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Simple analytical expressions for the analysis of the phase-dependent electromagnetically induced transparency in a double-Λ atomic scheme

J Dimitrijević and D Arsenović

Institute of Physics, University of Belgrade, Pregrevica 118, 10080 Belgrade, Serbia

E-mail: jelena.dimitrijevic@ipb.ac.rs

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Abstract
We study a double-Λ atomic scheme that interacts with four laser light beams so that a closed loop of radiation-induced transitions is formed. When specific relations for field phases, frequencies and amplitudes are satisfied, coherent superpositions (the so-called 'dark states') can be formed in a double-Λ, which leads to the well-known effect of electromagnetically induced transparency (EIT). If the interaction scheme in a double-Λ system is such that a closed loop is formed, the relative phase of the laser light fields becomes very important. We analyze here the effect of the lasers' relative phase on the EIT in double-Λ configuration of levels. The theoretical study of interactions of lasers with a double-Λ atomic scheme is commonly conducted by solving the optical Bloch equations (OBEs). We use here a perturbative method for solving OBEs, where the interaction of lasers with double-Λ is considered a perturbation. An advantage of the perturbative method is that it generally produces simpler solutions, and analytical expressions can be obtained. We present analytical expressions for the lower-order corrections of the EIT signal. Our results show that the EIT by the perturbative method can be approximated by the sum of products of complex Lorentzians. Through these expressions, we see in what way the relative phase affects the overall EIT profile.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Coherent effects in various excitation schemes have attracted much attention in recent decades. One of the most widely investigated effect is electromagnetically induced transparency (EIT) [1]. It has been indicated that in a closed-loop interaction scheme both the dynamics and steady state of EIT strongly depend on the lasers' relative phase [2, 3]. One of the most widely investigated closed-loop interaction schemes is double-Λ (see figure 1). Besides with EIT [4], it has been studied also in the context of phenomena such as four-wave mixing [5], lasers without inversion [6], slow light [7], quantum correlations [8] and so on.

The phase dependence of EIT in a double-Λ was demonstrated experimentally by Korsunsky and Kosachiov and a theoretical analysis was also presented [3]. Transient properties of phase-dependent EIT were studied in [9]. Applications were also achieved, for example, the creation of entanglement [10] and quantum-state transfer [11]. In this paper, we present a theoretical analysis of the phase-dependent EIT in a double-Λ atomic scheme by using the perturbative method.

2. The model

We solve the steady-state optical Bloch equations (OBEs)

$$\frac{\mathrm{i}}{\hbar}[\hat{H}_0, \hat{\rho}] + \frac{\mathrm{i}}{\hbar}[\hat{H}_\mathrm{I}, \hat{\rho}] + \hat{SE}\hat{\rho} + \gamma\hat{\rho} = \gamma\hat{\rho}_0 \qquad (1)$$

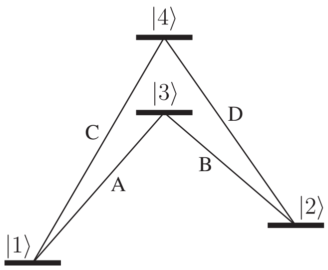

Figure 1. Double-Λ configuration of levels. The four laser light fields A, B, C and D couple states as indicated in the figure.

for the double-Λ interaction scheme given in figure 1. In equation (1), $\hat{H}_0$ is the Hamiltonian of the free double-Λ atom, $\hat{H}_{\mathrm{I}}$ describes the interaction with lasers and $\hat{SE}$ is the abbreviated spontaneous emission operator with the rate Γ for both excited levels. The term $\gamma\hat{\rho}$ describes the relaxation of all density matrix elements due to the atom's finite time of flight through the laser beam and $\gamma\hat{\rho}_0$ describes the flux of atoms to the laser beam, with equal population of two ground-state levels. The detailed system of OBEs for the four-level atom is given in appendix A.

