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Visibility of interference in Feynman's atomic light microscope

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Abstract
The visibility of the interference pattern in Feynman's atomic light microscope is studied theoretically using the evolution of the wave function of individual atoms passing through a double slit grating and interacting with laser light (of the wave vector k_i). Assuming that the slits' width (δ) is much smaller than the slits' distance (d), the expressions for visibility as functions of dk_i were derived for uniform and for Mandel distribution of transferred momentum during photon atom resonance scattering. The decreasing of oscillations of visibility with increasing dk_i was explained taking into account the wave functions of individual atoms and the statistics of transverse momentum transferred to atoms. The influence of nonnegligible slits' width on visibility was studied numerically. It is found that revivals are present for infinitesimally wide slits as well as when slits have finite width.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The realization by Chapman *et al* [1] of Feynman's light microscope gedanken experiment [2] inspired numerous theoretical studies, analyses and discussions [3–16]. An important finding of this experiment, revival of visibility beyond the limit $dk_i = \pi$, attracted special interest and analysis. Kokorowski *et al* [6] explained revivals as follows: 'Beneath an overall decay in coherence with distance, periodic coherence revivals are observed. This shape follows directly from the Fourier transform of the dipole radiation pattern for spontaneous emission. It has also been explained in terms of the ability of a single photon to provide which-path information [1]: the contrast drops to zero when the path separation is approximately equal to the resolving power of an ideal Heisenberg microscope $d \approx \lambda_i/2$, with revivals resulting from path ambiguity due to diffraction structure in the image'.

Arsenović *et al* [13] explained the decrease and revival of visibility by assuming that a wave is associated with each individual atom. The authors found how this wave evolves as the atom travels through the three-gratings Mach–Zehnder interferometer (MZI) and used this wave function to derive the expression for the dependence of visibility on dk_i. The experimental regain of visibility, induced by selecting a subset of atoms from the set of all those transmitted through the third grating, was explained by studying the dependence of visibility on the probability distribution of transferred momenta [14, 15] during photon atom scattering. From these results, Davidović and co-workers [13, 14] derived a general conclusion that individual atoms possess simultaneously wave and particle properties. However, when many atoms are collected, each arriving with a different quantum state, it might so happen that wave properties are not displayed.

In order to make the argumentation by Arsenović *et al* [13] and Božić *et al* [14] simpler and clearer, the method previously applied to atoms traveling through an MZI we apply here to atoms traveling through the double slit grating. The MZI was considered in [13–15], since it was used for experimental reasons, in the experiment by Chapman *et al* [1]. But double slit grating was used in Feynman's Gedanken light microscope, as well as in an atomic version of Feynman's light microscope proposed by Scully *et al* [17]. In this version, two micromaser cavities situated in front of the slits serve to get 'which way' information.

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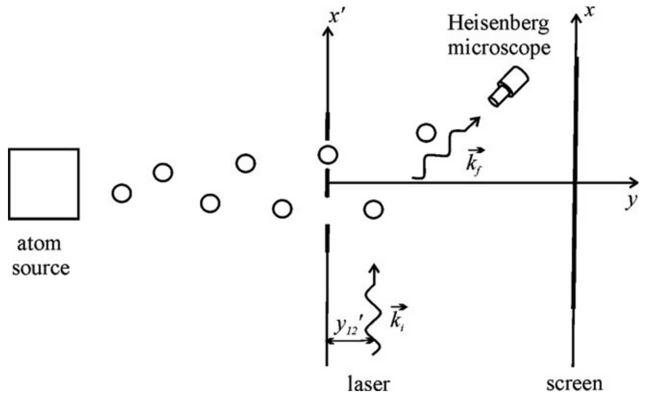

Figure 1. The scheme of Feynman's atomic microscope. After passing through a grating, atoms undergo resonance scattering with photons from a laser beam parallel to a grating at a distance y'_{12}. Atoms are detected at the screen at the distance y from the grating.

The paper by Scully *et al* [17] induced a vivid discussion [4] on the origin of the disappearance of interference in 'which-way' experiments: recoil (Heisenberg's uncertainty relations) versus decoherence (correlations between the measuring apparatus and the systems being observed). We hope that our study of visibility presented here might be useful for this discussion.

