Black Holes Without Singularities

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Abstract

We study the properties of a completely integrable deformation of the CGHS dilaton gravity model in two dimensions. The solution is shown to represent a singularity free black hole that at large distances asymptoticaly joins to the CGHS solution.

One of the fundamental unsolved problems in theoretical physics is the unification of quantum theory and gravity. One of the reason why this is so difficult stems from the complicated nonlinear structure of the equations of general relativity. These equations are much simpler in lower dimensions. For this reason there has recently been much activity related to the quantization of gravity in two and three dimensions. One of the most important results in 2d was the exactly solvable dilaton gravity model constructed by Callan, Giddings, Harvey and Strominger [1]. The CGHS model has 2d black hole solutions that are remarkably similar to the Schwarzschild solution of general relativity. Of the four fundamental interactions in nature, gravity is by far the weakest. For this reason, we can hope to see quantum effects only in the vicinity of classical singularities. Penrose and Hawking have shown that these singularities are endemic in general relativity. The general belief is that quantization will rid gravitation of singularities, just as in atomic physics it got rid of the singularity of the Coulomb potential. If this is indeed the case, then there must exist a non-singular gravitational effective action whose classical equations encode the full quantum theory. This effective action must have the Planck length $L_{\rm Planck}$ in it as an input parameter. For $L \gg L_{\rm Planck}$ the effective model must be indistinguishable from the classical gravity action. The search for such an effective model parallels Landau's treatment of phase transitions in ferromagnets. Landau chose (the simplest) effective action (Gibbs potential in statistical mechanics parlance) that led to a qualitatively correct discription of phase transitions.

An important result in dilaton gravity has been the work of Louis-Martinez and Kunstatter [2], who reduced the solution of the general dilaton gravity model to the solution of two ordinary integrals, i.e. to two quadratures. In a previous paper [3] we used their proceedure to construct a dilaton model that yields a black hole without a singularity. In this paper we will review the central results of this derivation, construct the deformed CGHS model and show that it leads to a maximal curvature proportional to L_{Planck}^{-1}.

The action of all dilaton gravity models can be put into the general form

$$S = \int d^2x\sqrt{-g}\left[\frac{1}{2}\,g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi) + D(\phi)R\right] \; . \tag{1}$$

The potentials $V(\phi)$ and $D(\phi)$ classify all the possible models. Performing a conformal scaling of the metric $\tilde{g}_{\alpha\beta} = e^{-2F(\phi)}g_{\alpha\beta}$, where the scaling factor $F(\phi)$ satisfies $F' = -1/4(D')^{-1}$ we can put the action into the simplified form

$$S = \int d^2x\sqrt{-\tilde{g}}\left[\tilde{\phi}\tilde{R} - \tilde{V}(\tilde{\phi})\right] \; , \tag{2}$$

where $\tilde{R}$ is the scalar curvature corresponding to $\tilde{g}_{\alpha\beta}$, and we have introduced the new dilaton field and potential according to $\tilde{\phi} = D(\phi)$ and $\tilde{V}(\tilde{\phi}) = e^{2F(\phi)}V(\phi)$. This form of the dilaton gravity action is obviously much easier to work with since we have lost the kinetic term for the dilaton field.

A well known property of two dimensional manifolds allows us to localy, i.e. patch by patch, choose conformally flat coordinates for which $\tilde{g}_{\alpha\beta} = e^{2\rho}\,\eta_{\alpha\beta}$. Louis-Martinez and Kunstatter [2] have shown that we can choose a coordinate system in which the solution of the general dilaton model is static and given by

$$x = -2\int\frac{d\tilde{\phi}}{W(\tilde{\phi}) + C} \tag{3}$$

$$e^{2\rho} = -\frac{C + W(\tilde{\phi})}{4} \; , \tag{4}$$

where the pre-potential $W(\tilde{\phi})$ is given by $\frac{dW}{d\tilde{\phi}} = \tilde{V}(\tilde{\phi})$, and C is an invariant. It is easy to show that $C < 0$, and that without loss of generality we can choose $C = -1$. As we can see, the above solution is given in terms of two quadratures: the first connecting F and D, and second one given in (3). A given model is completely integrable only if we can calculate both quadratures in closed form.

