Visibility of interference in Feynman's atomic light microscope

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Abstract
The visibility of the interference pattern in Feynman’s atomic light microscope is studied theoretically using the evolution of the wave function of individual atoms passing through a double slit grating and interacting with laser light (of the wave vector \( k_i \)). Assuming that the slits’ width (\( \delta \)) is much smaller than the slits’ distance (\( d \)), the expressions for visibility as functions of \( dk_i \) were derived for uniform and for Mandel distribution of transferred momentum during photon atom resonance scattering. The decreasing of oscillations of visibility with increasing \( dk_i \) was explained taking into account the wave functions of individual atoms and the statistics of transverse momentum transferred to atoms. The influence of nonnegligible slits’ width on visibility was studied numerically. It is found that revivals are present for infinitesimally wide slits as well as when slits have finite width.

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(Some figures may appear in colour only in the online journal)

1. Introduction
The realization by Chapman et al [1] of Feynman’s light microscope gedanken experiment [2] inspired numerous theoretical studies, analyses and discussions [3–16]. An important finding of this experiment, revival of visibility beyond the limit \( dk_i = \pi \), attracted special interest and analysis. Kokorowski et al [6] explained revivals as follows: ‘Beneath an overall decay in coherence with distance, periodic coherence revivals are observed. This shape follows directly from the Fourier transform of the dipole radiation pattern for spontaneous emission. It has also been explained in terms of the ability of a single photon to provide which-path information [1]: the contrast drops to zero when the path separation is approximately equal to the resolving power of an ideal Heisenberg microscope \( d \approx \lambda_i /2 \), with revivals resulting from path ambiguity due to diffraction structure in the image’.

Arsenović et al [13] explained the decrease and revival of visibility by assuming that a wave is associated with each individual atom. The authors found how this wave evolves as the atom travels through the three-gratings Mach–Zehnder interferometer (MZI) and used this wave function to derive the expression for the dependence of visibility on \( dk_i \). The experimental regain of visibility, induced by selecting a subset of atoms from the set of all those transmitted through the third grating, was explained by studying the dependence of visibility on the probability distribution of transferred momenta [14, 15] during photon atom scattering. From these results, Davidović and co-workers [13, 14] derived a general conclusion that individual atoms possess simultaneously wave and particle properties. However, when many atoms are collected, each arriving with a different quantum state, it might so happen that wave properties are not displayed.

In order to make the argumentation by Arsenović et al [13] and Božić et al [14] simpler and clearer, the method previously applied to atoms traveling through an MZI we apply here to atoms traveling through the double slit grating. The MZI was considered in [13–15], since it was used for experimental reasons, in the experiment by Chapman et al [1]. But double slit grating was used in Feynman’s Gedanken light microscope, as well as in an atomic version of Feynman’s light microscope proposed by Scully et al [17]. In this version, two micromaser cavities situated in front of the slits serve to get ‘which way’ information.
2. The influence of a grating and of subsequent photon–atom scattering on the wave function evolution

We start our description with the atom wave function \( \Psi(x, y, t) \) behind a grating at \( y = 0 \) (figure 1). This wave function can be expressed as a solution of the time-dependent free particle Schrödinger equation with the initial condition \( \Psi(x, y, 0^+) \). Assuming that the grating is one-dimensional and extends along the \( x \)-axis, thus neglecting the \( z \)-direction, the wave function behind the grating will read \[ \psi(x, y) = e^{iky} \psi(x, y), \] where \( h \omega = (\hbar^2 k^2 / 2m) \) and \( \psi(x, y) \) is a solution of the corresponding Helmholtz equation. This solution may be written in the Fresnel–Kirchhoff form:

\[
\psi(x, y, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \epsilon^{i(kx - \omega t)} e^{ikx} dx \psi(x', 0^+) \epsilon^{i(kx - \omega t)} e^{-ikx},
\]

for \( y \geq 0 \),

(2.2)

where \( \psi(x, 0^+) \) denotes the function \( \psi(x, y) \) just behind a grating. The form (2.2) is equivalent \[ \text{to the following form:} \]

\[
\psi(x, y) = \epsilon^{i(ky)} \int_{-\infty}^{\infty} \epsilon^{i(kx - \omega t)} e^{ikx} dx \psi(x', 0^+) \epsilon^{i(kx - \omega t)} e^{-ikx}, \quad \text{for } y \geq 0,
\]

