# Spin Liquid Behavior in Electronic Griffiths Phases

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**Abstract.** We study a possible mechanism of disorder-driven non-Fermi liquid behavior in heavy fermion systems. We present simple analytical arguments explaining the universal emergence of electronic Griffiths phases as precursors of disorder-driven metal-insulator transitions in correlated electronic systems. Then we examine the interplay of the Kondo effect and the RKKY interactions in electronic Griffiths phases using extended dynamical mean-field theory methods. We find that sub-Ohmic dissipation is generated for sufficiently strong disorder, leading to a suppression of the Kondo screening on a finite fraction of spins, and giving rise to universal spin-liquid behavior.

#### **INTRODUCTION**

In the last fifteen years a large number of heavy fermion alloys has been discovered whose properties clearly deviate from the predictions of the Fermi liquid theory [1]. The origin of this "non-Fermi liquid" (NFL) behavior is still an unresolved problem, and it is hoped that its solution will give us a better understanding of the physics of strongly correlated systems in general. Here we will concentrate on those heavy fermion alloys, like  $UCu_{5-x}Pd_x$ , whose thermodynamic and transport properties are believed to be dominated by disorder. Following the previous work [2] within an extended dynamical mean field theory (DMFT), we develop a simple model of non-Fermi liquid behavior in disordered Kondo lattice systems which can be solved even analytically. Within this model, rare sites with the lowest Kondo temperatures provide a leading singular thermodynamic response. We refer to the system described by this model as an electronic Griffiths phase. We then include random inter-site magnetic interactions and solve the model within an extended DMFT. We find that in a presence of even a moderate disorder, a fraction of the *f*-electrons decouple from the conduction bath forming a spin liquid, which gives a leading logarithmic contribution to the local dynamic magnetic susceptibility.

### **EFFECTIVE MODEL OF THE ELECTRONIC GRIFFITHS PHASE**

We introduce and solve an effective model of the electronic Griffiths phase as a DMFT model with a Gaussian distribution of random site energies. We should emphasize that

localization is not present in this effective model, but the Griffiths phase emerges in qualitatively the same fashion as in the more realistic calculations within "statistical" DMFT [1].

We consider the Anderson lattice model where the disorder is introduced by random site energies  $\varepsilon_i$  in the conduction band, as given by the Hamiltonian

$$H = -t \sum_{\langle ij \rangle \sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + \text{H.c.}) + \sum_{j\sigma} (\varepsilon_{j} - \mu) c^{\dagger}_{j\sigma} c_{j\sigma}$$
$$+ V \sum_{j\sigma} \left( c^{\dagger}_{j\sigma} f_{j\sigma} + \text{H.c.} \right) + \sum_{j\sigma} E_{f} f^{\dagger}_{j\sigma} f_{j\sigma} + U \sum_{j} f^{\dagger}_{j\uparrow} f_{j\uparrow} f^{\dagger}_{j\downarrow} f_{j\downarrow}, \qquad (1)$$

where  $f_{j\sigma}$  and  $c_{j\sigma}$  are annihilation operators for f- and conduction electrons, respectively. V is the hybridization parameter, and  $E_f$  is the f-electron energy. We assume  $U \rightarrow \infty$ , and choose a Gaussian distribution of random site energies for the conduction band

$$P(\varepsilon_i) = (2\pi W^2)^{-1/2} \exp\{-\frac{1}{2}\varepsilon_i^2/W^2\}.$$
 (2)

Within DMFT, which is formally exact in the limit of large coordination, the solution of the disordered Anderson lattice problem reduces to solving an ensemble of a single impurity problems supplemented by a self-consistency condition. DMFT equations can be solved analytically at zero temperature using mean field slave boson approach. Site dependent Kondo temperatures are equal to  $T_{Kj} \approx T_K^0 e^{-1/\lambda_j}$ , where the coupling constant is  $\lambda_j = t^2 \rho_o J/\varepsilon_j^2$ ,  $J = 2V^2/|E_f|$ ,  $\rho_o$  is the density of states at the Fermi level, and  $T_K^0$  is the Kondo temperature in the clean limit. From these equations, we can immediately find the desired distribution of local Kondo temperatures  $P(T_K) = P(\varepsilon(T_K))|d\varepsilon/dT_K|$ , which (up to a negligible logarithmic correction) is given asymptotically by

$$P(T_K) \propto (T_K/T_K^0)^{\alpha - 1}$$
, with  $\alpha(W) = t^2 \rho_o J/2W^2$ . (3)

