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Evolution of the wave function of an atom hit by a photon in a three-grating interferometer

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Abstract

In 1995, Chapman *et al* (1995 *Phys. Rev. Lett.* **75** 2783) showed experimentally that the interference contrast in a three-grating atom interferometer does not vanish in the presence of scattering events with photons, as required by the complementarity principle. In this work, we present an analytical study of this experiment by determining the evolution of an atom's wave function along the three-grating Mach–Zehnder interferometer under the assumption that the atom is hit by a photon after passing through the first grating. The consideration of a transverse wave function in momentum representation is essential in this study. As is shown, the number of atoms transmitted through the third grating is given by a simple periodic function of the lateral shift along this grating, both in the absence and in the presence of photon scattering. Moreover, the relative contrast (laser on/laser off) is shown to be a simple analytical function of the ratio d_p/λ_i, where d_p is the distance between atomic paths at the scattering locus and λ_i the scattered photon wavelength. We argue that this dependence, being in agreement with experimental results, can be considered as showing compatibility between the wave and corpuscle properties of atoms.

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1. Introduction

In an experiment performed by Chapman *et al* [1] in 1995, single photons were scattered off the atoms that passed through the first grating of a three-grating Mach–Zehnder interferometer [2]. The purpose of this experiment was to study the influence of photon scattering events on the atom interference. The dependence of the atom transmission through the third grating on the distance y'_{12} between the place where the scattering event occurred and the first grating (figure 1) was then investigated. For each value of y'_{12}, the transmission was measured as a function of the lateral shift Δx_3 of the third grating, showing that the relative fringe contrast of the transmission depended on the ratio d_p/λ_i, where λ_i is the scattered photon wavelength, and $d_\mathrm{p} = y'_{12}\lambda/d$ is the distance between two atomic paths at the scattering locus; in the latter relation d is the grating constant, $\lambda = h/mv = 2\pi/k$ is the atomic de Broglie wavelength and v and k are the atomic initial velocity and wave number, respectively.

The experiment showed that the contrast decreases to zero for $d/\lambda_i \approx 0.5$, and several revivals with decreasing relative maxima follow as d/λ_i increases [1, 2]. Chapman *et al* associated the loss of coherence with complementarity and the subsequent revival with the spatial resolution function of a single scattered photon. Moreover, they also considered that their experiment addresses the following questions: Where is the coherence lost and how might it be regained? These questions, in particular revivals of contrast, have been the subject of discussions and studies [3–6].

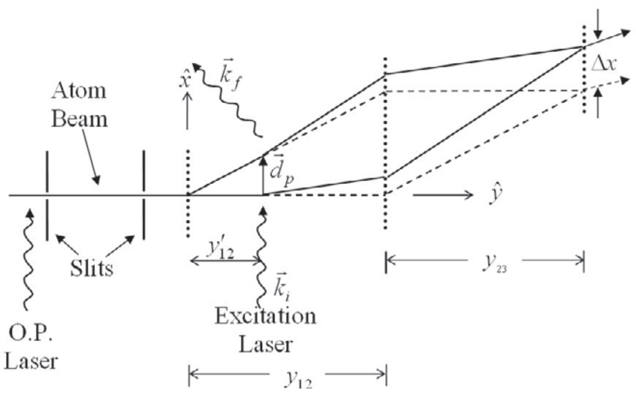

Figure 1. Sketch of the experimental three-grating interferometer used by Chapman *et al* [1, 2].

Here, we propose an explanation for the experimental results observed by Chapman *et al* [1] by determining the evolution of the wave function of an atom in a three-grating interferometer in two cases: (i) the atom moves freely between the gratings and (ii) the atom is hit by a photon between the first and second grating. The consideration of a transverse wave function in momentum representation is essential in our explanation.