2. The influence of a grating and of subsequent photon–atom scattering on the wave function evolution

We start our description with the atom wave function $\Psi(x, y, t)$ behind a grating at $y = 0$ (figure 1). This wave function can be expressed as a solution of the time-dependent free particle Schrödinger equation with the initial condition $\Psi(x, y = 0^+, 0)$. Assuming that the grating is one-dimensional and extends along the x-axis, thus neglecting the z-direction, the wave function behind the grating will read [13]

$$\Psi(x, y, t) = \mathrm{e}^{\mathrm{i}ky}\mathrm{e}^{-\mathrm{i}\omega t}\psi(x, y), \tag{2.1}$$

where $\hbar\omega = (\hbar^2k^2/2m)$ and $\psi(x, y)$ is a solution of the corresponding Helmholtz equation. This solution may be written in the Fresnel–Kirhoff form:

$$\psi(x, y) = \sqrt{\frac{k}{2\pi y}}\mathrm{e}^{-\mathrm{i}\pi/4}\mathrm{e}^{\mathrm{i}ky}\int_{-\infty}^{+\infty} \mathrm{d}x'\psi(x', 0^+)\mathrm{e}^{\mathrm{i}(k(x-x')^2/2y)}, \quad \text{for } y \geqslant 0, \tag{2.2}$$

where $\psi(x, 0^+)$ denotes the function $\psi(x, y)$ just behind a grating. The form (2.2) is equivalent [18] to the following form:

$$\psi(x, y) = \frac{\mathrm{e}^{\mathrm{i}ky}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{d}k_x c(k_x)\, \mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{-\mathrm{i}k_x^2 y/2k}, \quad \text{for } y \geqslant 0, \tag{2.3}$$

where $c(k_x)$ is the probability amplitude of transverse momentum determined by the incident wave function $\psi(x, 0^-)$ and the transmission function $T(x)$ of the grating:

$$c(k_x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}x\, \psi(x, 0^+)\, \mathrm{e}^{-\mathrm{i}k_x x} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{d}x\, T(x)\psi(x, 0^-)\, \mathrm{e}^{-\mathrm{i}k_x x}. \tag{2.4}$$

If the slits are totally transparent and walls between the slits totally absorbing, the transmission function is written as a sum of functions $t_j(x)$

$$T(x) = \sum_{x_j} t_j(x) \tag{2.5}$$

such that

$$\begin{aligned} t_j(x) &= 1, \quad \text{for} \quad x \in [x_j, -\delta/2,\ x_j + \delta/2], \\ t_j(x) &= 0, \quad \text{for} \quad x \notin [x_j, -\delta/2,\ x_j + \delta/2]. \end{aligned} \tag{2.6}$$

x_j denotes the center of the jth slit. Function $c(k_x)$ for such a grating with n slits reads

$$c_n(k_x) = \frac{\sqrt{2}}{\sqrt{\pi n\delta}}\frac{\sin(k_x\delta/2)}{k_x}\frac{\sin(k_x dn/2)}{\sin(k_x d/2)}. \tag{2.7}$$

To determine the evolution of a wave function in Feynman's microscope we need the function $c(k_x)$ for $n = 2$:

$$c_2(k_x) = \frac{2}{\sqrt{\pi\delta}}\frac{\sin(k_x\delta/2)}{k_x}\cos\frac{k_x d}{2}. \tag{2.8}$$

To a very good approximation, the particle motion parallel to the y-direction can be treated as a classical motion with constant velocity, which means that the relation $y = vt$ is applicable. We are also going to consider wave functions such that $c(k_x)$ has nonnegligible values only whenever $k_x^2 \ll k_y^2 \approx k^2 \equiv k_x^2 + k_y^2$. Taking this into account, it is useful to introduce [18] the time-dependent wave function of the transverse motion

$$\psi^{\mathrm{tr}}(x, t) \equiv \psi(x, y)\, \mathrm{e}^{-\mathrm{i}ky}\,\big|_{y=vt}\,. \tag{2.9}$$

Substituting (2.3) into (2.9) yields

$$\psi^{\mathrm{tr}}(x, t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}k_x c(k_x)\, \mathrm{e}^{\mathrm{i}k_x x}\, \mathrm{e}^{-\mathrm{i}\hbar k_x^2 t/2m} \tag{2.10}$$

from which $c(k_x)$ acquires the meaning of the probability amplitude of the particle transverse momentum. Therefore,

$$c(k_x, t) = c(k_x)\, \mathrm{e}^{-\mathrm{i}\hbar k_x^2 t/2m} \tag{2.11}$$

will be the time-dependent wave function in the momentum representation, which allows one to express (2.9) as

$$\psi^{\mathrm{tr}}(x, t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}k_x c(k_x, t)\, \mathrm{e}^{\mathrm{i}k_x x}. \tag{2.12}$$