(2.3)

where \( c(k_x) \) is the probability amplitude of transverse momentum determined by the incident wave function \( \psi(x, 0^-) \) and the transmission function \( T(x) \) of the grating:

\[
c(k_x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \psi(x, 0^-) e^{-ik_x x}
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx T(x) \psi(x, 0^-) e^{-ik_x x}.
\]

If the slits are totally transparent and walls between the slits totally absorbing, the transmission function is written as a sum of functions \( t_j(x) \)

\[
T(x) = \sum_{j} t_j(x)
\]

such that

\[
t_j(x) = 1, \quad \text{for } x \in [x_j, -\delta/2, x_j + \delta/2],
\]

\[
t_j(x) = 0, \quad \text{for } x \notin [x_j, -\delta/2, x_j + \delta/2].
\]

(2.6)

\( x_j \) denotes the center of the \( j \)-th slit. Function \( c(k_x) \) for such a grating with \( n \) slits reads

\[
c_n(k_x) = \frac{\sqrt{n}}{\sqrt{\pi n\delta}} \frac{\sin(k_x \delta/2)}{k_x} \frac{\sin(k_x d/2)}{k_x d/2}.
\]

(2.7)

To determine the evolution of a wave function in Feynman's microscope we need the function \( c(k_x) \) for \( n = 2 \):

\[
c_2(k_x) = \frac{2}{\sqrt{\pi \delta}} \frac{\sin(k_x \delta/2)}{k_x} cos \frac{k_x d}{2}.
\]

(2.8)

To a very good approximation, the particle motion parallel to the \( y \)-direction can be treated as a classical motion with constant velocity, which means that the relation \( y = vt \) is applicable. We are also going to consider wave functions such that \( c(k_x) \) has nonnegligible values only whenever \( k_x^2 \ll k^2 \approx k^2_1 + k^2_2 \). Taking this into account, it is useful to introduce \[ \text{[18]} \] the time-dependent wave function of the transverse motion

\[
\psi^*(x, t) \equiv \psi(x, y) e^{-iky} \bigg|_{y=vt}.
\]

(2.9)

Substituting (2.3) into (2.9) yields

\[
\psi^*(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk_x c(k_x) e^{ik_x x} e^{-ik^2 t/2m}
\]

(2.10)

from which \( c(k_x) \) acquires the meaning of the probability amplitude of the particle transverse momentum. Therefore,

\[
c(k_x) = c(k_x) e^{-ik^2 t/2m}
\]

(2.11)

will be the time-dependent wave function in the momentum representation, which allows one to express (2.9) as

\[
\psi^*(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk_x c(k_x, t) e^{ik^2 x}.
\]

(2.12)

On the other hand, one can also make an estimation of the value of the wave function when it has evolved far from the grating, i.e. for large values of \( y \), where one may assume
\(x \gg x'\) in (2.2) and neglect the term \(x'^2\) in the exponential under the integral, which yields

\[
\psi(x, y) = \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} e^{i(x/k)2y} \times \int_{-\infty}^{\infty} dx' \psi(x', 0^+) e^{-i(x'/y)k}. \quad (2.13)
\]

By taking into account (2.3) one finds

\[
\psi(x, y) = \sqrt{\frac{\Delta k}{\sqrt{y}}} e^{-i\pi/4} e^{i(kx'/2y)} e^{i(kx/y)2} e^{i(x/k)2y}, \quad (2.14)
\]

which can be used to find the time-dependent transverse wave function in the far field

\[
\psi^r(x, t) = \frac{\Delta k}{\hbar} e^{-i\pi/4} e^{i(m/2\hbar)} e^{i(x/m)2t} \times (x/m). \quad (2.15)
\]

It follows from (2.14) and (2.15) that the \(x\)-dependence of \(|\psi(x, y)|^2\) and \(|\psi^r(x, t)|^2\) in the far field is governed by the functions \(c(kx/y)^2\) and \(c(x/m\hbar)^2\), respectively.