This expression has exactly the form expected for a Griffiths phase, where the exponent characterizing the local energy scale distribution assumes a parameter-dependent (tunable) form.

The average susceptibility is equal to  $\chi(T) = \int_0^\infty P(T_K)\chi(T, T_K)dT_K$ , with  $\chi(T, T_K) = C/(T + aT_K)$ . At weak disorder, the exponent  $\alpha$  is large and the distribution  $P(T_K)$  is regular,  $\chi(0) = \chi_o + C_2/(\alpha - 1)$ , but NFL behavior emerges once  $\alpha \leq 1$ , which corresponds to  $W \geq W_{nfl} = \sqrt{t^2 \rho_o J/2}$ . For  $\alpha = 1$  the magnetic susceptibility has a logarithmic divergence,  $\chi(T) \propto \ln(1/T)$ , characteristic of marginal Fermi liquid behavior, while for  $\alpha < 1$  a power law divergence is obtained,  $\chi(T) \propto T^{\alpha-1}$  as  $T \to 0$ . The same singularity also leads to an anomalous behavior in the transport properties [1].

In the above DMFT formulation, we had to choose a special form of disorder distribution in order to obtain the desired power-law distribution of Kondo temperatures. On the other hand, from numerical simulations of lattices with finite coordination, it has been established that the emergence of the Griffiths phase is a universal phenomenon [1] which appears even for a bounded distribution of site energies. We have shown [3] that the specific form of randomness is universally generated by renormalizations due to disorder-induced fluctuations of the conduction bath: The renormalized distribution  $P(\tilde{\varepsilon}_i)$  will have universal Gaussian tails even if the bare distribution  $P(\varepsilon_i)$  is bounded.

## RKKY INTERACTIONS IN THE ELECTRONIC GRIFFITHS PHASE

The simplicity of this DMFT effective model makes it possible to describe all the qualitative features of the solution using simple analytical arguments, thus eliminating the need for large scale numerical computations in the description of the electronic Griffiths phase. This is crucial in order to address more complicated issues, such as the role of additional RKKY interactions in disordered Kondo alloys. According to the existing picture, all the spins with  $T_K < T$  will not be Kondo screened, thus providing a large contribution to thermodynamic response. These decoupled spins will, however, not act as free local moments, but will feature dynamics dominated by frustrating intersite RKKY interactions. The simplifications introduced by our effective model open an attractive avenue to incorporate both the Kondo effect and the RKKY interaction in a single theory.

We concentrate on an exactly solvable model within an extended DMFT[4]. In addition to the effective model term describing the Kondo impurity, a new term appears into the local action coming from infinite-ranged randomly distributed RKKY interactions:

$$\mathscr{A}_{j} = \mathscr{A}_{j}^{0} - \frac{J^{2}}{2} \int_{0}^{\beta} d\tau \int_{0}^{\beta} d\tau' \chi(\tau - \tau') \mathbf{S}_{j}(\tau) \cdot \mathbf{S}_{j}(\tau').$$
(4)

In this expression,  $\mathbf{S}_j$  and  $\mathbf{s}_j = \frac{1}{2} \sum_{\alpha\beta} c_{j\alpha}^{\dagger} \sigma_{\alpha\beta} c_{j\beta}^{\dagger}$  represent a localized spin and the conduction electron spin density at site *j*, respectively. On a level of a single impurity this exactly the Bose-Fermi Kondo model. The Kondo spin is embedded into the fermionic bath of the conduction electrons and the bosonic bath describing spin fluctuations. However, for the Kondo lattice we need to study an ensemble of such Bose-Fermi impurity models. The bosonic and the conduction bath are determined by the self-consistency conditions  $\chi(\tau) = \overline{\langle T_{\tau} \mathbf{S}_j(\tau) \cdot \mathbf{S}_j(0) \rangle_{\mathscr{A}_j}}$  and  $G_c(\tau) = -\overline{\langle T_{\tau} c_{j\sigma}(\tau) c_{j\sigma}^{\dagger}(0) \rangle_{\mathscr{A}_j}}$ .