On the other hand, one can also make an estimation of the value of the wave function when it has evolved far from the grating, i.e. for large values of y, where one may assume

$x \gg x'$ in (2.2) and neglect the term x'^2 in the exponential under the integral, which yields

$$\psi(x, y) = \sqrt{\frac{k}{2\pi y}}\, \mathrm{e}^{-\mathrm{i}\pi/4}\, \mathrm{e}^{\mathrm{i}ky}\, \mathrm{e}^{\mathrm{i}(x^2k/2y)} \times \int_{-\infty}^{\infty} \mathrm{d}x' \psi(x', 0^+)\, \mathrm{e}^{-\mathrm{i}(x'x/y)k}. \quad (2.13)$$

By taking into account (2.3) one finds

$$\psi(x, y) = \frac{\sqrt{k}}{\sqrt{y}}\, \mathrm{e}^{-\mathrm{i}\pi/4}\, \mathrm{e}^{\mathrm{i}(kx^2/2y)} c\left(\frac{kx}{y}\right) \mathrm{e}^{\mathrm{i}ky}, \quad (2.14)$$

which can be used to find the time-dependent transverse wave function in the far field

$$\psi^{\mathrm{tr}}(x, t) = \sqrt{\frac{m}{\hbar t}} \mathrm{e}^{-\mathrm{i}\pi/4} \mathrm{e}^{\mathrm{i}x^2 m/2\hbar t} c\left(x \frac{m}{\hbar t}\right). \quad (2.15)$$

It follows from (2.14) and (2.15) that the x-dependence of $|\psi(x, y)|^2$ and $|\psi^{\mathrm{tr}}(x, t)|^2$ in the far field is governed by the functions $|c(kx/y)|^2$ and $|c(xm/\hbar t)|^2$, respectively.

The photon–atom scattering event induced by laser light at a distance y'_{12} from a grating leads to a change of the atomic transverse momentum, Δk_x, and therefore to a shift of the wave function in the momentum representation. Hence, after an atom absorbs and re-emits again a photon somewhere along the x-axis at a time t'_{12} and a distance $y'_{12} = vt'_{12} = (\hbar k/m)t'_{12}$ from the grating, the transverse atomic wave function for $y > y'_{12}$ takes the form [13]

$$\psi^{\mathrm{tr}}_{\Delta k_x}(x, y) = \mathrm{e}^{\mathrm{i}\Delta k_x (x+\Delta x_0) - \mathrm{i}\Delta k_x^2 y/k} \times \int_{-\infty}^{+\infty} \mathrm{d}k'_x c(k'_x)\, \mathrm{e}^{-\mathrm{i}k'^2_x y/2k}\, \mathrm{e}^{\mathrm{i}k'_x (x+\Delta x_0 - y\Delta k_x/k)}, \quad (2.16)$$

where

$$\Delta x_0 = \frac{\Delta k_x \hbar t'_{12}}{m} = \frac{\Delta k_x y'_{12}}{k}. \quad (2.17)$$

From (2.16) one obtains the space-dependent wave function

$$\psi_{\Delta k_x}(x, y) = \mathrm{e}^{\mathrm{i}ky} \psi^{\mathrm{tr}}_{\Delta k_x}(x, y) = \frac{\mathrm{e}^{\mathrm{i}ky}}{\sqrt{2\pi}} \cdot \mathrm{e}^{\mathrm{i}\Delta k_x (x+\Delta x_0) - \mathrm{i}\Delta k_x^2 y/k} \times \int_{-\infty}^{+\infty} \mathrm{d}k'_x c(k'_x)\, \mathrm{e}^{-\mathrm{i}k'^2_x y/2k}\, \mathrm{e}^{\mathrm{i}k'_x (x+\Delta x_0 - y\Delta k_x/k)}. \quad (2.18)$$

