Faraday patterns in Bose-Einstein condensates

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Overview

- Basic theory of Faraday patterns
- First theoretical prediction of Faraday patterns in Bose-condensed gases
  - Full 3D simulations
  - Modeling based on Mathieu equation
  - Multiple scale analysis
- Observation of Faraday patterns in Bose-condensed gases
- Faraday patterns in low-density condensates
  - The non-polynomial Schrödinger equation
  - Analytical solution based on Mathieu equation
  - Full 3D simulations
  - Fast Fourier Transforms – measuring the periodicity of these patterns
- Faraday patterns in high-density condensates
- Conclusions

Faraday patterns, fundamentals

- Naturally, the first work is due to Faraday. The ***Appendix*** of his much-celebrated paper is now a classic: *When the upper surface of a plate vibrating so as to produce sound is covered with a layer of water, the water usually presents a beautifully crispated appearance…the crispations being produced more readily and beautifully when there is a certain quantity than when there is less. For small crispations, the water should flow upon the surface freely. Large crispations require more water than small ones. Too much water sometimes interferences with the beauty of the appearance, but the crispation is not incompatible with much fluid, for the depth may amount to eight, ten, or twelve inches, and is probably unlimited.* (crispation=1. (A) curled condition; curliness; (an) undulation. Now rare. SOED)

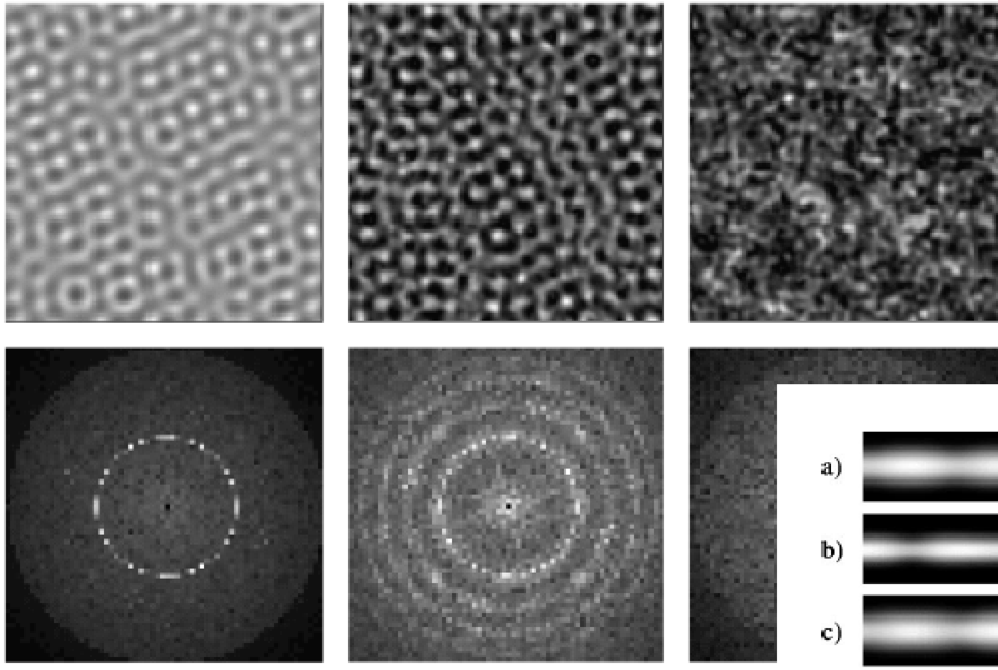
- Faraday patterns became a standard topic in nonlinear physics due to experiments with liquids in the 80s

- Faraday patterns were unknown to the BEC community until the seminal paper of Staliunas *et al.* (*PRL* **89**, 210406) in 2002. Their main point was that by periodically modulating the scattering length of a magnetically trapped 3D one excites a series of patterns similar to those in fluid mechanics.

Faraday patterns, fundamentals

- The group formed around Staliunas published two main papers, one in 2002 (*PRL* **89**, 210406) and one in 2004 (*PRA* **70**, 011601).
- ***The one in 2002*** uses full 3D simulations to show the patterns in the density profile of the condensate but no systematic computations are performed. They use the Mathieu equations only to show that there is an instability.
- ***The one in 2004*** addresses cigar-shaped and pancake-like condensates, *i.e.*, quasi one-dimensional and quasi two-dimensional setups. They show the formation of the waves through direct integration of the GP and give analytical arguments based on multiple scale analysis. It is very important to notice that in this paper the modulation is on the trapping potential ***not*** on the scattering length.
- As far as the proof of concept goes Staliunas *et al.* have paternity of these ideas in the BEC community.