First we examine a behavior of the disordered Kondo lattice in a presence of a given bosonic bath

$$\chi(\tau) \sim 1/\tau^{2-\varepsilon}.$$
 (5)

Fermi liquid behavior is recovered for  $\varepsilon = 0$ , but for  $\varepsilon > 0$  (sub-Ohmic dissipation), and for sufficiently small bare Kondo temperature  $T_K$ , the spin decouples from the conduction electrons. Within an electronic Griffiths phase, however, the disordered Kondo lattice has a very broad distribution of local Kondo temperatures  $P(T_K) \sim (T_K)^{\alpha-1}$ . Therefore, for  $\varepsilon > 0$  and arbitrarily weak coupling to the bosonic bath (i.e. weak RKKY interaction), a fraction of the spins will decouple.

To obtain a sufficient condition for decoupling, we examine the stability of the Fermi liquid solution, by considering the limit of infinitesimal RKKY interactions. To leading order we replace  $\chi(\tau) \longrightarrow \chi_o(\tau) \equiv \chi(\tau; J = 0)$ , and the calculation reduces to the "bare model". The resulting bosonic bath, which is an average over the site-dependent local dynamic spin susceptibility,  $\chi_o(i\omega_n) = \int dT_K P(T_K) \chi(T_K, i\omega_n)$ , has a Fermi liquid form in the presence of weak disorder. However, for stronger randomness,  $W > W^* \approx \sqrt{t^2 \rho_o J_K}/2$  corresponding to  $\alpha < 2$ , the power law distribution of energy scales within a Griffiths phase produces sub-Ohmic dissipation, corresponding to  $\varepsilon = 2 - \alpha > 0$ . Note

that the estimate based on the bare theory sets an *upper bound* for the true critical disorder strength, i.e.  $W_c < W^* = W_{nfl}/\sqrt{2}$  (here,  $W_{nfl} \approx \sqrt{t^2 \rho_o J_K/2}$  is the threshold for NFL behavior in the bare model, corresponding to  $\alpha = 1$ ). We emphasize that within the electronic Griffiths phase, such decoupling emerges for  $W > W_c$  even for arbitrarily small J, in contrast to the clean case where much stronger RKKY interactions  $(J > J_c \approx 10T_K)$  are required to destroy the Kondo effect.

The actual value  $\varepsilon$  has to be self-consistently determined, since the solution of each impurity model depends sensitively on  $\varepsilon$ , and the solutions of the impurity models in its turn determine the form of the bosonic bath. We find that the decoupled spins provide a dominant contribution to the thermodynamic response which assumes a marginal Fermi liquid form,  $\chi(T) \sim \ln(1/T)$ . We emphasize that this leading singularity has a universal form throughout the two fluid phase.

To obtain quantitative results, we have solved the extended DMFT equations in the large-*N* limit. Our most important results can be summarized in a form of a phase diagram. For the disorder strength smaller than a critical value  $W^*$ , we find the Fermi liquid phase. For stronger disorder decoupled spins are formed, and a universal marginal Fermi liquid behavior emerges at low temperatures. Spin glass phase boundary is estimated from the mean-field condition  $\sqrt{\langle \chi_i^2 \rangle} J/\sqrt{2} = 1$ .



**FIGURE 1.** Phase diagram obtained with  $J_K = 0.8$ , J = 0.05 and  $\mu = -0.1$  corresponding to  $T_K(\varepsilon_i = 0) = 0.1$  and  $W_c \approx 0.1$ . The inset shows the fraction of decoupled spins as a function of disorder.

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