In analogy to the approximation (2.14) for (2.3), the wave function (2.18) can also be approximated in the far field by the simpler form

$$\psi_{\Delta k_x}(x, y) = \mathrm{e}^{\mathrm{i}ky} \frac{\sqrt{k}}{\sqrt{y}} \mathrm{e}^{-\pi/4} \mathrm{e}^{-\mathrm{i}(\Delta k_x^2 y/2k)} \mathrm{e}^{\mathrm{i}(k(x+\Delta x_0)^2/2y)} \times c\left(\frac{k(x+\Delta x_0)}{y} - \Delta k_x\right). \quad (2.19)$$

From this expression it follows that the overall form of the modulus square of the wave function $|\psi_{\Delta k_x}(x, y)|^2$ is the same as the form of $|\psi(x, y)|^2 = |\psi_0(x, y)|^2$. But for $\Delta k_x \neq 0$, at the distance $y > y'_{12}$ from the first grating, this form is shifted along the x-axis by

$$\Delta x(\Delta k_x) = -\Delta x_0 + y\frac{\Delta k_x}{k} = \frac{\Delta k_x}{k}(y - y'_{12}). \quad (2.20)$$

If the scattering happens just behind a double-slit grating ($y'_{12} = 0$), it follows from (2.17) that $\Delta x_0 = 0$. Consequently, the shift is given by

$$\Delta x(\Delta k_x) = y\frac{\Delta k_x}{k}. \quad (2.21)$$

3. The dependence of visibility on dk_i and on the probability distribution of transferred momentum

The probability density that an atom that has undergone a change of momentum Δk_x during the photon–atom scattering process is found at a point x at the screen situated at a distance y from the grating is

$$P(y, \Delta k_x, x) = |\Psi(x, y, t)|^2 = \left|\psi_{\Delta k_x}(x, y)\right|^2. \quad (3.1)$$

If it were possible to select all atoms that have undergone photon–atom scattering with a given change of transverse momentum Δk_x we would detect characteristic double-slit distribution of atoms on the screen shifted by $\Delta x(\Delta k_x)$, which is described by the expression

$$I_{\Delta k_x}(x, y) = \frac{k}{y}\frac{\delta}{\pi}\frac{\sin^2[\delta/2(kx/y - \Delta k_x)]}{[\delta/2(kx/y - \Delta k_x)]^2} \times \cos^2\left[\frac{d}{2}\left(\frac{kx}{y} - \Delta k_x\right)\right]. \quad (3.2)$$

If slits are infinitesimally small ($\delta \ll d$), the probability amplitude (2.8) may be approximated by its first approximation, which reads

$$c_2(k_x) \cong \frac{\sqrt{\delta}}{\sqrt{\pi}} \cos\frac{k_x d}{2}. \quad (3.3)$$

In this case $I_{\Delta k_x}$ is approximated by the following simple periodic function, shifted with respect to the point of symmetry by $\Delta x(\Delta k_x)$:

$$I_{\Delta k_x}(x, y) = \frac{k}{y}\frac{\delta}{\pi}\left[1 + \cos\left(d\left(\frac{kx}{y} - \Delta k_x\right)\right)\right]. \quad (3.4)$$

In this case visibility is constant, $V = 1$.

But in the real experiment one detects atoms which have undergone any change of momentum in a certain interval. In the case of the resonance photon–atom experiment, as in the experiment [1], this interval is $[0, 2k_i]$, where k_i is the wave vector of photons from the laser. Therefore, the number of atoms around a point (x, y) is proportional to the integral of (3.2) over all possible values of Δk_x, taking into account the distribution $P(\Delta k_x)$ of transferred momentum:

$$I(x, y) = \int_0^{2k_i} I_{\Delta k_x}(x, y) \times P(\Delta k_x) \times d(\Delta k_x). \quad (3.5)$$

By substituting (3.2) into (3.5), we find

$$I(x,y)=\frac{\delta}{\pi}\frac{k}{y}\int_0^{2k_i}\frac{\sin^2[\delta/2(kx/y-\Delta k_x)]}{[\delta/2(kx/y-\Delta k_x)]^2}\times\cos^2\left[\frac{d}{2}\left(\frac{kx}{y}-\Delta k_x\right)\right]\times P(\Delta k_x)d(\Delta k_x). \tag{3.6}$$

This very complicated integral cannot be integrated analytically, but only numerically.