2. Evolution of the wave diffracted by a grating

Consider an initial stationary atomic monochromatic wave, spreading along the y-axis, that strikes a one-dimensional grating parallel to the x-axis at $y = 0$,

$$\Psi(x, y, t) = \mathrm{e}^{-\mathrm{i}\omega t}\psi^i(x, y) = B^i\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{e}^{\mathrm{i}ky}, \qquad y < 0, \tag{1}$$

where B^i is a constant. After reaching the grating, this incident wave is being transformed into

$$\Psi(x, y, t) = \mathrm{e}^{-\mathrm{i}\omega t}\psi(x, y), \qquad y \geqslant 0, \tag{2a}$$

$$\psi(x, y) = \frac{\mathrm{e}^{\mathrm{i}ky}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\mathrm{d}k_x c(k_x)\,\mathrm{e}^{\mathrm{i}k_x x}\,\mathrm{e}^{-\mathrm{i}k_x^2 y/2k}, \qquad y \geqslant 0. \tag{2b}$$

Here, we consider gratings such that the function $c(k_x)$ has non-negligible values only for $k^2 \gg k_x^2$ [7, 8]. Under this assumption, $\psi(x, y)$ satisfies the Helmholtz equation. The function $c(k_x)$ gives the probability amplitude of transverse momenta and is determined by the boundary conditions at the grating. If the grating is completely transparent inside the slits (union of slit areas is denoted by A) and completely absorbing outside them, $c(k_x)$ is given by the following equation [7, 8]:

$$\begin{aligned} c(k_x) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\mathrm{d}x'\psi(x', 0^+)\,\mathrm{e}^{-\mathrm{i}k_x x'} \\ &= \frac{1}{\sqrt{2\pi}}\int_A \mathrm{d}x'\psi^i(x', 0^-)\,\mathrm{e}^{-\mathrm{i}k_x x'}, \end{aligned} \tag{3}$$

where $\psi(x', 0^+)$ is the wave function just behind the first grating and $\psi^i(x', 0^-)$ is the wave function just before the first grating.

As shown by Arsenović *et al* [9], the solution of the Helmholtz equation, $\psi(x, y)$, given by (2*b*), is equivalent to the Fresnel–Kirchhoff solution

$$\psi(x, y) = \sqrt{\frac{k}{2\pi y}}\mathrm{e}^{-\mathrm{i}\pi/4}\,\mathrm{e}^{\mathrm{i}ky}\int_{-\infty}^{+\infty}\mathrm{d}x'\,\psi(x', 0^+)\,\mathrm{e}^{\mathrm{i}k(x-x')^2/2y}. \tag{4}$$

The latter form is very useful because one can easily show from it that there exists direct proportionality between the functions $\psi(x, y)$ and $c(kx/y)$ in the region far from the grating:

$$\psi(x, y) = \frac{\sqrt{k}}{\sqrt{y}}\mathrm{e}^{-\mathrm{i}\pi/4}\,\mathrm{e}^{\mathrm{i}kx^2/2y}\,c\left(\frac{kx}{y}\right)\mathrm{e}^{\mathrm{i}ky}. \tag{5}$$

The solution given in (2*a*) and (2*b*) suggests that, behind the grating, the atom continues propagating with the initial longitudinal momentum, since a change of it is negligible. However, there is a probability density $|c(k_x)|^2$ that an atom acquires a transverse momentum $p_x = \hbar k_x$. This justifies [4, 5] the substitution of y by $\hbar kt/m$ in the integrand of (2*b*) and defining the so-called *wave function of the transverse motion*,

$$\begin{aligned} \psi^{\mathrm{tr}}(x, t = ym/\hbar k) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\mathrm{d}k_x c(k_x, t)\,\mathrm{e}^{\mathrm{i}k_x x} \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\mathrm{d}k_x c(k_x)\mathrm{e}^{-\mathrm{i}k_x^2\hbar t/2m}\mathrm{e}^{\mathrm{i}k_x x}, \end{aligned} \tag{6}$$

where $c(k_x, t)$ is the time-dependent transverse wave function in momentum representation,

$$\begin{aligned} c(k_x, t) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\mathrm{d}x\psi^{\mathrm{tr}}(x, t)\mathrm{e}^{-\mathrm{i}k_x x} \\ &= c(k_x)\mathrm{e}^{-\mathrm{i}k_x^2\hbar t/2m}. \end{aligned} \tag{7}$$

