Short-time Effective-action Approach for Numerical Studies of Rotating Ideal BECs

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ABSTRACT

Recently, we have developed an efficient recursive approach for analytically calculating the short-time expansion of the propagator to extremely high orders for a general many-body quantum system [1]. Here we apply this technique for a numerical study of thermodynamical properties of a rotating ideal Bose gas of ^{87}Rb atoms in an anharmonic trap [2]. First, the energy spectrum of the system is obtained by the exact diagonalization of the discretized short-time propagator. Then the condensation temperature, ground-state occupancy, density profiles and the time-of-flight absorption pictures are calculated for varying rotation frequencies, including the critical and over-critical regime. The obtained results improve previous semiclassical calculations [3].

FAST ROTATING BECs

- Response to rotation is one of the hallmarks of superfluidity
- Fast-rotating Bose-Einstein condensates are challenging subject from both experimental and theoretical point of view
- Experimentally, it is a delicate matter to achieve fast rotation and to keep the spatial confinement of atoms
- Recent experiment [2] resolved this by introducing an additional anharmonic part into the common harmonic trapping potential for the ensemble of $N_a = 3\times 10^5$ ^{87}Rb atoms:

$$V_{BEC} = \frac{M}{2}\,(\omega_\perp^2 - \Omega^2) r_\perp^2 + \frac{M}{2}\,\omega_z^2 z^2 + \frac{k}{4}\,r_\perp^4,$$

$$\omega_\perp = 2\pi\times 64.8\ \mathrm{Hz}, \omega_z = 2\pi \times 11.0\ \mathrm{Hz}, k_{exp} = 2.6\times 10^{-11}\ \mathrm{Jm}^{-4}.$$

- This type of setup allows fast rotating frequencies close to the critical frequency, i.e. $r = \Omega/\omega_\perp \sim 1$
- The small quartic anharmonicity in *x-y* plane keeps the condensate spatially confined, even for the critical frequency

PROPERTIES OF IDEAL BECs

- Within the grand-canonical ensemble, free energy of the ideal Bose gas can be expressed by the cumulant expansion

$$\mathcal{F} = -\frac{1}{\beta}\ln \mathcal{Z} = -\frac{1}{\beta}\sum_{m=1}^{\infty}\frac{e^{m\beta\mu}}{m}\mathcal{Z}_1(m\beta)$$

- Number of particles for *T*<*Tc*, with $\mu = E_0$

$$N = N_0 + \sum_{m=1}^{\infty}(e^{m\beta E_0}\mathcal{Z}_1(m\beta) - 1)$$

- Definition of the condensation temperature

$$\frac{N_0}{N} = 1 - \frac{1}{N}\sum_{m=1}^{\infty}(e^{m\beta_c E_0}\mathcal{Z}_1(m\beta_c) - 1) = 0$$

- Numerically calculated *Tc* slightly lower than the semiclassical result, as expected [4,5].
- Energy eigenvalues and eigenstates are obtained using the approach from Ref. [6], which can be substantially improved [7] by applying the effective action approach [1]
- Density profiles for low temperatures

$$n(\vec r) = N_0|\psi_0(\vec r)|^2 + \sum_{n\geq 1} N_n|\psi_n(\vec r)|^2$$

- Density profiles for mid-range temperatures

$$n(\vec r) = N_0|\psi_0(\vec r)|^2 + \sum_{m\geq 1}\left[e^{m\beta E_0}A(\vec r, 0; \vec r, m\beta\hbar) - |\psi_0(\vec r)|^2\right]$$

- Time-of-flight graphs for density profiles for low temperatures

$$n(\vec r, t) = N_0|\psi_0(\vec r, t)|^2 + \sum_{n\geq 1} N_n|\psi_n(\vec r, t)|^2$$

where $\psi_n(\vec r, t) = \int \frac{d^3\vec k\, d^3\vec R}{(2\pi)^3}\, e^{-i\omega_{\vec k}t + i\vec k\cdot\vec r - i\vec k\cdot\vec R}\,\psi_n(\vec R)$

- Time-of-flight graphs for density profiles for mid-range temperatures

$$n(\vec r, t) = N_0|\psi_0(\vec r, t)|^2 + \sum_{m\geq 1}\left[e^{m\beta E_0}\int\frac{d^3\vec k_1\, d^3\vec k_2\, d^3\vec R_1\, d^3\vec R_2}{(2\pi)^6}\times\right.$$
$$\left. e^{-i(\omega_{\vec k_1} - \omega_{\vec k_2})t + i(\vec k_1 - \vec k_2)\cdot\vec r - i\vec k_1\cdot\vec R_1 + i\vec k_2\cdot\vec R_2}\, A(\vec R_1, 0; \vec R_2, m\beta\hbar) - |\psi_0(\vec r, t)|^2\right]$$