The CGHS model is an example of a completely integrable dilaton gravity model. The standard form of the CGHS action is

$$S = \int d^2x\sqrt{-g}\,e^{-2\varphi}\left(R + 4g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi + 4\lambda^2\right) \; . \tag{5}$$

The simple field redefinition $\phi = \sqrt{8}\, e^{-\varphi}$ puts this into the general form for dilaton gravity actions given in (1). Further, a conformal scaling with $F(\phi) = -\ln\phi$ gives us the simplified form of the CGHS action

$$S = \int d^2x \sqrt{-\tilde{g}} \left(\tilde{\phi}\tilde{R} + \frac{1}{2}\,\lambda^2 \right) \; . \tag{6}$$

The CGHS model is completely integrable. A simple application of the Louis-Martinez and Kunstatter proceedure gives us the general solution. For the scalar curvature we find $R = -32\, A^{-1}$, where we have introduced $A = 8/\lambda^2 \left(e^{\frac{\lambda^2}{4}x} - 1 \right)$. The metric for the general dilaton model, given in terms of F and ρ, is simply $ds^2 = e^{2(F+\rho)}(-dt^2 + dx^2)$. In the case of CGHS we get

$$e^{2(F+\rho)} = \frac{\lambda^2}{64}\, \frac{e^{\frac{\lambda^2}{4}x}}{e^{\frac{\lambda^2}{4}x} - 1} \; , \tag{7}$$

which vanishes for $x = -\infty$. For stationary metrics the equation $g_{00} = 0$ determines the horizon. Therefore, in these coordinates the CGHS black hole has a horizon at $x = -\infty$. The curvature, on the other hand, is well behaved at this point. As with the Schwartzschild black hole one can now find coordinates which are well behaved at the horizon. In this way one finally obtains information about the global character of the manifold.

We now proceed to construct a new dilaton gravity model that satisfies the following requirements: it is completely integrable, for $x \to \infty$ it goes over into the CGHS model and is singularity free. As we have seen, dilaton gravity models are specified by giving the two potentials $D(\phi)$ and $V(\phi)$. It is very difficult to see how one should deform these potentials from their CGHS form in order to satisfy the above criteria. Note, however, that the models are also uniquely determined by giving $F(\phi)$ and $\tilde{V}(\tilde{\phi})$. This is much better for us since we have now untangled the two integrability requirements: $F(\phi)$ determines the first quadrature and $\tilde{V}(\tilde{\phi})$ the second. Deformations of a given model correspond to changes of both of these functions. In this paper we will look at a simpler problem. We shall keep $\tilde{V}(\tilde{\phi})$ fixed, i.e. it will have the same value as in the CGHS model. We will only deform $F(\phi)$. By doing this we are guaranteed that the second (and more difficult) quadrature is automatically solved. From our second requirement we see that for large x the dilaton field $\phi(x)$ must be near to its CGHS form. Specifically, $x \to \infty$ corresponds to $\phi \to \infty$. Thus, our second requirement imposes that for $\phi \to \infty$ we have $F(\phi) \to -\ln\phi$. $F(\phi)$ must also be such that the first quadrature is exactly solvable. To do this we choose

$$F(\phi) = -\frac{1}{\alpha} \ln \left(\frac{1 + \beta\phi^\alpha}{\beta} \right) \; , \tag{8}$$

with $\alpha > 0$. The α and β values parametrize our class of deformations. The first quadrature gives

$$D(\phi) = \begin{cases} \frac{1}{8}\phi^2 + \frac{1}{4\beta}\ln\phi & \text{for } \alpha = 2 \\ \frac{1}{8}\phi^2 + \frac{1}{4\beta(2-\alpha)}\phi^{2-\alpha} & \text{for } \alpha \neq 2\ . \end{cases} \tag{9}$$

On the other hand, the potential $V(\phi)$ is now simply

$$V(\phi) = -\frac{1}{2}\lambda^2 \left(\frac{1+\beta\phi^\alpha}{\beta}\right)^{\frac{2}{\alpha}} \ . \tag{10}$$

The choice of α corresponds to a choice of explicit model, while β just sets a scale for the dilaton field. Rather than work here with the general deformed model we will now concentrate on the simplest model in this class; the one corresponding to the choice $\alpha = 4$. The action for this model is

$$S = \int d^2x\sqrt{-g}\left(\frac{1}{2}\,g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{2}\,\lambda^2\left(\frac{1+\beta\phi^4}{\beta}\right)^{\frac{1}{2}} + \frac{1}{8}\left(\phi^2 - \frac{1}{\beta\phi^2}\right)R\right) \ . \tag{11}$$