As can be seen, $\psi^{\mathrm{tr}}(x, t)$ has the form of a non-stationary solution of the one-dimensional free-particle time-dependent Schrödinger equation. The solution (2*a*) is then a product [7–9] of a longitudinal plane wave and a non-stationary transverse wave function,

$$\Psi(x, y, t) = \mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{e}^{\mathrm{i}ky}\psi^{\mathrm{tr}}(x, t). \tag{8}$$

3. Evolution of the diffracted wave after the atom is hit by a photon

We shall now use the above atomic wave function behind the grating and its interpretation to determine the atomic wave function after the atom absorbed and re-emitted a photon somewhere along the x-axis at a time t'_{12} and a distance $y'_{12} = vt'_{12} = (\hbar k/m)t'_{12}$ from the first grating. As a result of the scattering with the photon, there is a change of the atomic transverse momentum Δk_x, which also leads to a change of the wave function in the momentum representation. We denote the wave function after the photon–atom scattering event in momentum representation as $c_{\Delta k_x}(k_x, t)$. It has to satisfy

$$\left|c_{\Delta k_x}(k_x, t'_{12})\right|^2 = \left|c(k_x - \Delta k_x, t'_{12})\right|^2. \tag{9}$$

From this relation, it follows that

$$c_{\Delta k_x}(k_x, t'_{12}) = c(k_x - \Delta k_x, t'_{12}) e^{\mathrm{i} f(\Delta k_x, k_x)}, \tag{10}$$

where $f(\Delta k_x, k_x)$ is (for now) an unknown phase function. The corresponding transverse wave function at time t'_{12} is then given by

$$\psi^{\mathrm{tr}}_{\Delta k_x}(x, t'_{12}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}k_x c_{\Delta k_x}(k_x, t'_{12})\, e^{\mathrm{i}k_x x}, \tag{11}$$

which should satisfy

$$\left|\psi^{\mathrm{tr}}_{\Delta k_x}(x, t'_{12})\right|^2 = \left|\psi^{\mathrm{tr}}(x, t'_{12})\right|^2. \tag{12}$$

Using (10), one can show that the latter condition will be fulfilled if

$$f(\Delta k_x, k_x) = 0. \tag{13}$$

After substituting (10) and (13) into (11), one finds that, just after the photon–atom scattering event, the atomic wave function becomes

$$\psi^{\mathrm{tr}}_{\Delta k_x}(x, t'_{12}) = \frac{1}{\sqrt{2\pi}} e^{-\mathrm{i}\Delta k_x^2 \hbar t'_{12}/2m} \times \int_{-\infty}^{+\infty} \mathrm{d}k_x c(k_x - \Delta k_x) e^{-\mathrm{i}k_x^2 \hbar t'_{12}/2m}\, e^{\mathrm{i}k_x(x+\Delta x_0)}, \tag{14}$$

where we have introduced the magnitude

$$\Delta x_0 = \frac{\Delta k_x \hbar t'_{12}}{m} = \frac{\Delta k_x y'_{12}}{k}. \tag{15}$$

Assuming that the function (14) keeps the same form for $t > t'_{12}$, we may write

$$\psi^{\mathrm{tr}}_{\Delta k_x}(x, t) = \frac{1}{\sqrt{2\pi}} e^{-\mathrm{i}\Delta k_x^2 \hbar t/2m} \times \int \mathrm{d}k_x c(k_x - \Delta k_x) e^{-\mathrm{i}\Delta k_x^2 \hbar t/2m}\, e^{\mathrm{i}k_x(x+\Delta x_0)}. \tag{16}$$