NUMERICAL RESULTS: Global properties of BECs

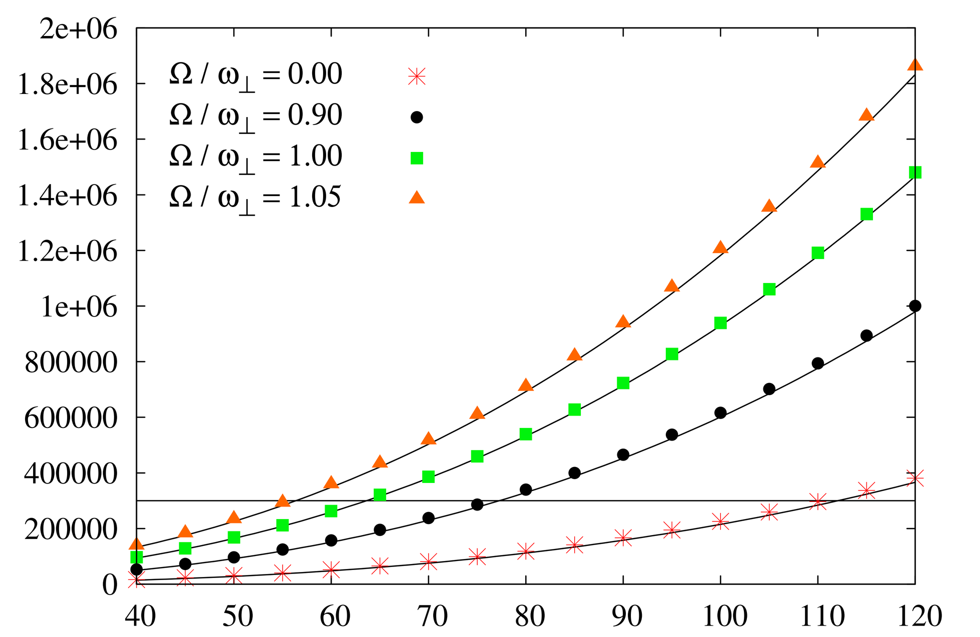

Fig. 1: Number of particles as a function of $T[\mathrm{nK}]$, calculated with $p=18$ effective action. The horizontal line shows the number of bosons in the experiment. Solid lines give semiclassical results from Ref. [3].

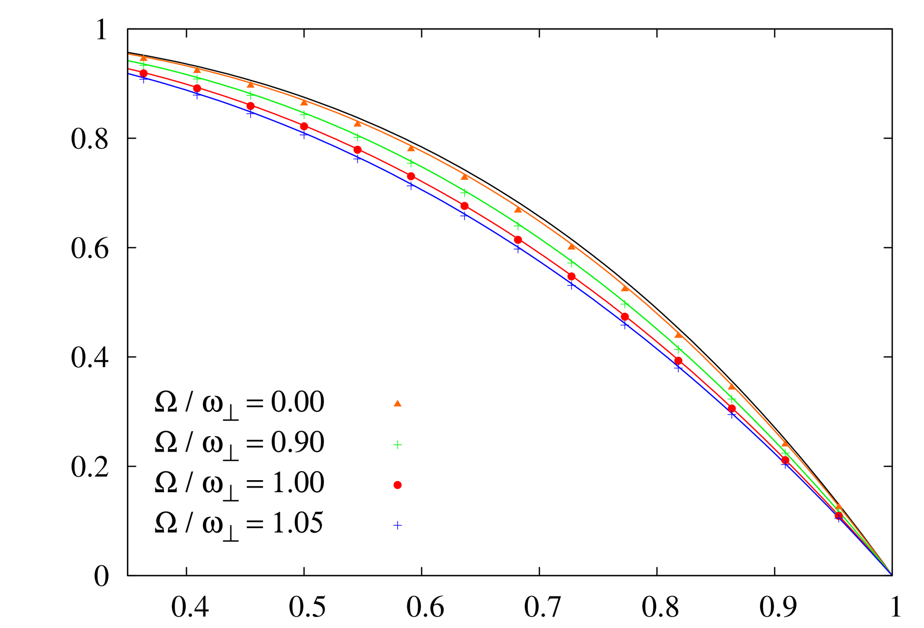

Fig. 2: Ground state occupancy as a function of T/T_c, calculated with $p=18$ effective action. Solid lines represent semiclassical values from Ref. [3].