The Rabi frequencies are Ω_K and the lasers' light detunings from the corresponding atomic frequencies are Δ_K, where K stands for a laser A, B, C or D, as shown in figure 1. Detuning between ground levels 1 and 2 is $\Delta_{\mathrm{R}} \equiv \Delta_A - \Delta_B = \Delta_C - \Delta_D$ and between excited levels is $\Delta_{\mathrm{E}} = \Delta_C - \Delta_A = \Delta_D - \Delta_B$. The relative, constant phase between lasers is $\Phi = (\varphi_A - \varphi_B) - (\varphi_C - \varphi_D)$, where φ_K are the lasers' phases.

The details of the perturbative method are described in [12]. We here apply a perturbative method to the system of four lasers interacting with a four-level atom, where the interaction with all four lasers is taken to be a perturbation, i.e. the sum on the rhs of equation (A.1). The solution of the density matrix (elements of which are sorted in a column x) by the perturbative method represents the sum of the unperturbed part x_0 and the series of successive corrections x_n, where n is the iteration number.

3. Results and discussion

The solution obtained by the perturbative method is such that the first appearance of narrow resonances is in the second correction of the density matrix and that only elements which show such behavior are ground-level coherences $\rho_{12}^{x_2}$ and $\rho_{21}^{x_2}$. The analytical expression for $\rho_{21}^{x_2}$ is

$$\rho_{21}^{x_2}(\Delta_{\mathrm{R}}) = -\frac{2(2\gamma+\Gamma-\mathrm{i}\Delta_{\mathrm{R}})}{\gamma-\mathrm{i}\Delta_{\mathrm{R}}} \times \left[\frac{\mathrm{e}^{-\mathrm{i}(\varphi_A-\varphi_B)}\Omega_A\Omega_B}{(2\gamma+\Gamma-2\mathrm{i}\Delta_A)(2\gamma+\Gamma+2\mathrm{i}\Delta_A-2\mathrm{i}\Delta_{\mathrm{R}})} + \frac{\mathrm{e}^{-\mathrm{i}(\varphi_C-\varphi_D)}\Omega_C\Omega_D}{(2\gamma+\Gamma-2\mathrm{i}\Delta_C)(2\gamma+\Gamma+2\mathrm{i}\Delta_C-2\mathrm{i}\Delta_{\mathrm{R}})}\right] \tag{2}$$

and $\rho_{12}^{x_2}$ is the complex conjugate. The rhs of equation (2) represents the sum of products of complex Lorentzians (CL),