By changing now the integration variable $k'_x = k_x - \Delta k_x$ and using the relation $\hbar t/m = y/k$, equation (16) transforms into

$$\psi^{\mathrm{tr}}_{\Delta k_x}(x, y) = \frac{1}{\sqrt{2\pi}} e^{\mathrm{i}\Delta k_x(x+\Delta x_0) - \mathrm{i}\Delta k_x^2 y/k} \times \int_{-\infty}^{+\infty} \mathrm{d}k'_x c(k'_x) e^{-\mathrm{i}k'^2_x y/2k} e^{\mathrm{i}k'_x(x+\Delta x_0 - y\Delta k_x/k)}. \tag{17}$$

Then, after multiplying (17) by $e^{\mathrm{i}ky}$, we obtain the space-dependent wave function, which is the continuation of (2*b*) for $y > y'_{12}$, i.e.

$$\psi_{\Delta k_x}(x, y) = e^{\mathrm{i}ky} \psi^{\mathrm{tr}}_{\Delta k_x}(x, y). \tag{18}$$

In analogy to the approximation (5) for (2*b*) and (4), the wave function (18) can also be approximated in the far field by the simpler form,

$$\psi_{\Delta k_x}(x, y) = e^{\mathrm{i}ky} \frac{\sqrt{k}}{\sqrt{y}} e^{-\mathrm{i}\pi/4} e^{-\mathrm{i}(\Delta k_x^2 y/2k)} e^{\mathrm{i}(k(x+\Delta x_0)^2/2y)} \times c\left(\frac{k(x+\Delta x_0)}{y} - \Delta k_x\right). \tag{19}$$