NUMERICAL RESULTS: Condensate density profiles

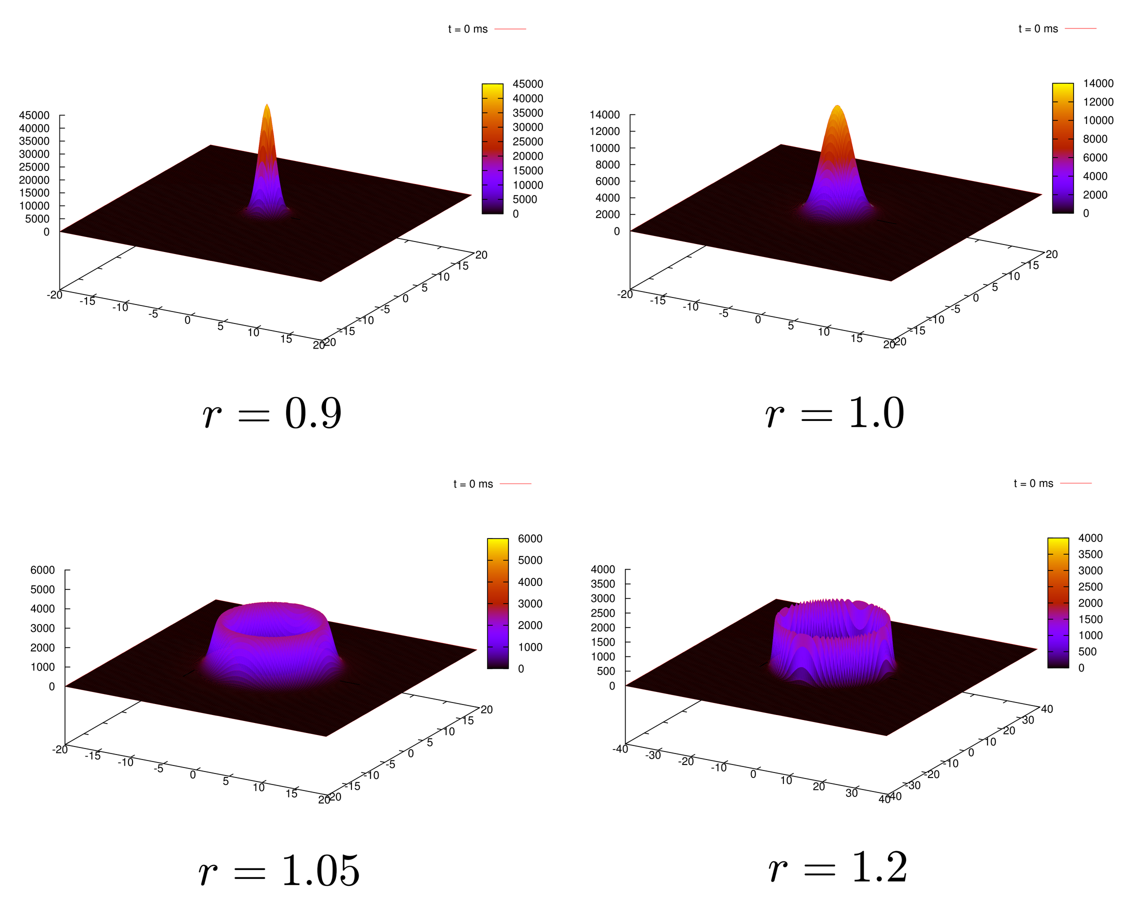

Fig. 3: Density profiles in *x-y* plane for condensates at *T* = 10 nK rotating close to the critical frequency, calculated with effective action of the order $p=21$. Linear size of profiles is $54\mu m$ for *r*=0.9, 1.0, 1.05 and $108\mu m$ for *r* =1.2.

SHORT-TIME EFFECTIVE ACTION APPROACH

- Recently introduced short-time effective-action approach [1,8-10] allows efficient calculation of general many-body transition amplitudes with high precision
- Short-time amplitudes are written in the form

$$A(\vec q_1, 0; \vec q_2, t) = \frac{1}{(2\pi t)^{d/2}}\exp\left[-\frac{\vec\delta^2}{2t} - t\,W\right]$$

where *W* is the effective potential and $\vec\delta = \vec q_1 - \vec q_2$

- Effective potential can be expressed as a double series in the time of propagation and discretized velocity $\vec\delta$, as we have shown previously [8-10]
- A set of recursive relations for the effective potential is derived in Ref. [1], and analytic expressions for expansion of *W* up to very high order *p* are obtained
- Such effective actions can be numerically used for accurate calculation of short-time transition amplitudes, due to rapid convergence

$$A(\vec q_1, 0; \vec q_2, t) = A^{(p)}(\vec q_1, 0; \vec q_2, t) + O(t^p).$$

- Coupled with the exact diagonalization of the evolution operator [7], this approach also allows highly efficient calculation of energy eigenvalues and eigenstates of few-body systems

