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Comparison of a double-Λ atomic scheme with single- and two-fold coupled transitions

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Abstract
A four-level double-Λ atomic scheme, i.e. two Λ systems sharing the same two ground levels, that interacts with four laser light fields is studied theoretically. The peculiarity is that each of the two ground states can be coupled to each excited state by two laser light fields. A certain energy difference exists between excited-state levels. We consider this energy small enough so that a laser resonant to either transition can also couple the other transition. We test whether coupling of the more detuned laser is not non-negligible. Multiply connected states were also recently analyzed (and a comparison with experiment was presented), but for the simpler, two- and three-level atomic schemes (Stacey *et al* 2008 *J. Phys. B: At. Mol. Opt. Phys.* **41** 085502). Theoretical treatment of the double-Λ atomic scheme is commonly done by solving optical Bloch equations (OBEs). When the rotating-wave approximation (RWA) is applied, OBEs become, by their form, a set of linear differential equations with constant coefficients. Theoretical treatment of the interaction scheme treated here leads to OBEs with coefficients that are not constant, but oscillate with time, even after RWA is applied. Under certain assumptions, the approximation can be used where the time-dependent coefficients are averaged over their periods. The method yields a new system of equations (similar to standard OBEs), but with more independent variables, and can also be solved in the usual way. The results presented here analyze the validity of this approximation by comparing the results for the double-Λ atomic scheme with single- and two-fold coupled transitions. We test whether in the limit of large-energy splitting between excited-state levels both approaches lead to similar results.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Excitations of various atomic schemes by lasers were studied in systems as simple as two level and three level, such as Λ, V or ladder configuration, to complex multilevel systems with or without Zeeman splitting. These interactions have been shown to give rise to interesting effects such as coherent population trapping [2], electromagnetically induced transmission [3] and electromagnetically induced absorption [4]. The double-Λ atomic scheme, i.e. two Λ systems sharing the same two ground levels (see figure 1), has also been shown to be of interest as the basis for many investigations and applications [5, 6]. In this paper, we study a double-Λ atomic scheme where each of the transitions between two ground and two excited levels is coupled by two lasers (see figure 1(b)). We test whether optical Bloch equations (OBEs) can be solved for this interaction scheme and compare the results with the standard, single-fold coupled double-Λ, i.e. each transition driven by only one laser (figure 1(a)).

2. Theoretical model for the two-fold coupled double-Λ atomic scheme

We solve OBEs:

$$\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}[\hat{H}_0, \hat{\rho}(t)] - \frac{\mathrm{i}}{\hbar}[\hat{H}_\mathrm{I}, \hat{\rho}(t)] - \hat{SE}\hat{\rho}(t) - \gamma\hat{\rho}(t) + \gamma\hat{\rho}_0 \tag{1}$$

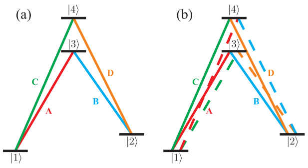

Figure 1. Double-Λ configuration of levels with single-fold (a) and two-fold (b) coupled states. Four lasers A, B, C and D couple atomic states as indicated in the figure, where additional couplings are denoted by dashed lines.

for the double-Λ schemes given in figure 1. In equation (1), $\hat{H}_0$ is the Hamiltonian of the free, four-level atom and $\hat{H}_\mathrm{I}$ is the interaction Hamiltonian of an atom interacting with four laser fields. $\hat{SE}$ represents the operator of spontaneous emission with rate Γ for both excited levels, $\gamma\hat{\rho}$ is the relaxation due to time of flight and $\gamma\hat{\rho}_0$ describes the continuous flux of atoms to the laser beam, with equally populated ground-state levels.

Following the procedure described in [1], we next discuss whether OBEs with constant coefficients can be obtained for the interaction scheme given in figure 1(b). In the following, expressions for coherences ρ_{ij} will be presented, while for ρ_{ji} the corresponding complex conjugates need to be taken. The following substitution is made to optical coherences ρ_{13}, ρ_{14}, ρ_{23} and ρ_{24}:

$$\rho_{13} = \tilde{\rho}_{13}^{A}\mathrm{e}^{\mathrm{i}\omega_A t} + \tilde{\rho}_{13}^{C}\mathrm{e}^{\mathrm{i}\omega_C t}, \quad \rho_{14} = \tilde{\rho}_{14}^{C}\mathrm{e}^{\mathrm{i}\omega_C t} + \tilde{\rho}_{14}^{A}\mathrm{e}^{\mathrm{i}\omega_A t},$$
$$\rho_{23} = \tilde{\rho}_{23}^{B}\mathrm{e}^{\mathrm{i}\omega_B t} + \tilde{\rho}_{23}^{D}\mathrm{e}^{\mathrm{i}\omega_D t}, \quad \rho_{24} = \tilde{\rho}_{24}^{D}\mathrm{e}^{\mathrm{i}\omega_D t} + \tilde{\rho}_{24}^{B}\mathrm{e}^{\mathrm{i}\omega_B t}, \quad (2)$$

where ω_K (K is a laser) are lasers' frequencies which satisfy the multi-photon resonance condition:

$$\omega_A - \omega_B = \omega_C - \omega_D. \quad (3)$$

In a closed-loop interaction scheme, such as double-Λ, the steady state can be reached only if this condition is satisfied [6].

The next step is to write explicitly equations for the density matrix elements from the interaction part of Liouville's equation (equation (1)) with substituted optical coherences (equation (2)). This yields the new set equations, where we next analyze equations for coherences ρ_{12} and ρ_{34}. For these coherences there appear three groups of terms on the rhs that oscillate with specific frequency, $\mathrm{e}^{\mathrm{i}t(\omega_A-\omega_B)}$, $\mathrm{e}^{\mathrm{i}t(\omega_C-\omega_B)}$, $\mathrm{e}^{\mathrm{i}t(\omega_A-\omega_D)}$ for ρ_{12} and $\mathrm{e}^{\mathrm{i}t(\omega_A-\omega_C)}$, $\mathrm{e}^{\mathrm{i}t(\omega_C-\omega_A)}$, $\mathrm{e}^{\mathrm{i}t0}$ for ρ_{34}. These oscillatory terms introduce substitutions for coherences ρ_{12} and ρ_{34} as follows:

$$\rho_{12} = \tilde{\rho}_{12}^{AB}\mathrm{e}^{\mathrm{i}(\omega_A-\omega_B)t} + \tilde{\rho}_{12}^{CB}\mathrm{e}^{\mathrm{i}(\omega_C-\omega_B)t} + \tilde{\rho}_{12}^{AD}\mathrm{e}^{\mathrm{i}(\omega_A-\omega_D)t}$$
$$\rho_{34} = \tilde{\rho}_{34}^{CA}\mathrm{e}^{\mathrm{i}(\omega_C-\omega_A)t} + \tilde{\rho}_{34}^{0} + \tilde{\rho}_{34}^{AC}\mathrm{e}^{\mathrm{i}(\omega_A-\omega_C)t}. \quad (4)$$

New variables, given by equations (2) and (4), are next inserted into equation (1). This yields equations of the form $\sum_k \mathrm{e}^{\mathrm{i}\omega_k^{ij}t}c_k^{ij} = 0$ (one for each matrix element ρ_{ij}) or equivalently the new set of differential and algebraic equations, $c_k^{ij} = 0$. The approximation we use consists in omitting the algebraic subset, which is the same as the standard rotating-wave approximation for the OBEs with single-fold couplings. The difference is that the latter procedure introduces more approximations, i.e. more terms have to be neglected. The subset of differential equations

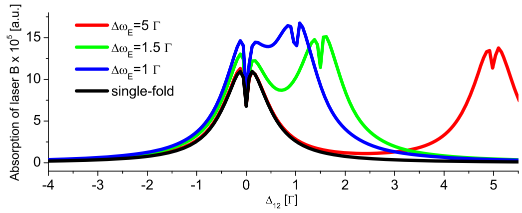

Figure 2. Steady-state absorption of laser B for three different energy splittings between excited levels for the two-fold coupled transitions. Absorption for the single-fold coupled transition is also presented by the black curve. Steady-state equations are normalized to Γ and we take Γ = 1. We take the relaxation rate $\gamma = 0.03\,\Gamma$, detunings $\Delta_{13}^A = \Delta_{14}^C = 0$, $\Delta_{23}^B = \Delta_{24}^D = -\Delta_{12}$ and Rabi frequencies $\Omega_{13}^A = \Omega_{13}^C = 0.05\,\Gamma$, $\Omega_{14}^A = \Omega_{14}^C = 0.055\,\Gamma$, $\Omega_{23}^B = \Omega_{23}^D = 0.01\,\Gamma$, $\Omega_{24}^B = \Omega_{24}^D = 0.011\,\Gamma$. We take all lasers' initial phase equal to 0.

yields the new set of modified OBEs which we use to describe the interaction of lasers with a two-fold coupled double-Λ atomic scheme. The explicitly written system of modified OBEs will be published elsewhere.