4. Visibility in the case of infinitesimally small slits

Fortunately, assuming that slits are infinitesimally small, the analytic integration of the integral in (3.6) is possible for distributions $P(\Delta k_x)$ which are of physical interest. Using the approximation (3.3) and assuming that $P(\Delta k_x)$ is normalized to 1, the integral (3.6) is approximated by

$$\begin{aligned}I(x,y)&\cong\frac{\delta}{\pi}\frac{k}{y}\left[1+V_{\rm c}(d,k_i)\cos\left(d\frac{kx}{y}\right)+V_{\rm s}(d,k_i)\sin\left(d\frac{kx}{y}\right)\right]\\&\equiv\frac{\delta}{\pi}\frac{k}{y}\left[1+V\cos\left(d\frac{kx}{y}-\phi\right)\right],\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}V_{\rm c}(k_i,d)&=\int_0^{2k_i}{\rm d}(\Delta k_x)\cos(d\cdot\Delta k_x)^*P(\Delta k_x),\ V_{\rm s}(k_i,d)\\&=\int_0^{2k_i}{\rm d}(\Delta k_x)\sin(d\cdot\Delta k_x)^*P(\Delta k_x).\end{aligned} \tag{4.2}$$

Visibility and the phase of the interference pattern described by (4.1) are given by

$$V(k_i,d)=\sqrt{V_{\rm c}^2(k_i,d)+V_{\rm s}^2(k_i,d)},\qquad tg\phi=\frac{V_{\rm s}(k_i,d)}{V_{\rm c}(k_i,d)}. \tag{4.3}$$

For uniform distribution over the interval $[0, 2k_i]$, $P_{\rm u}(\Delta k_x)=1/2k_i$, from (4.2), we find

$$V_{\rm c,u}(k_i,d)=\frac{\sin(2dk_i)}{2dk_i},\qquad V_{\rm s,u}(k_i,d)=\frac{1-\cos(2dk_i)}{2dk_i}. \tag{4.4}$$

By combining (4.3) and (4.4), we obtain visibility and phase for the uniform distribution of transferred momentum:

$$V_{\rm u}(k_i,d)=\frac{|\sin(dk_i)|}{dk_i},\qquad \phi_{\rm u}=dk_i. \tag{4.5}$$

The graph of visibility for the uniform distribution is presented in figure 2. This is an oscillatory function of dk_i with decreasing maxima. Zeros of this function are for $dk_i=\pi,\pi,3\pi$ ....This is understandable. For $dk_i=\pi,2\pi,3\pi,\ldots,$ the integration in (4.2) is over integer periods of the function of Δk_x whose period is equal to $2\pi/d$. Visibility is a product of $P(\Delta k_x)=1/2k_i$, which is inversely proportional to k_i and of an integral which does not increase

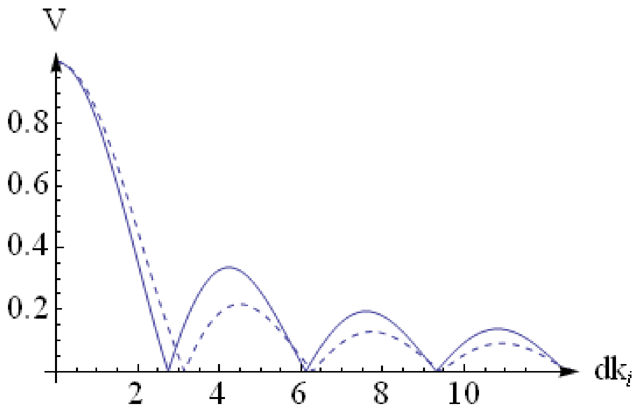

Figure 2. The dependence of visibility on dk_i for uniform (- - -, equation (4.5)) and Mandel's distribution (——, equation (4.9)) of transferred momentum, assuming that $\delta \ll d$ and $y'_{12}=0$.

with k_i (but oscillates) despite the fact that the range of integration is proportional to k_i.

If photons are resonantly scattered by atoms which just passed through the grating, the distribution of transferred momentum to the atoms, $P_{\rm M}(\Delta k_x)$, was determined by Mandel [19]. It reads [19, 20]

$$P_{\rm M}(\Delta k_x)=\left(\frac{3}{8k_i}\right)\left[1+\left(1-\frac{\Delta k_x}{k_i}\right)^2\right]. \tag{4.6}$$