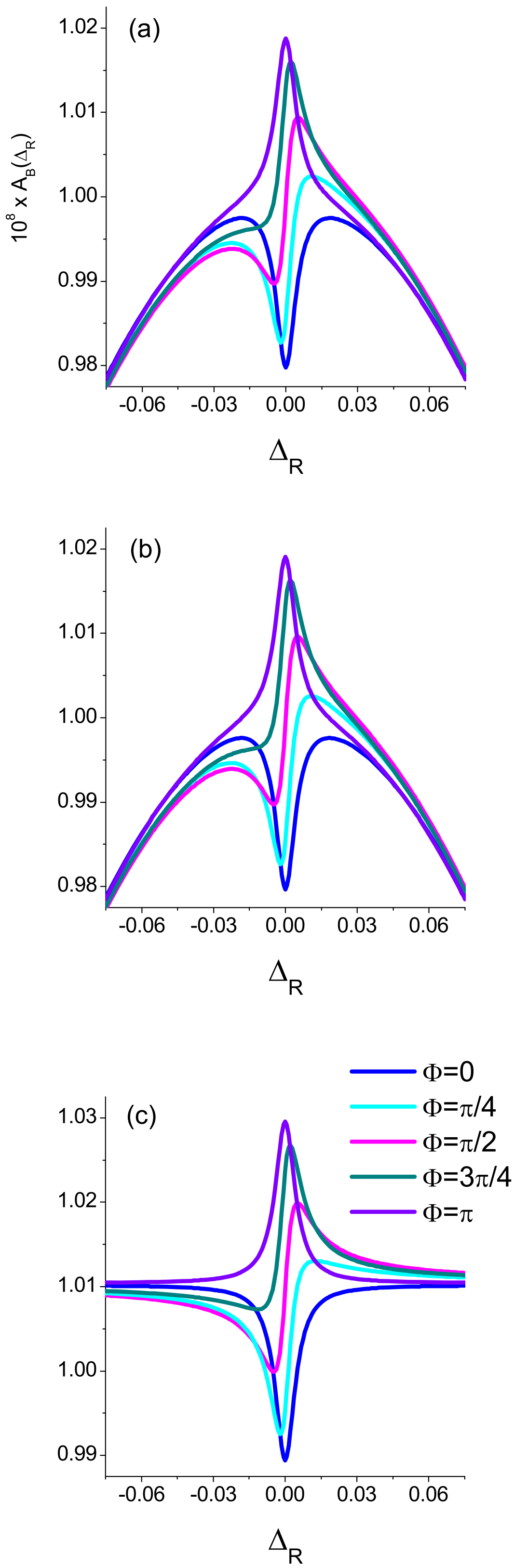

Figure 2. Numerical results for the calculated $A_B(\Delta_{\mathrm{R}})$ for five different values of the overall phase Φ: (a) the exact solution, (b) the perturbative solution $A_B^{x_1+x_3}(\Delta_{\mathrm{R}})$ and (c) the approximative expression for $A_B^{x_1+x_3}(\Delta_{\mathrm{R}})$ given by equation (4). The results are for steady-state OBEs. Equations (parameters) are normalized with Γ, i.e. we take Γ = 1. We take the relaxation rate $\gamma = 0.005\,\Gamma$, the Rabi frequencies $\Omega_A = 0.001\,\Gamma$, $\Omega_B = 0.0001\,\Gamma$, $\Omega_C = 0.005\,\Gamma$ and $\Omega_D = 0.0005\,\Gamma$, detunings Δ_A, Δ_C are equal to zero, while we vary $\Delta_{\mathrm{R}} = -\Delta_B = -\Delta_D$ around 0.

where terms within square brackets in equation (2) contain very wide CLs (since $\Gamma \gg \gamma$ and $\Gamma \gg \Delta_{A,B,C,D}$) and each can be approximated with $\frac{1}{\Gamma}$. This yields a simple analytical expression for $\rho_{21}^{x_2}$ in the form of one very narrow CL:

$$\rho_{21}^{x_2}(\Delta_\mathrm{R}) \cong n\mathrm{CL}(\Delta_\mathrm{R})$$
$$= -2\frac{\mathrm{e}^{-\mathrm{i}(\varphi_A-\varphi_B)}\Omega_A\Omega_B + \mathrm{e}^{-\mathrm{i}(\varphi_C-\varphi_D)}\Omega_C\Omega_D}{(\gamma - \mathrm{i}\Delta_\mathrm{R})\Gamma}. \quad (3)$$

These two narrow resonances (identified as the real part of $n\mathrm{CL}(\Delta_\mathrm{R})$) are, by the iterative procedure, transferred to all higher-order corrections and lead in the end to the development of EIT.

In figure 2, we present the results for the steady-state, phase-dependent EIT in a double-Λ configuration. As a spectroscopic signal we take the dependence of the laser's B the absorbtion on the detuning Δ_R. It is calculated as the imaginary part of linear susceptibility, i.e. absorption coefficient, $A_B = N\,\mathrm{Im}(\mathrm{e}^{-\mathrm{i}\varphi_B}\Omega_B\rho_{23})$. The constant N stands for the atomic concentration, and is irrelevant in this study, i.e. we take $N = 1$.

The results of the perturbative method show that the narrow EIT resonance appears after including higher-order ($n \geqslant 3$) corrections of ρ_{23}. From figures 2(a) and (b), we see that already the sum up to the third correction of A_B (or ρ_{23}) shows numerically good agreement with the exact numerical solution obtained by solving OBEs. The analytical expression for the $\rho_{23}^{x_1+x_3}$ is just too long and we do not present it here. Approximating again wide CLs with $\frac{1}{\Gamma}$ yields the following expression for the absorption of laser B:

$$A_B^{x_1+x_3}(\Delta_\mathrm{R}) \cong$$
$$\times\,\mathrm{Im}\left[-\frac{4\mathrm{i}(\Omega_A^2\Omega_B^2+\Omega_B^2\Omega_D^2+\Omega_B^4+\mathrm{e}^{\mathrm{i}\Phi}\Omega_A\Omega_B\Omega_C\Omega_D)}{\Gamma^3}\right.$$
$$+\frac{\mathrm{i}\Omega_B^2}{\Gamma} - \frac{2\mathrm{i}\,\Omega_B^2(-\Omega_A^2+\Omega_B^2-\Omega_C^2+\Omega_D^2)}{\gamma\Gamma^2}$$
$$\left.-\frac{4\mathrm{i}(\Omega_A^2\Omega_B^2+\mathrm{e}^{\mathrm{i}\Phi}\Omega_A\Omega_B\Omega_C\Omega_D)}{\Gamma^2(\gamma-\mathrm{i}\Delta_\mathrm{R})}\right], \quad (4)$$

where only the last term on the rhs of equation (4) depends on Δ_R and others are constant. This term represents the sum of two CLs (up to the constant equal to the narrow resonance given by equation (3)) and can be of opposite sign. One of these CLs is phase independent and the other has $\mathrm{e}^{\mathrm{i}\Phi}$ as a multiplicative factor. Summing these two resonances yields different profiles, which can, for some values of relative phase Φ, completely change the sign of resonance. For comparison, numerical results for the expression given by equation (4) are shown in figure 2(c).

In conclusion, we have used the perturbative method to analyze phase-dependent EIT in a double-Λ atomic scheme. We have obtained a simple expression for the laser's absorption signal, i.e. the sum of two CLs which can (up to the constant) simulate the variation of EIT with the change of the lasers' relative phase.

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Appendix. Optical Bloch equations

OBEs for elements of the density matrix ρ describing the interacting four-level atom can be written as

$$\dot{\rho}_{ij} = \mathrm{i}\sum_k(\rho_{ik}R_{kj} - R_{ik}\rho_{kj}) + \mathrm{i}\,D_{ij}\rho_{ij} + G_{ij} \quad (\mathrm{A}.1)$$
$$-\gamma\rho_{ij} + \frac{\gamma}{2}\delta_{ij}(\delta_{i1}+\delta_{i2}), \quad i, j = 1, 2, 3, 4.$$

The notation of levels is as in figure 1. Matrices R, D and G describe certain terms of equation (1). Spontaneous emission is given through matrix G:

$$G = \Gamma\begin{pmatrix} \frac{1}{2}(\rho_{33}+\rho_{44}) & 0 & -\frac{1}{2}\rho_{1,3} & -\frac{1}{2}\rho_{1,4} \\ 0 & \frac{1}{2}(\rho_{33}+\rho_{44}) & -\frac{1}{2}\rho_{2,3} & -\frac{1}{2}\rho_{2,4} \\ -\frac{1}{2}\rho_{3,1} & -\frac{1}{2}\rho_{3,2} & -\rho_{3,3} & -\rho_{3,4} \\ -\frac{1}{2}\rho_{4,1} & -\frac{1}{2}\rho_{4,2} & -\rho_{4,3} & -\rho_{4,4} \end{pmatrix},$$

the elements of matrix D are detunings of lasers from the corresponding atomic frequencies:

$$D = \begin{pmatrix} 0 & -\Delta_\mathrm{R} & -\Delta_A & -\Delta_C \\ \Delta_\mathrm{R} & 0 & -\Delta_B & -\Delta_D \\ \Delta_A & \Delta_B & 0 & -\Delta_\mathrm{E} \\ \Delta_C & \Delta_D & \Delta_\mathrm{E} & 0 \end{pmatrix}$$

and R is a matrix with Rabi frequencies describing the interaction part of the Liouville equation

$$R = \begin{pmatrix} 0 & 0 & \mathrm{e}^{\mathrm{i}\varphi_A}\Omega_A & \mathrm{e}^{\mathrm{i}\varphi_C}\Omega_C \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\varphi_B}\Omega_B & \mathrm{e}^{\mathrm{i}\varphi_D}\Omega_D \\ \mathrm{e}^{-\mathrm{i}\varphi_A}\Omega_A & \mathrm{e}^{-\mathrm{i}\varphi_B}\Omega_B & 0 & 0 \\ \mathrm{e}^{-\mathrm{i}\varphi_C}\Omega_C & \mathrm{e}^{-\mathrm{i}\varphi_D}\Omega_D & 0 & 0 \end{pmatrix}.$$

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