The system of modified OBEs has 32 equations with 32 unknowns, 16 + 12 from the rhss of equations (2) and (4) (with corresponding complex-conjugates) and four are populations. It also introduces new quantities, modified Rabi frequencies Ω_{ij}^K and detunings Δ_{ij}^K, where i and j refer to levels and K stands for a laser. From figure 1 we see that for lasers $K = A, C$, possible values for (i, j) are (1, 3) or (1, 4), while for the lasers $K = B, D$, one can have $(i, j) = (2, 3)$ or (2, 4). The Rabi frequencies are given by $\Omega_{ij}^K = \mu_{ij}E_K/\hbar$, where μ_{ij} are dipole moments of the $|i\rangle \to |j\rangle$ transitions and E_K are complex amplitudes of the lasers' K electric fields. One-photon lasers' detunings are defined as $\Delta_{ij}^K = \omega_K + \omega_i - \omega_j$, where $\hbar\omega_1, \ldots, \hbar\omega_4$ are energies of the four atomic states. The two-photon ground and excited detunings, which follow from the multi-photon resonance condition given by equation (3), are also introduced:

$$\Delta_{12} \equiv \Delta_{14}^C - \Delta_{24}^D = \Delta_{13}^C - \Delta_{23}^D,$$
$$\Delta_{34} \equiv \Delta_{14}^C - \Delta_{13}^A = \Delta_{24}^D - \Delta_{23}^B. \quad (5)$$

3. Results and discussion

In figure 2, we compare the results for the absorption of laser B for the single- and two-fold coupled double-Λ for different values of the energy difference between excited levels. Results are presented for the steady-state regime. The laser's B absorption is given as a function of the two-photon ground detuning Δ_{12}. For single-fold coupled double-Λ we calculate it as $N\,\mathrm{Im}(\Omega_B\rho_{23})$ and for two-fold as $N\mathrm{Im}(\rho_{23}^B\Omega_{23}^B + \rho_{24}^B\Omega_{24}^B)$. The constant N stands for the atomic concentration, and is irrelevant in this study, i.e. we take $N = 1$. Absorption of laser B for the two-fold coupled states exhibits two one-photon absorption profiles, one corresponding to the $2 \to 3$ and another to the $2 \to 4$ transition. From figure 2 we also see that both of these profiles show superimposed narrow electromagnetically induced transparency resonances.

If the energy splitting between excited levels, $\Delta\omega_\mathrm{E} = \omega_4 - \omega_3$, is large enough, laser B cannot drive transitions to both excited states. Under such an assumption solutions of equations for the two-fold case should downgrade to standard single-connected, for values of Δ_{12} around 0 where the single-coupled laser's B absorption shows. Results presented in figure 2 clearly show that an increase of $\Delta\omega_{\mathrm{E}}$ leads to identical results between single- and two-fold double-Λ OBEs (compare red and black curves in figure 2 for values of Δ_{12} around 0).

Numerically, with an increase of $\Delta\omega_{\mathrm{E}}$, the new set of variables reduces to the density matrix elements of the single-fold coupled double-Λ interaction scheme. Optical coherences connecting states coupled by the more detuned lasers tend to zero, while others tend to their counterparts for the single-connected equations. For example, the influence of the laser B on the transition $2 \rightarrow 4$ cannot be neglected for small enough $\Delta\omega_E$, as is obvious from figure 2 for values of Δ_{12} around 0. As $\Delta\omega_{\mathrm{E}}$ increases, density matrix element $\tilde{\rho}_{24}^{B}$ tends to zero, $\tilde{\rho}_{24}^{B} \rightarrow 0$, while $\tilde{\rho}_{23}^{B}$ approaches the density matrix element of the single-fold coupled double-Λ, $\tilde{\rho}_{23}^{B} \rightarrow \tilde{\rho}_{23}$.

In conclusion, we studied the interaction of the two-fold coupled double-Λ with four lasers. Our results show that modified OBEs, which we use to treat this interaction scheme, have the expected properties. The laser's absorption shows two one-photon absorption profiles corresponding to both couplings. Also, numerical solutions of modified OBEs have the correct limit to the solutions with single-fold excitations as the energy difference between excited levels increases.

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