Therefore, in order to determine visibility in this case it is necessary to evaluate the integrals

$$\begin{aligned}V_{\rm c,M}(k_i,d)&=\frac{3}{8k_i}\int_0^{2k_i}{\rm d}(\Delta k_x)\cos(d\cdot\Delta k_x)\times\left(2-2\frac{\Delta k_x}{k_i}+\left(\frac{\Delta k_x}{k_i}\right)^2\right),\\V_{\rm s,M}(k_i,d)&=\frac{3}{8k_i}\int_0^{2k_i}{\rm d}(\Delta k_x)\sin(d\cdot\Delta k_x)\times\left(2-2\frac{\Delta k_x}{k_i}+\left(\frac{\Delta k_x}{k_i}\right)^2\right).\end{aligned} \tag{4.7}$$

The above integrals may be computed analytically. The result is

$$\begin{aligned}V_{\rm c,M}(d,k_i)&=\frac{2}{d^3k_i^2}\cos dk_i\times\left[2dk_i\cos dk_i-2\sin dk_i+2d^2k_i^2\sin 2dk_i\right],\\V_{\rm s,M}(d,k_i)&=\frac{2}{d^3k_i^2}\sin dk_i\times\left[2dk_i\cos dk_i-2\sin dk_i+2d^2k_i^2\sin 2dk_i\right].\end{aligned} \tag{4.8}$$

Using the above expressions we find that in the case of very small slits, the intensity at the screen, at distance y from a

