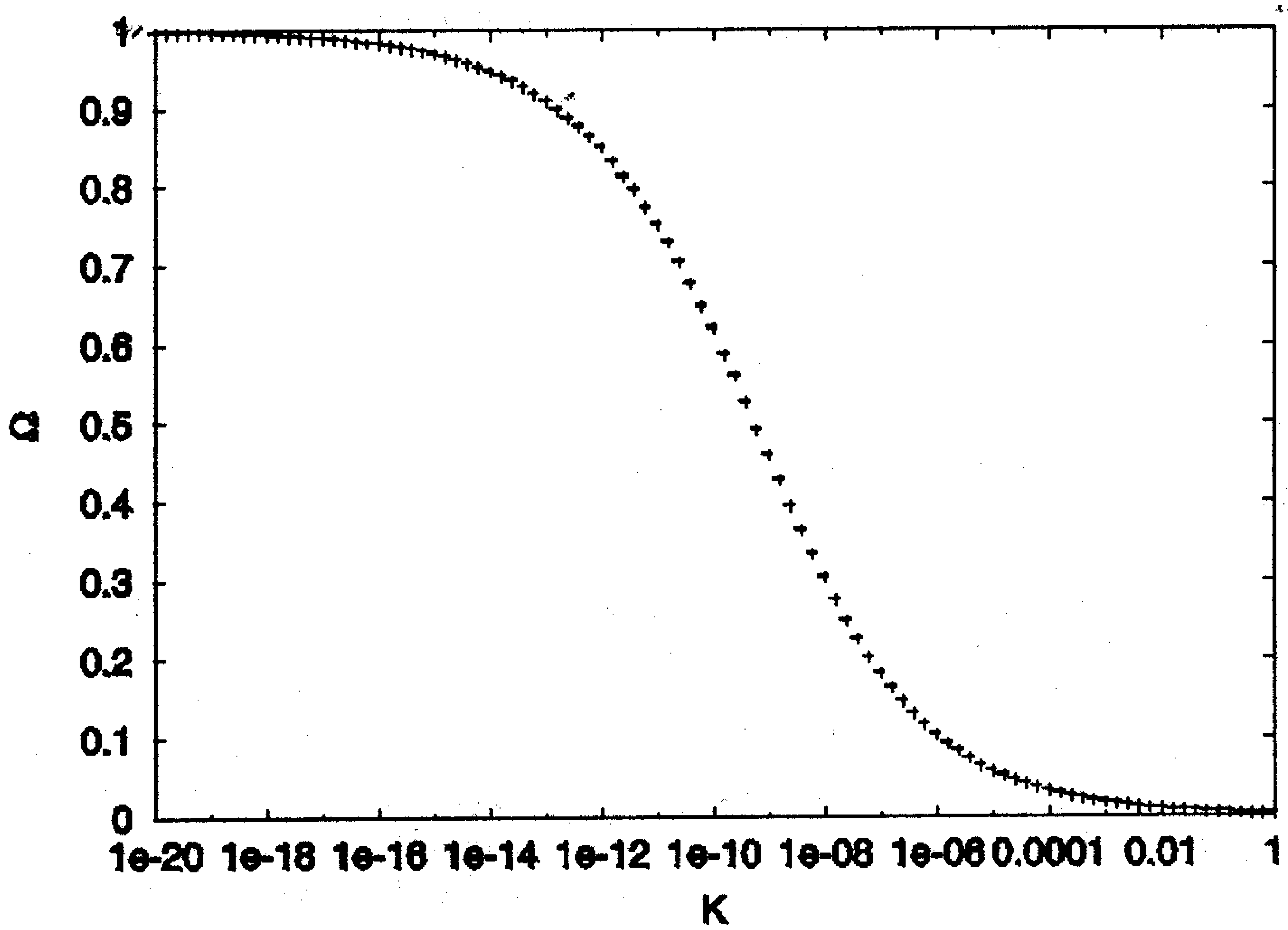


Fig. 1. $\Omega$ as a function of $K$ for $N = 10^3$ initial particles, and triangular $\rho(r)$ peaked at 1.

It is rather easy to fit the data obtained using a triangular $\rho(r)$ peaked at 1 (see Balaž *et al.* 1999b, eq. (3)) to a simple law. We find

$$\Omega \equiv \frac{n}{N} = \frac{1}{1 + AN^{\alpha}K^{\beta}} , \tag{1}$$

where $\alpha = 0.737(6)$, $\beta = 0.251(2)$, while $A = 2.10(5)$. When the initial mass distribution $\rho(r)$ is varied within a class of triangular distributions by changing the position of the peak, the exponents $\alpha$ and $\beta$ remain the same within the error bars. The same happens when we abandon the triangular distribution all together and use quadratic, gaussian, or $r/(1+r^4)$ distributions instead. These findings support the claim that the above values of scaling exponents $\alpha$ and $\beta$ are universal within a single hump $\rho(r)$ universality class, and possibly even further.