NUMERICAL RESULTS: Time-of-flight graphs for condensate density profiles

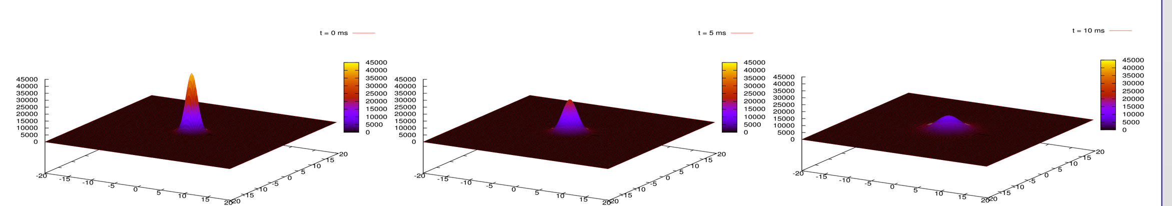

Fig. 4: Time evolution of density profiles in *x-y* plane for condensate at *T* = 10 nK rotating at *r* =0.9 frequency, calculated with effective action of the order $p=21$. Linear size of condensate density profiles is $54\mu m$.

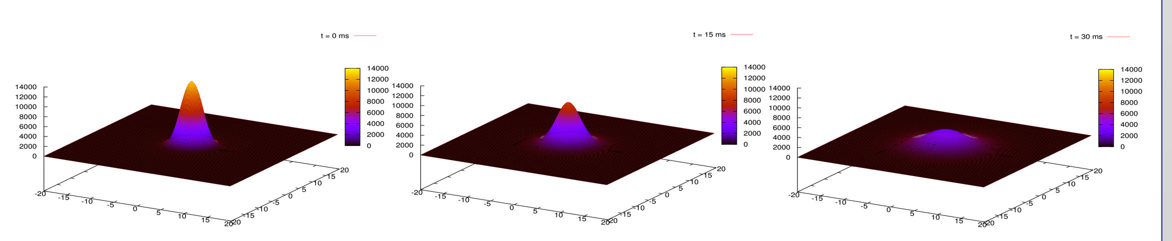

Fig. 5: Time evolution of density profiles in *x-y* plane for condensate at *T* = 10 nK rotating at *r* =1.0 frequency, calculated with effective action of the order $p=21$. Linear size of condensate density profiles is $54\mu m$.

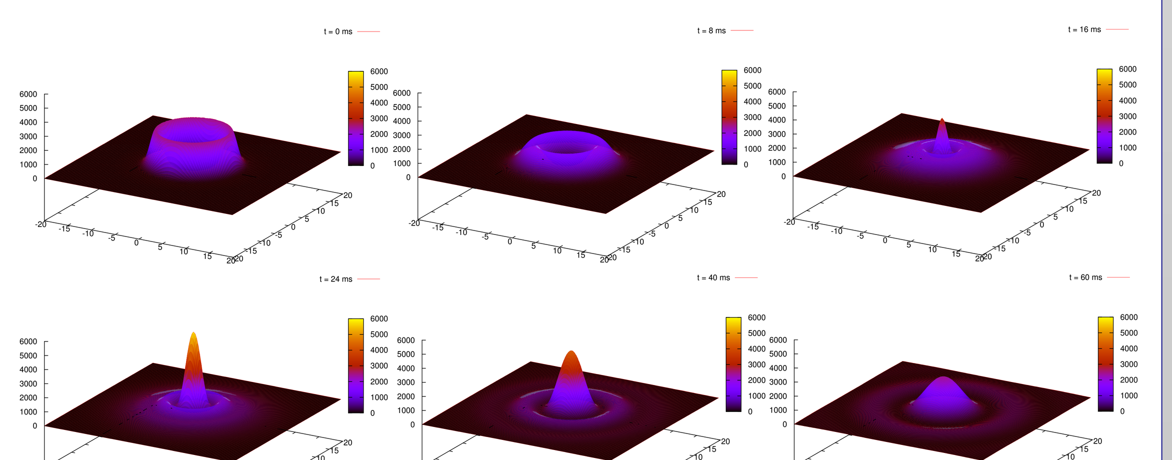

Fig. 6: Time evolution of density profiles in *x-y* plane for condensate at *T* = 10 nK rotating at *r* =1.05 frequency, calculated with effective action of the order $p=21$. Linear size of condensate density profiles is $54\mu m$.

- We observe oscillation of the density profile at the origin for overcritical rotation, which considerably increases typical time-scale of free expansion

ACKNOWLEDGEMENTS

This work was supported in part by the Serbian Ministry of Science, under project no OI141035. I. Vidanović has received support under FP6 project CX-CMCS. Numerical results were obtained on the AEGIS e-Infrastructure, supported in part by FP7 projects EGEE-III and SEE-GRID-SCI. We acknowledge support within the bilateral Serbian-German research project PI-BEC, funded jointly by the Serbian Ministry of Science and German Academic Exchange Service (DAAD).

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