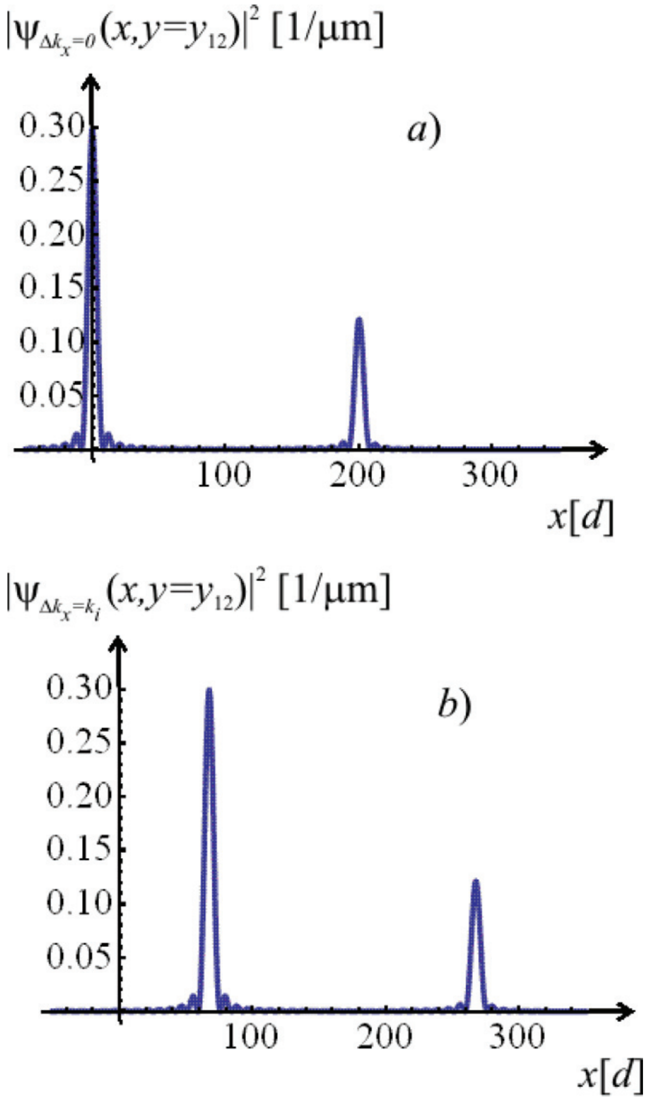

Figure 2. The function $|\psi_{\Delta k_x}(x, y = y_{12})|^2$ when the laser is off (a), with $\Delta k_x = 0$, and when the laser is on (b), with $y'_{12} = 5kd/8k_i$ and $\Delta k_x = k_i$. The parameters considered are $v = 1400\,\mathrm{m\,s^{-1}}$, $k = m_{\mathrm{Na}} \cdot v/\hbar = 5.09067 \times 10^{11}\,\mathrm{m^{-1}}$, $k_i = 2\pi/(589\,\mathrm{nm}) = 1.06675 \times 10^7\,\mathrm{m^{-1}}$, $y_{12} = y_{23} = 0.65\,\mathrm{m}$, $d = 2 \times 10^{-7}\,\mathrm{m}$, $\delta = 1 \times 10^{-7}\,\mathrm{m}$ and $n = 24$.

Assuming that the beam incident on the first grating is a plane wave that illuminates n slits, from (3) we find that

$$c(k_x) = \frac{\sqrt{2}}{\sqrt{\pi n \delta}} \frac{\sin(k_x \delta/2)}{k_x} \frac{\sin(k_x dn/2)}{\sin(k_x d/2)}, \tag{20}$$

where d is the grating period and δ is the slit width.

The wave function $\psi_{\Delta k_x}(x, y = y_{12})$ that reaches the second grating has two narrow maxima, each one covering several slits. The square modulus of this function is shown in figure 2(a) for the laser off and in figure 2(b) for the laser on.

4. The wave function behind the second grating

In order to determine the wave function behind the second grating, it is convenient to apply the form (4) of the atomic wave function. Thus, we have

$$\psi(x,y)=\sqrt{\frac{k}{2\pi y}}\,\mathrm{e}^{-\mathrm{i}\pi/4}\,\mathrm{e}^{\mathrm{i}ky}\int_{-\infty}^{+\infty}\mathrm{d}x'\,\psi(x',y_{12}^{+0})\mathrm{e}^{\mathrm{i}\left(k(x-x')^2/2y\right)},\qquad y>y_{12},\tag{21}$$

where $\psi(x', y_{12}^{+0})$ is the wave function just after the second grating.

If the laser is off ($\Delta k_x = 0$), the wave function does not depend on y'_{12}. We then find that the square modulus of the wave function striking the third grating has the form shown in figure 3(a): it oscillates with period d. If the laser is turned on, the function has the same form again, but it undergoes a shift along the x-axis (see figure 3(b)) for an amount that depends on Δk_x.

5. Transmission through the third grating

In the experiment of Chapman *et al* [1] the corresponding patterns were obtained by counting the number of atoms transmitted through the third grating. So, in order to compare the above analytical results with experimental data, it is necessary to evaluate the number of transmitted atoms through the third grating for various values of its lateral shift Δx_3. The transmission is evaluated by integrating first the intensity in the region of the first maximum (i.e. in the range of x shown in figure 3) for fixed values of the lateral shift and transferred impulses $\Delta k_x$to the atom during the photon scattering, i.e.

$$T(y'_{12},\Delta k_x,\Delta x_3)=\int_{\text{slits}}\left|\psi_{\Delta k_x}(x,y=y_{12}+y_{23})\right|^2\mathrm{d}x.\tag{22}$$

The numerical results we have obtained for different values of y'_{12} and Δk_x show that the function $T(y'_{12}, \Delta k_x, \Delta x_3)$ has the following simple periodic form:

$$T(y'_{12},\Delta k_x,\Delta x_3)=a+b\,\cos(2\pi\,\Delta x_3/d+d_{\mathrm{p}}\Delta k_x),\tag{23}$$

where a and b are constants that do not depend on y'_{12} and Δk_x, and the quantity

$$d_{\mathrm{p}}=(2\pi/kd)y'_{12}\tag{24}$$

is the distance between the paths (the lines of maxima of the atomic wave function) at the place of scattering with a photon.