The photon–atom scattering event induced by laser light at a distance \(y\) from the first grating, this form is shifted along the \(y\)-axis by

\[
\Delta x(\Delta k_x) = -\Delta x_0 + y \frac{\Delta k_x}{k} (y - y'_1). \quad (2.20)
\]

If the scattering happens just behind a double-slit grating \((y'_{12} = 0)\), it follows from (2.17) that \(\Delta x_0 = 0\). Consequently, the shift is given by

\[
\Delta x(\Delta k_x) = y k^{-1}. \quad (2.21)
\]

3. The dependence of visibility on \(dk_x\) and the probability distribution of transferred momentum

The probability density that an atom that has undergone a change of momentum \(\Delta k_x\) during the photon–atom scattering process is found at a point \(x\) at the screen situated at a distance \(y\) from the grating is

\[
P(y, \Delta k_x, x) = |\psi(x, y, t)|^2 = \left|\psi_{\Delta k}(x, y)\right|^2. \quad (3.1)
\]

If it were possible to select all atoms that have undergone photon–atom scattering with a given change of transverse momentum \(\Delta k_x\) we would detect characteristic double-slit distribution of atoms on the screen by \(\Delta x(\Delta k_x)\), which is described by the expression

\[
I_{\Delta k}(x, y) = \frac{k}{y} \sin^2\left[\delta/2(kx/y - \Delta k_x)]\right] \times \cos^2\left[\frac{d}{2} \left(\frac{kx}{y} - \Delta k_x\right)\right]. \quad (3.2)
\]

If slits are infinitesimally small \((\delta \ll d)\), the probability amplitude (2.8) may be approximated by its first approximation, which reads

\[
c_2(k_x) \equiv \frac{\sqrt{\delta}}{\sqrt{\pi}} \cos \left(\frac{k_x d}{2}\right). \quad (3.3)
\]

In this case \(I_{\Delta k}\) is approximated by the following simple periodic function, shifted with respect to the point of symmetry by \(\Delta x(\Delta k_x)\):

\[
I_{\Delta k}(x, y) = \frac{k}{y} \sin^2\left[\delta/2(kx/y - \Delta k_x)]\right] \times \left[1 + \cos\left(d \left(\frac{kx}{y} - \Delta k_x\right)\right)\right]. \quad (3.4)
\]

In this case visibility is constant, \(V = 1\).

But in the real experiment one detects atoms which have undergone any change of momentum in a certain interval. In the case of the resonance photon–atom experiment, as in the experiment [1], this interval is \([0, 2k]\), where \(k_{\pi}\) is the wave vector of photons from the laser. Therefore, the number of atoms around a point \((x, y)\) is proportional to the integral of (3.2) over all possible values of \(\Delta k_x\), taking into account the distribution \(P(\Delta k_x)\) of transferred momentum:

\[
I(x, y) = \int_{0}^{2k} I_{\Delta k}(x, y) \times P(\Delta k_x) \times d(\Delta k_x). \quad (3.5)
\]
By substituting (3.2) into (3.5), we find

\[
I(x, y) = \frac{\delta k}{\pi y} \int_0^{2\delta} \sin^2\left[\frac{\delta}{2}(kx/y - \Delta k_i)\right] \cos^2\left[\frac{d}{2}(kx/y - \Delta k_i)\right] P(\Delta k_i) d(\Delta k_i).
\]

This very complicated integral cannot be integrated analytically, but only numerically.

\[\text{(3.6)}\]

4. Visibility in the case of infinitesimally small slits

Fortunately, assuming that slits are infinitesimally small, the analytic integration of the integral in (3.6) is possible for distributions \(P(\Delta k_i)\) which are of physical interest. Using the approximation (3.3) and assuming that \(P(\Delta k_i)\) is normalized to 1, the integral (3.6) is approximated by

\[
I(x, y) \approx \frac{\delta k}{\pi y} \left[1 + V_c(d, k) \cos\left(\frac{kx}{y}\right) + V_s(d, k) \sin\left(\frac{kx}{y}\right)\right]
\]

\[\text{(4.1)}\]

where

\[
V_c(k_i, d) = \int_0^{2\delta} d(\Delta k_i) \cos(d \cdot \Delta k_i) P(\Delta k_i), \quad V_s(k_i, d) = \int_0^{2\delta} d(\Delta k_i) \sin(d \cdot \Delta k_i) P(\Delta k_i).
\]

\[\text{(4.2)}\]

Visibility and the phase of the interference pattern described by (4.1) are given by

\[
V(k_i, d) = \sqrt{V_c^2(k_i, d) + V_s^2(k_i, d)}, \quad \tan \phi = \frac{V_c(k_i, d)}{V_s(k_i, d)}.
\]

\[\text{(4.3)}\]

For uniform distribution over the interval \([0, 2k_i]\), \(P_u(\Delta k_i) = 1/2k_i\), from (4.2), we find

\[
V_{c,u}(k_i, d) = \frac{\sin(2d k_i)}{2d k_i}, \quad V_{s,u}(k_i, d) = \frac{1 - \cos(2d k_i)}{2d k_i}.
\]

\[\text{(4.4)}\]