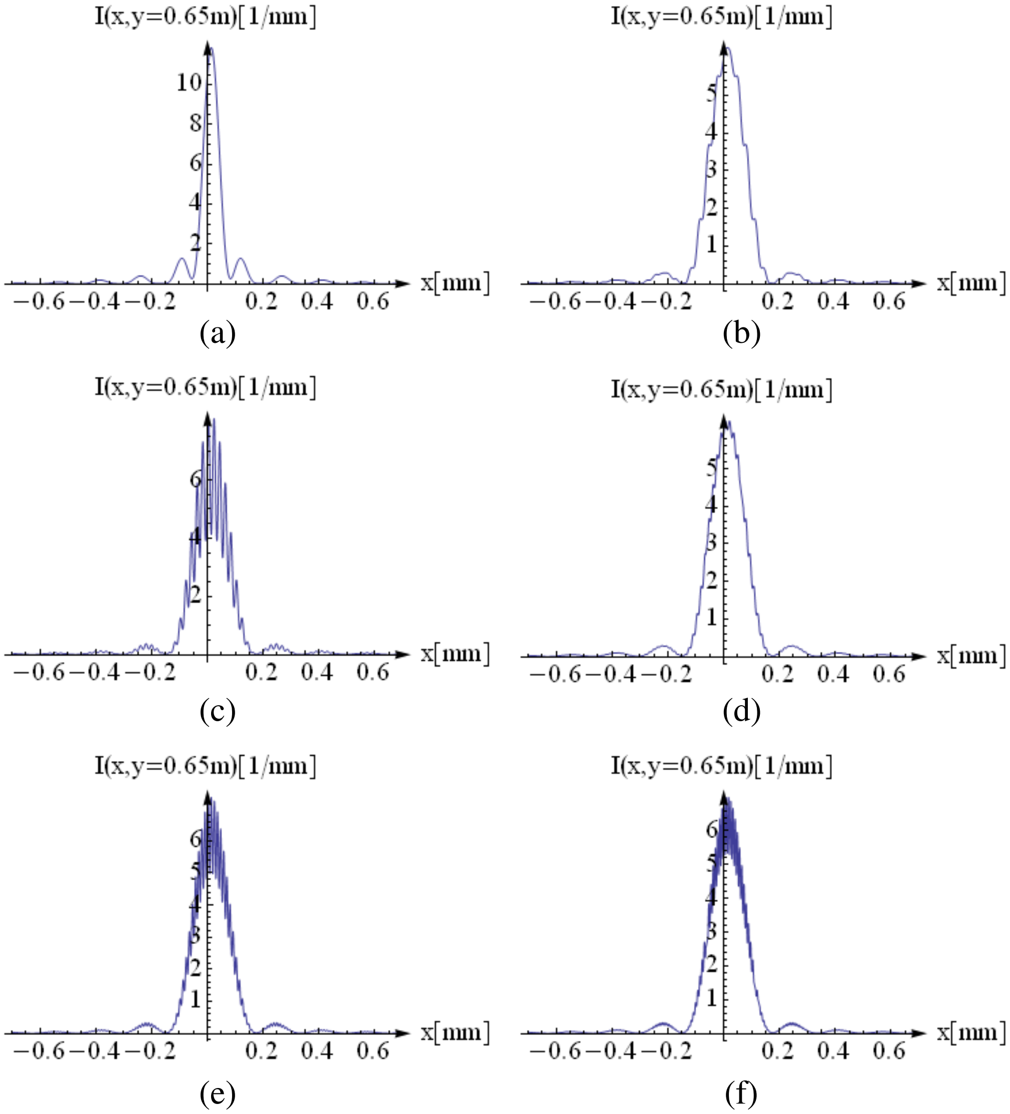

Figure 3. The graphs of the function $I(x, y)$ given in (3.6) for six values of the slits' separation d and for Mandel's distribution $P_M(\Delta k_x)$. The values of d in figures 3(a)–(f) are $d = 1.2\delta$, 5.1δ, 7.9δ, 11.3δ, 14.3δ and 20.3δ, respectively. The values of other quantities are $y = 0.65$ m, $\delta = 0.5 \times 10^{-7}$ m, $k_i = 1.0621 \times 10^7$ 1 m^{-1} and $k = 5.09067 \times 10^{11}$ liter m^{-1}.

grating, is a simple periodic function (4.1) of the coordinate x, with visibility and phase given by

$$V_M(k_i, d) = \frac{3}{2}\frac{1}{dk_i}\left|\left(1 - \frac{1}{d^2k_i^2}\right)\sin(dk_i) + \frac{1}{dk_i}\cos(dk_i)\right|,$$

$$\phi_M = dk_i \qquad (4.9)$$

The function (4.9) is identical to the function obtained by Arsenović *et al* [13] for visibility of interference in an MZI, which agrees very well with the experimental curve [1]. Visibility curve (4.9) is graphically presented in figure 2.

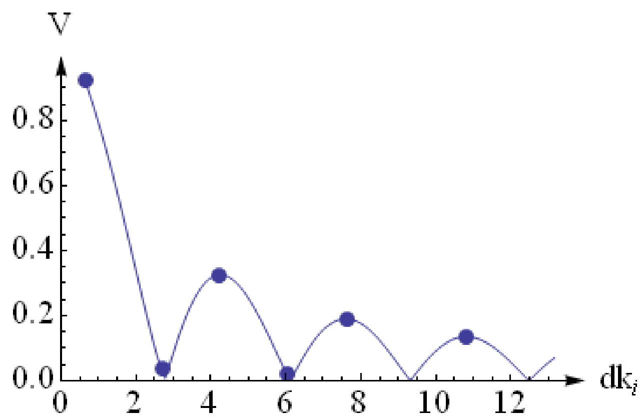

Figure 4. Visibility as a function of d, evaluated from the definition (5.4), using numerically evaluated $I(x, y)$ defined in (3.6), for $P(\Delta k_x) = P_M(\Delta k_x)$ given in (4.6). The values of d below the black points in increasing order correspond to the values of d for which graphs are presented in figures 3(a)–(f).

5. The influence of the finite width of the slits on visibility

In order to investigate the influence of the finite width of the slits on the interference curve and visibility, we shall evaluate

Table 1. Summary and comparison of two different interpretations of the cause of quantum interference and of the loss and revivals of it.

	The Bohr-like interpretation	The de Broglie-like interpretation
Cause of interference	Impossibility to get 'which way' information	Particle motion is influenced by the evolution of its wave function
Cause of the decrease and loss of visibility	Availability of 'which way' information	The shift of the atomic wave function which depends on the change of atoms' transverse momentum, during a photon atom scattering event. The statistical distribution of these shifts in many such events causes visibility to be an oscillatory function of dk_i with decreasing amplitude.
Cause of revivals	Path ambiguity due to diffraction structure in the image	The visibility is an oscillatory function of dk_i with decreasing amplitude of oscillations. So, the initial decreasing part belongs to the first oscillation, the first revival is the second oscillation, the second revival is the third oscillation, etc.
Conclusion about the nature of duality	Wave and particle properties are complementary	Wave and particle properties are coexistent (compatible).

the intensity $I(x, y)$ by approximating the function $c_2(k_x)$ by the function

$$c_2(k_x) \cong \frac{\sqrt{\delta}}{\sqrt{\pi}}\left[1-\frac{1}{6}\left(\frac{k_x\delta}{2}\right)^2\right]\cos\frac{k_x d}{2}. \tag{5.1}$$