Note that for $\beta \to \infty$ this goes over into the action of the CGHS model. As we have seen, β is just a scale for ϕ, hence, this is just a re-statement of our second requirement. From our construction we see that (11) corresponds, for each finite value of β, to a model that satisfies our first two requirements. All that is left is to check that the theory is indeed free of singularities. Being in two dimensions all that we need to check is the scalar curvature. A simple but tedious calculation now gives

$$\begin{aligned} R \ &= \ \sqrt{2}\,\lambda^2\left(\frac{1}{\beta} + A^2\right)^{-\frac{7}{4}}\left(A + \sqrt{\frac{1}{\beta} + A^2}\right)^{\frac{1}{2}} \cdot \\ &\quad \cdot\left\{\frac{16}{\beta\lambda^2} + \frac{3}{\beta}A - \frac{8}{\lambda^2}A^2 + \left(\frac{1}{\beta} - \frac{8}{\lambda^2}A\right)\sqrt{\frac{1}{\beta} + A^2}\right\} \ . \end{aligned} \tag{12}$$

For $\beta \to \infty$ we indeed find that R goes over into the CGHS result. From (12) we see that the curvature of the deformed CGHS model is indeed not singular. As may be seen in Figure 1, the deformed model has maximal curvature at $x = 0$. Its value is

$$R_{\text{max}} = \sqrt{2}\,\left(16\beta^{\frac{1}{2}} + \lambda^2\right) \ . \tag{13}$$

At right infinity the deformed model tends to the CGHS result. On the other hand, at left infinity both the CGHS model and its deformation tend to a de Sitter space $R = \Lambda$. However, for CGHS we have $\Lambda = 4\lambda^2$, while for the deformed model the constant is a complicated function of β and λ. Rather than writing it out let us only give the result for large β when we have $\Lambda = 2^{-10}\lambda^8\beta^{-\frac{3}{2}}$. We have just determined that the $x \to -\infty$ and $\beta \to \infty$ limits do not commute. Therefore, imposing that our model joins to CGHS at right infinity doesn't automaticaly guarantee a similar joininig at left infinity.

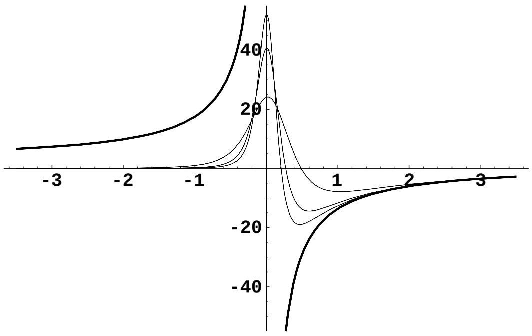

Figure 1: $R(x)$ for the CGHS model (thick line) and deformations with $\beta = 1$, 3 and 5. As β increases the deformations for $x > 0$ join CGHS. The plot is for $\lambda^2 = 1$

We are now in the position of trying to interpret the meaning of our deformed CGHS model. Obviously, one possibility is to think of (11) as the classical action of a model with scale $\frac{1}{\beta}$. However, it seems more natural to interpret our model as an effective action. $\frac{1}{\beta}$ then naturaly comes about from quantization, while $\beta \to \infty$ corresponds to the semi classical limit. Our model should thus be the effective action corresponding to the quantization of the CGHS model. Quantization gives $S \sim \hbar$, and essentialy dimensional analysis (in units $G = c = 1$) gives $\phi^2 \sim \hbar$, as well as $\frac{1}{\beta} \sim \hbar^2$. Therefore, if we are to interpret our model as an effective action then $\beta = \kappa \hbar^{-2}$, where κ is a constant of the order of unity. We see then that the maximal curvature (13) is proportional to $\frac{1}{\hbar}$, i.e. represents a non-perturbative effect. Expanding our model in $\hbar$ we find

$$S_{\mathrm{eff}} = S_{\mathrm{cghs}} - \frac{1}{8\kappa}\,\hbar^2 \int d^2x\,\sqrt{-g}\,\left(R - 2\lambda^2\right)\phi^{-2} + o(\hbar^4)\ . \tag{14}$$

The leading correction to CGHS is of the form of the Jackiw-Teitelboim action for 2d gravity. It would be very interesting to get this result by quantizing some fundamental 2d theory. To do this we would need to start from the CGHS model coupled to some matter fields. We would then have to integrate out the matter. The last step would be to calculate the effective action. It is probably impossible to do this exactly, however, we could hope to do this perturbatively and compare with (14).

References

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