By combining (4.3) and (4.4), we obtain visibility and phase for the uniform distribution of transferred momentum:

\[
V_k(k_i, d) = \left|\frac{\sin(d k_i)}{d k_i}\right|, \quad \phi_k = d k_i.
\]

The graph of visibility for the uniform distribution is presented in figure 2. This is an oscillatory function of \(d k_i\) with decreasing maxima. Zeros of this function are for \(d k_i = \pi, 2\pi, 3\pi \ldots\). This is understandable. For \(d k_i = \pi, 2\pi, 3\pi, \ldots\), the integration in (4.2) is over integer periods of the function of \(\Delta k_i\), whose period is equal to \(2\pi/d\).

Visibility is a product of \(P(\Delta k_i) = 1/2k_i\), which is inversely proportional to \(k_i\) and of an integral which does not increase with \(k_i\) (but oscillates) despite the fact that the range of integration is proportional to \(k_i\).

If photons are resonantly scattered by atoms which just passed through the grating, the distribution of transferred momentum to the atoms, \(P_M(\Delta k_i)\), was determined by Mandel [19]. It reads [19, 20]

\[
P_M(\Delta k_i) = \left(\frac{3}{8k_i}\right) \left[1 + \left(\frac{\Delta k_i}{k_i}\right)^2\right].
\]

\[\text{(4.6)}\]

Therefore, in order to determine visibility in this case it is necessary to evaluate the integrals

\[
V_{c,M}(k_i, d) = \frac{3}{8k_i} \int_0^{2\delta} d(\Delta k_i) \cos(d \cdot \Delta k_i) \times \left(2 - 2\frac{\Delta k_i}{k_i} + \left(\frac{\Delta k_i}{k_i}\right)^2\right),
\]

\[
V_{s,M}(k_i, d) = \frac{3}{8k_i} \int_0^{2\delta} d(\Delta k_i) \sin(d \cdot \Delta k_i) \times \left(2 - 2\frac{\Delta k_i}{k_i} + \left(\frac{\Delta k_i}{k_i}\right)^2\right).
\]

\[\text{(4.7)}\]

The above integrals may be computed analytically. The result is

\[
V_{c,M}(d, k_i) = \frac{2}{d^3 k_i^2} \cos d k_i 	imes \left[2 d k_i \cos d k_i + 2 d^2 k_i^2 \sin 2 d k_i\right],
\]

\[
V_{s,M}(d, k_i) = \frac{2}{d^3 k_i^2} \sin d k_i \times \left[2 d k_i \cos d k_i + 2 d^2 k_i^2 \sin 2 d k_i\right].
\]

\[\text{(4.8)}\]

Using the above expressions we find that in the case of very small slits, the intensity at the screen, at distance \(y\) from a
Figure 3. The graphs of the function $I(x, y)$ given in (3.6) for six values of the slits’ separation $d$ and for Mandel’s distribution $P_M(\Delta k_x)$. The values of $d$ in figures 3(a)–(f) are $d = 1.2\delta, 5.1\delta, 7.9\delta, 11.3\delta, 14.3\delta$ and $20.3\delta$, respectively. The values of other quantities are $y = 0.65\, \text{m}$, $\delta = 0.5 \times 10^{-7}\, \text{m}$, $k_i = 1.0621 \times 10^7\, \text{m}^{-1}$ and $k = 5.09067 \times 10^{11}\, \text{liter m}^{-1}$.

5. The influence of the finite width of the slits on visibility

In order to investigate the influence of the finite width of the slits on the interference curve and visibility, we shall evaluate the grating, a simple periodic function (4.1) of the coordinate $x$,

$$V_M(k_i, d) = \frac{3}{2} \frac{1}{dk_i} \left| \sin(dk_i) + \frac{1}{dk_i} \cos(dk_i) \right|,$$

$$\phi_M = dk_i$$

The function (4.9) is identical to the function obtained by Arsenović et al [13] for visibility of interference in an MZI, which agrees very well with the experimental curve [1]. Visibility curve (4.9) is graphically presented in figure 2.