Next, we have to integrate over all possible values of the transferred momentum taking into account the probability distribution of the transferred momentum, $P_1(\Delta k_x)$. As shown by Mandel and Wolf [10], this distribution is given by

$$P_1(\Delta k_x)=\frac{3}{8k_i}\left[1+\left(1-\frac{\Delta k_x}{k_i}\right)^2\right].\tag{25}$$

Consequently,

$$\begin{aligned}T(y'_{12},\Delta x_3)&=\int_0^{2k_i}\mathrm{d}(\Delta k_x)\,P_1(\Delta k_x)\,T(y'_{12},\Delta k_x,\Delta x_3)\\&=\int_0^{2k_i}\mathrm{d}(\Delta k_x)\,\frac{3}{8k_i}\left[1+\left(1-\frac{\Delta k_x}{k_i}\right)^2\right]\\&\quad\times(a+b\,\cos(2\pi\,\Delta x_3/d+d_{\mathrm{p}}\Delta k_x)),\end{aligned}\tag{26}$$

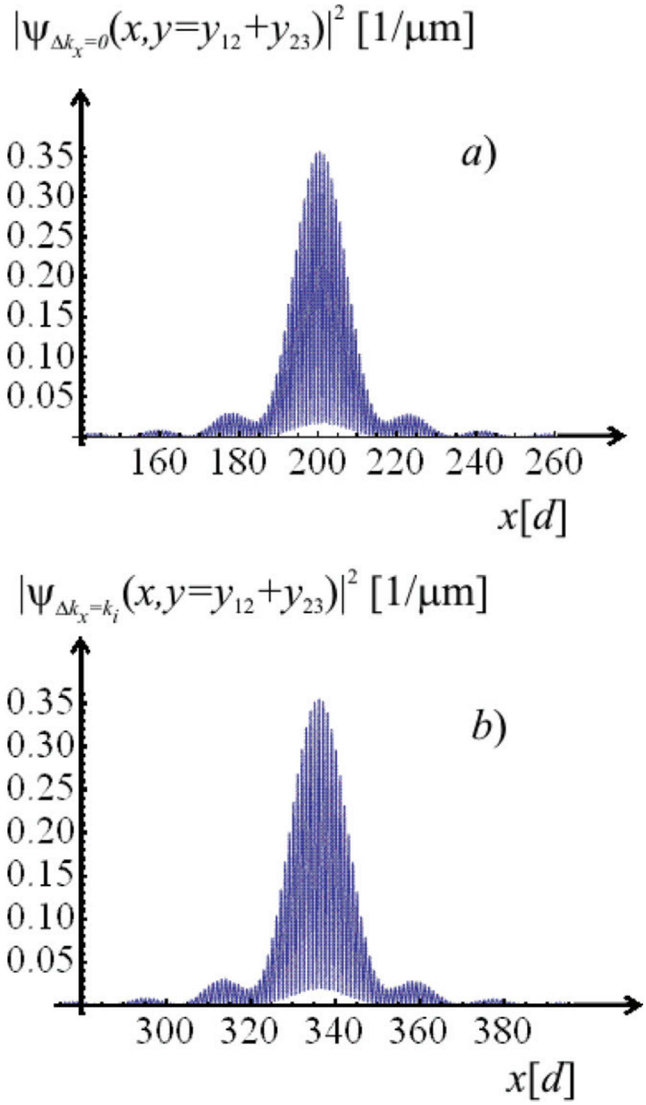

Figure 3. The function $|\psi_{\Delta k_x}(x, y = y_{12} + y_{23})|^2$ when the laser is off (a), with $\Delta k_x = 0$, and when the laser is on (b), with $y'_{12} = 5kd/8k_i$ and $\Delta k_x = k_i$. The parameters considered are the same as in figure 2. The period of the fast oscillations observed is the same as the grating period.