The quantity $\Omega$ is a global property of condensates. A more detailed understanding of their structure is achieved by studying their distributions in mass and distance from the star. The typical mass distribution, i.e. the fraction of condensates at mass $m$, is shown in Figure 2 for $K = 10^{-7}$. It is immediately obvious that the mass distribution of condensates lighter then a certain mass scale $m^*$ can be fitted to a simple power law

$$n(m_i)/n = BN^{-\tau}m^{-\tau}. \tag{2}$$

The condensates that scale according to this law are designated as light, and those that do not as heavy. The mass scale $m^*$ is thus the dividing line distinguishing between light and heavy condensates. For the triangular $\rho(r)$ peaked at 1, the scaling exponent takes the value $\tau = 1.2(2)$. Again, the same value (within error bars) is obtained for all other density distributions considered, i.e. it is universal.

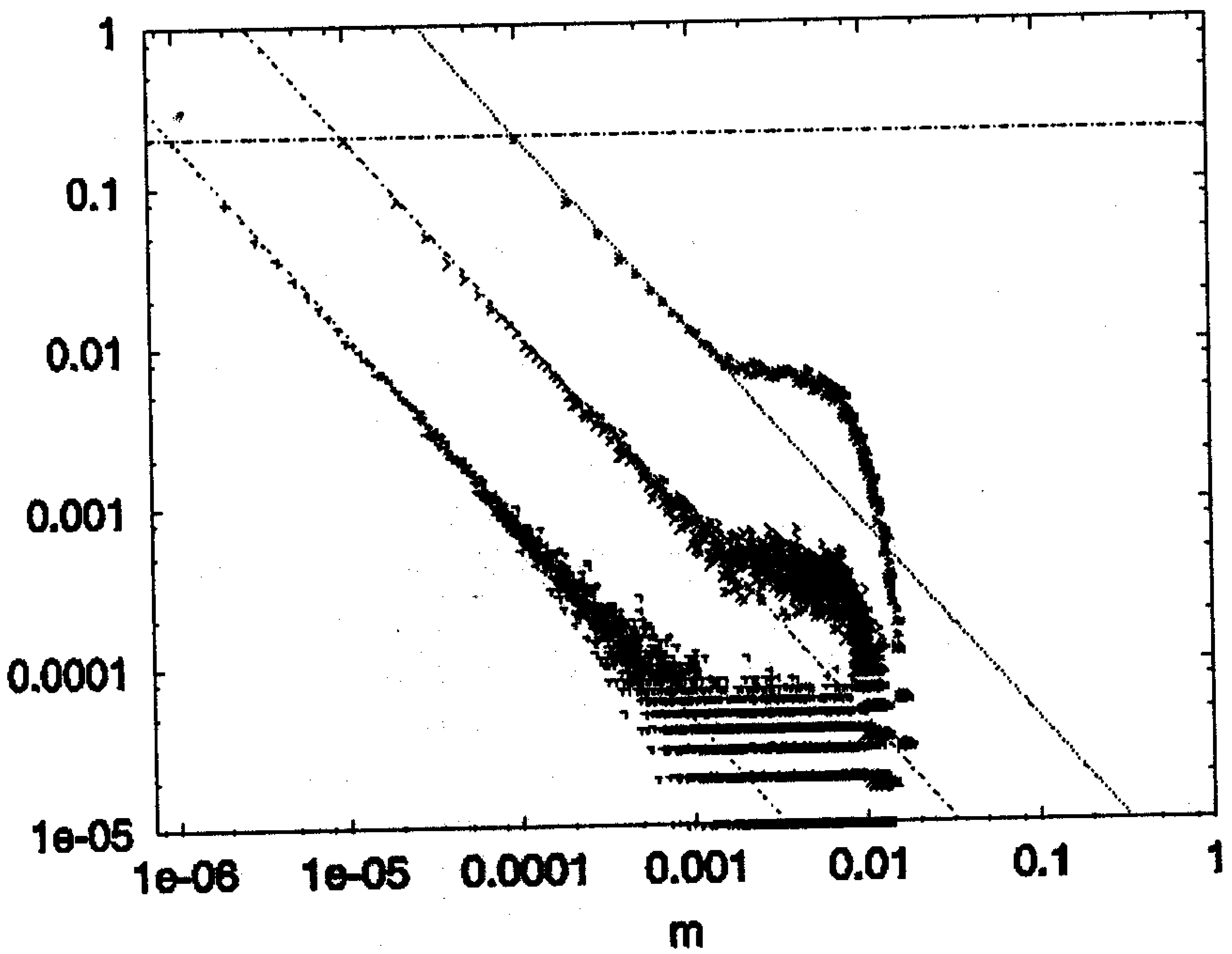


Fig. 2. Plot of the relative number of condensates of mass $m_i = i/N$ (averaged over 100 runs), for $K = 10^{-7}$ and $N = 10^4$, $10^{4.5}$, $10^5$, and $10^{5.5}$ initial particles.