Figure 4. Visibility as a function of $d$, evaluated from the definition (5.4), using numerically evaluated $I(x, y)$ defined in (3.6), for $P(\Delta k_x) = P_M(\Delta k_x)$ given in (4.6). The values of $d$ below the black points in increasing order correspond to the values of $d$ for which graphs are presented in figures 3(a)–(f).
and (5.4) distance $y$ Therefore, when $\delta$ function of the coordinate $x$ In this case, as it may be seen from the following expressions, $I_{\text{M}}(d, x, \delta, k) \approx \sin \left(\frac{k x}{y}\right) \cdot V_{\text{M}, \text{M}} \left(\frac{k x}{y}\right)$. (5.2)

$$V_{\text{M}, \text{M}} \left(\frac{k x}{y}\right) = \int_{0}^{2\pi} d(\Delta k_{x}) \cos(d \cdot \Delta k_{x}) \times \left(1 - \frac{\delta^{2}}{6 \cdot 4} \left(\frac{k x}{y} - \Delta k_{x}\right)^{2} \right) \left(\frac{3}{8k_{i}}\right) \times \left(2 - 2 \frac{\Delta k_{x}}{k_{i}} + \left(\frac{\Delta k_{x}}{k_{i}}\right)^{2}\right),$$

$$V_{\text{M}, \text{M}} \left(\frac{k x}{y}\right) = \int_{0}^{2\pi} d(\Delta k_{x}) \sin(d \cdot \Delta k_{x}) \times \left(1 - \frac{\delta^{2}}{6 \cdot 4} \left(\frac{k x}{y} - \Delta k_{x}\right)^{2} \right) \left(\frac{3}{8k_{i}}\right) \times \left(2 - 2 \frac{\Delta k_{x}}{k_{i}} + \left(\frac{\Delta k_{x}}{k_{i}}\right)^{2}\right).$$

Therefore, when $\delta$ is not negligible, intensity at the screen at distance $y$ is not a periodic function of $x$, but quasi-periodic.

The function $I(x, y)$, evaluated by numerical integration of the function under the integral sign in (3.6) with Mandel distribution $d$ distribution $4.6$, is graphically presented in figure 3 for six values of the slits’ distance $d$, keeping the slits’ width constant. One clearly sees that $I(x, y)$ is a quasi-periodic function with amplitudes of oscillations strongly dependent on $d$. For this quasi-periodic function $I(x, y)$, it seems appropriate to define visibility by taking into account the central maximum $I_{\text{max}}$ (which is the largest of all local maxima), its first neighboring maximum $I_{2\text{max}}$, and the minimum $I_{\text{min}}$ in between these two maxima. So, we use the following definition of visibility:

$$V = \frac{(I_{\text{max}} + I_{2\text{max}}) / 2 - I_{\text{min}}}{(I_{\text{max}} + I_{2\text{max}}) / 2 + I_{\text{min}}}. \quad (5.4)$$

We determined $I_{\text{max}}$, $I_{2\text{max}}$, and $I_{\text{min}}$ from the numerically evaluated intensity given in (3.6) for a large number of values of the slits distance $d$, keeping $\delta$, $k_{i}$, $k$ fixed. Visibility evaluated in this way is presented in figure 4. By comparing figures 2 and 4, we see that visibility, which we evaluated by taking into account oscillations around the central maximum of (3.6), turns out to have the same graphical form as (4.9), obtained from the first order approximation (4.1) with respect to $\delta/d$ of $I(x, y)$.

### 6. Conclusions

By determining the time evolution of the wave function of the single atom in Feynman’s atomic light microscope, we found the functional dependence of visibility of interference on the product $dk_{i}$. In the case of infinitesimally small slit widths an analytic expression for visibility was obtained. By numerical simulation we found that the dependence described by this analytic expression is valid for nonnegligible slit widths, too. Since revivals exist for infinitesimally small slits as well as when slits have finite width, we conclude that the existence of revivals does not depend on the width of the slits and...
diffraction structure. So, the assertion that [6] ‘revivals result from path ambiguity due to diffraction structure in the image’ is questionable.

The expression obtained here for visibility is the same as that found by Arsenović et al [13] for visibility of interference in a Mach Zehnder atomic interferometer. The MZI was used in the experimental realization [1] of Feynman’s atomic light microscope. The theoretical description exposed here of Feynmann’s double slit light microscope supports de Broglie’s understanding of wave–particle duality [21]. According to de Broglie’s interpretation wave and particle properties are coexistent (compatible). A comparison of the reasoning leading to this conclusion with Bohr’s argument [22] that wave and particle properties are complementary is presented in table 1.

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