After analytical integration of (26), we obtain

$$T(y'_{12},\Delta x_3)=a+bB\cos\left(2\pi\,\Delta x_3/d+d_{\mathrm{p}}k_i\right),\tag{27}$$

where

$$B=\frac{3}{4\pi}\frac{\lambda_i}{d_{\mathrm{p}}}\times\left[\left(1-\frac{1}{(2\pi)^2}\frac{\lambda_i^2}{d_{\mathrm{p}}^2}\right)\sin\left(2\pi\frac{d_{\mathrm{p}}}{\lambda_i}\right)+\frac{1}{2\pi}\frac{\lambda_i}{d_{\mathrm{p}}}\cos\left(2\pi\frac{d_p}{\lambda_i}\right)\right].\tag{28}$$

As is apparent from (27), the contrast when the laser is off and on is determined by the quantities a, b and B, as

$$C_0=\left|\frac{b}{a}\right|,\qquad C=\frac{T_{\max}-T_{\min}}{T_{\max}+T_{\min}}=\left|\frac{b}{a}B\right|,\tag{29}$$

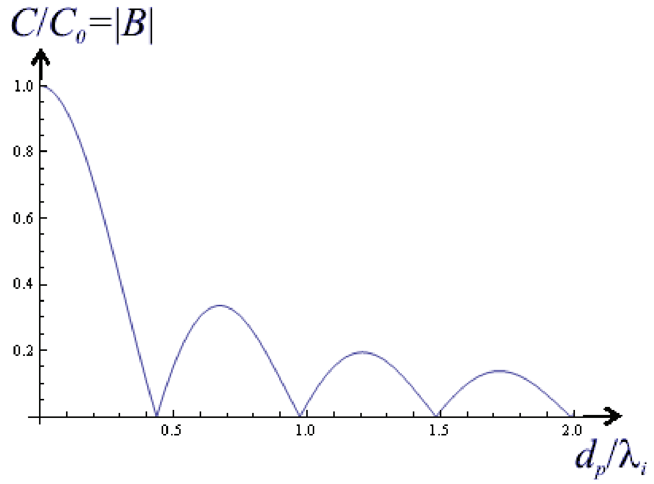

Figure 4. Relative contrast as a function of d_p/λ_i.

with the relative contrast being

$$C/C_0 = |B|. \tag{30}$$

The relative contrast displayed in figure 4 is an analytic function of the ratio d_p/λ_i.

6. Conclusions

Our description and explanation of the experiment by Chapman *et al* [1, 2] is based on the assumption that there is a wave associated with an atom. The evolution of the wave is determined by the Schrödinger equation, the boundary conditions imposed by the gratings and the interaction between the atom and a photon. As shown here, an initial harmonic atomic wave is transformed by the first grating into a wave with narrow maxima at the points along and in the close vicinity of three particular paths (although only two of them are of relevance in this experiment) and negligible values at any other point. The two maxima move together; in other words, the wave is coherent. At the grating, the particle associated with the wave acquires randomly a new value for its momentum, which directs the particle towards one of the paths along which it moves following the time evolution of a wave field. The photon scattering that takes place between the first and second gratings causes the change of the atomic transverse momentum. Consequently, the atomic wave function is shifted along the x-axis, but without destroying the coherence, and the contrast of the transmission function will not depend neither on the point of scattering nor on the photon wavelength.

The dependence of the transmission on the ratio d_p/λ_i is obtained after integrating over all possible values of transferred momenta. In this explanation, wave and particle properties are compatible since both are present and play a role. Within the model presented here, the behavior of contrast can be explained for all values of d_p/λ_i. Moreover, the problem of explaining the so-called revivals of the coherence after it was 'lost' at $d_p/\lambda_i \cong 0.5$ does not appear, as required by complementarity.

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