The best fit of our model (in particular the heavy condensates) to Solar system data is obtained for $K = 0.1$ (Balaž *et al.* 1999b). The mass distribution of condensates for that value of $K$ is shown in Figure 3. The distinction between light and heavy condensates is apparent again, the only difference being that $m^*$ has developed a stronger $N$ dependence. In order for heavy condensates span four orders of magnitude in mass we must have $m^* < 10^{-4}$. This is achieved for $N > 10^6$. If we further impose that light and heavy condensates be well separated, then at least $N = 10^8$ initial bodies are needed.

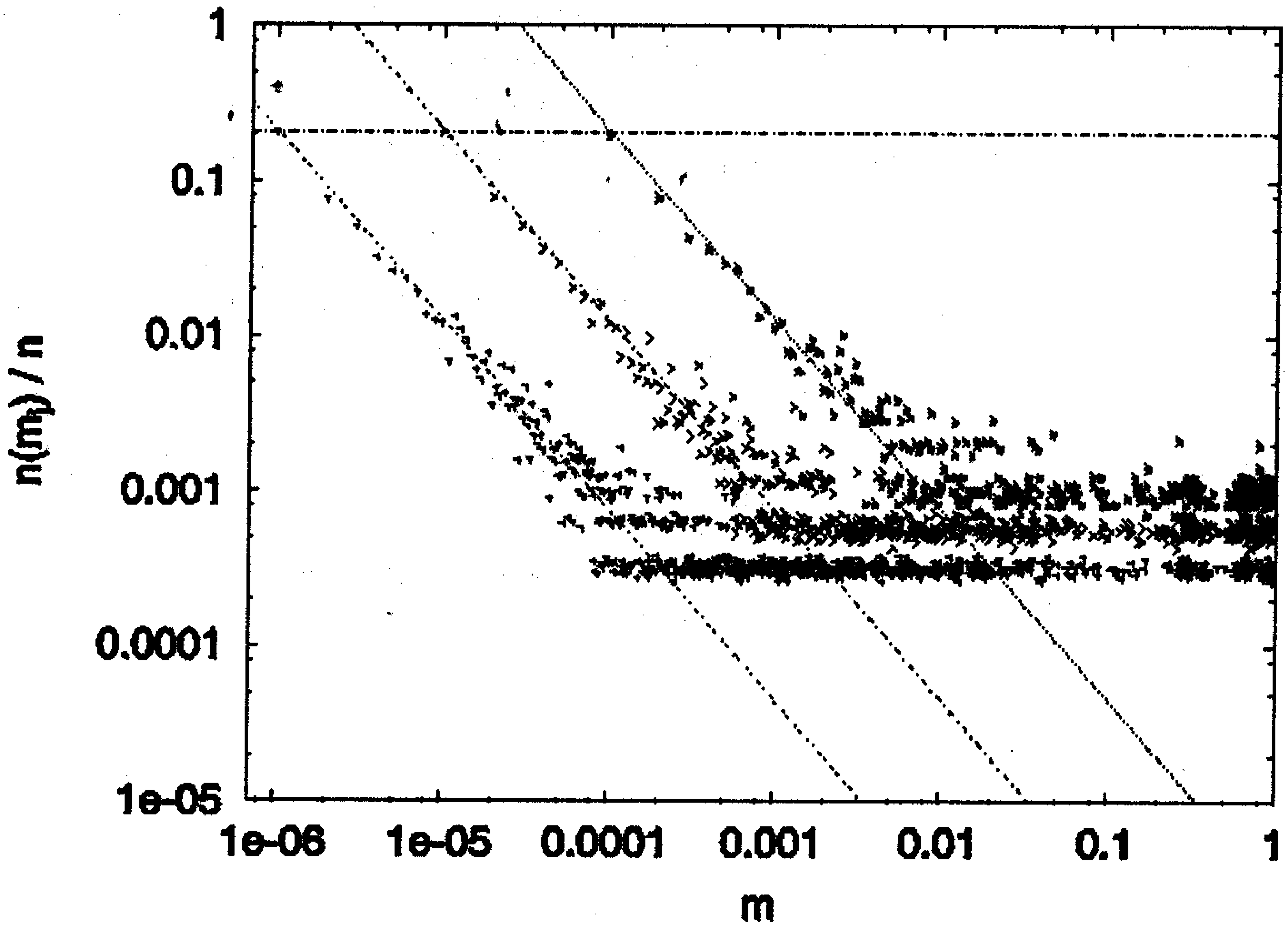


Fig. 3. The same as in Figure 2, for $K = 0.1$, the value that best fits our solar system.

The radial distribution of light condensates, i.e. the fraction of condensates found at distance $r$ from the star, also displays a power law behaviour $\Lambda(r) \propto r^{-\zeta}$. For the triangular $\rho(r)$ peaked at 1, and $K = 0.1$ the scaling exponent is $\zeta = 1.85(5)$. However, unlike the previously considered exponents, this exponent is *not* universal.

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