In this case, as it may be seen from the following expressions, there are many more integrals to be evaluated. One also sees that the resulting function will not be a simple periodic function of the coordinate x, but quasi-periodic.

$$I(x, y) = \frac{k}{y}\frac{2}{\pi\delta}\left[1+\cos\left(d\frac{kx}{y}\right)\cdot V_{\mathrm{c,M}}\left(d, k_i, \delta, \frac{kx}{y}\right)\right.$$
$$\left.+\sin\left(d\frac{kx}{y}\right)\cdot V_{\mathrm{s,M}}\left(d, k_i, \delta, \frac{kx}{y}\right)\right], \tag{5.2}$$

$$V_{\mathrm{c,M}}\left(d, k_i, \delta, \frac{kx}{y}\right) = \int_0^{2k_i}\mathrm{d}(\Delta k_x)\cos(d\cdot\Delta k_x)$$
$$\times\left(1-\frac{\delta^2}{6\cdot 4}\left(\frac{kx}{y}-\Delta k_x\right)^2\right)^2\left(\frac{3}{8k_i}\right)$$
$$\times\left(2-2\frac{\Delta k_x}{k_i}+\left(\frac{\Delta k_x}{k_i}\right)^2\right),$$

$$V_{\mathrm{s,M}}\left(d, k_i, \delta, \frac{kx}{y}\right) = \int_0^{2k_i}\mathrm{d}(\Delta k_x)\sin(d\cdot\Delta k_x)$$
$$\times\left(1-\frac{\delta^2}{6\cdot 4}\left(\frac{kx}{y}-\Delta k_x\right)^2\right)^2\left(\frac{3}{8k_i}\right)$$
$$\times\left(2-2\frac{\Delta k_x}{k_i}+\left(\frac{\Delta k_x}{k_i}\right)^2\right). \tag{5.3}$$

Therefore, when δ is not negligible, intensity at the screen at distance y is not a periodic function of x, but quasi-periodic. The function $I(x, y)$, evaluated by numerical integration of the function under the integral sign in (3.6) with Mandel distribution (4.6), is graphically presented in figure 3 for six values of the slits' distance d, keeping the slits' width constant. One clearly sees that $I(x, y)$ is a quasi-periodic function with amplitudes of oscillations strongly dependent on d. For this quasi-periodic function $I(x, y)$, it seems appropriate to define visibility by taking into account the central maximum $I_{1\max}$ (which is the largest of all local maxima), its first neighboring maximum $I_{2\max}$, and the minimum $I_{\min}$ in between these two maxima. So, we use the following definition of visibility:

$$V = \frac{[(I_{1\max}+I_{2\max})/2]-I_{\min}}{[(I_{1\max}+I_{2\max})/2]+I_{\min}}. \tag{5.4}$$

We determined $I_{1\max}$, $I_{2\max}$ and $I_{\min}$ from the numerically evaluated intensity given in (3.6) for a large number of values of the slits distance d, keeping δ, k_i , k fixed. Visibility evaluated in this way is presented in figure 4. By comparing figures 2 and 4, we see that visibility, which we evaluated by taking into account oscillations around the central maximum of (3.6), turns out to have the same graphical form as (4.9), obtained from the first order approximation (4.1) with respect to δ/d of $I(x, y)$.

6. Conclusions

By determining the time evolution of the wave function of the single atom in Feynman's atomic light microscope, we found the functional dependence of visibility of interference on the product dk_i. In the case of infinitesimally small slit widths an analytic expression for visibility was obtained. By numerical simulation we found that the dependence described by this analytic expression is valid for nonnegligible slit widths, too. Since revivals exist for infinitesimally small slits as well as when slits have finite width, we conclude that the existence of revivals does not depend on the width of the slits and diffraction structure. So, the assertion that [6] 'revivals result from path ambiguity due to diffraction structure in the image' is questionable.

The expression obtained here for visibility is the same as that found by Arsenović *et al* [13] for visibility of interference in a Mach Zehnder atomic interferometer. The MZI was used in the experimental realization [1] of Feynman's atomic light microscope. The theoretical description exposed here of Feynmann's atomic double slit light microscope supports de Broglie's understanding of wave–particle duality [21]. According to de Broglie's interpretation wave and particle properties are coexistent (compatible). A comparison of the reasoning leading to this conclusion with Bohr's argument [22] that wave and particle properties are complementary is presented in table 1.

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