Faraday patterns, fundamentals



(a) (b)

FIG. 2. Evolution of patterns in parametrical
as obtained by numerical integration of E
potential with periodic boundary conditions,
$\omega = 1.5\pi$. Upper row: distribution in physica
row: distributions in momentum space. The
take at times: (a) $t = 100$, (b) $t = 200$, (c) t =
component in momentum space pictures is ren

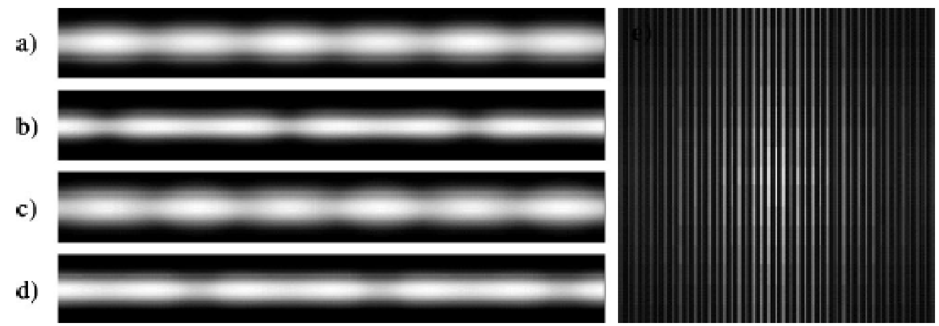


FIG. 1. (a)– (d) Sequence of BEC density as taken at every 1/2 of the trap modulation period (from top to bottom); (e) BEC density in momentum space (density of the spatial Fourier spectrum of the BEC wave function) corresponding to snapshot (a). Plots are obtained by numerical integration of Eq. (2) with periodic boundary conditions in both directions, and with the trapping potential in the vertical (Y) direction. The trap modulation frequency is $\omega = 0.62(\omega_{\text{breath}} \approx 1.77)$. Other parameters are $\alpha = 0.5$, $\gamma = 0.01$, and $\mu = 1.54$. The spatial grid is 256×32 (aspect ratio: 8:1). The size of integration space along the horizontal (X) coordinate is 176. The mode $n = 3$ of periodic boundaries (along the X axis) is excited.

Faraday patterns, experimental results

P. Engels, C. Atherton, and M. A. Hoefer, PRL 98, 095301 (2007).

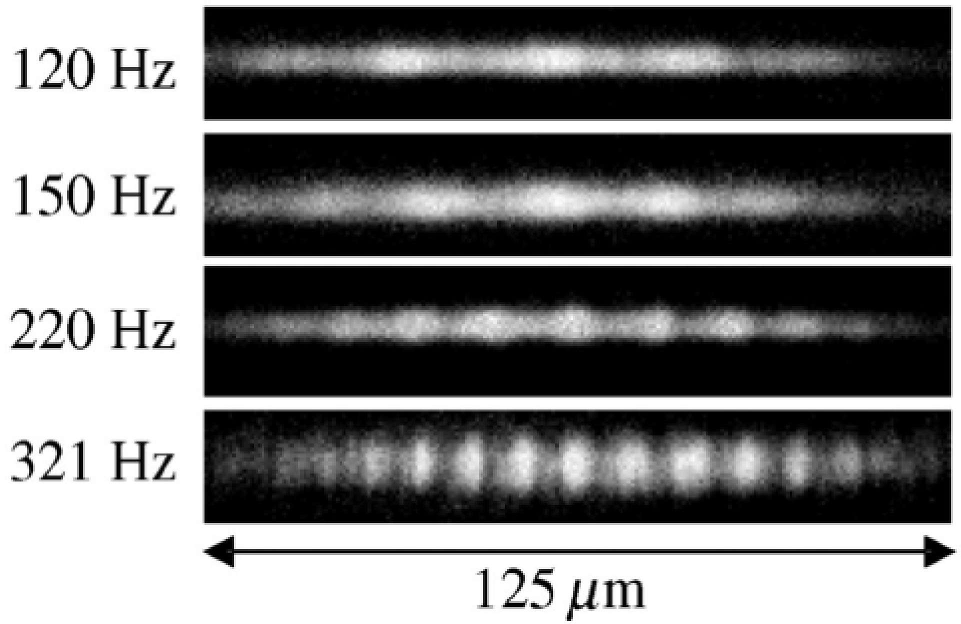


FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

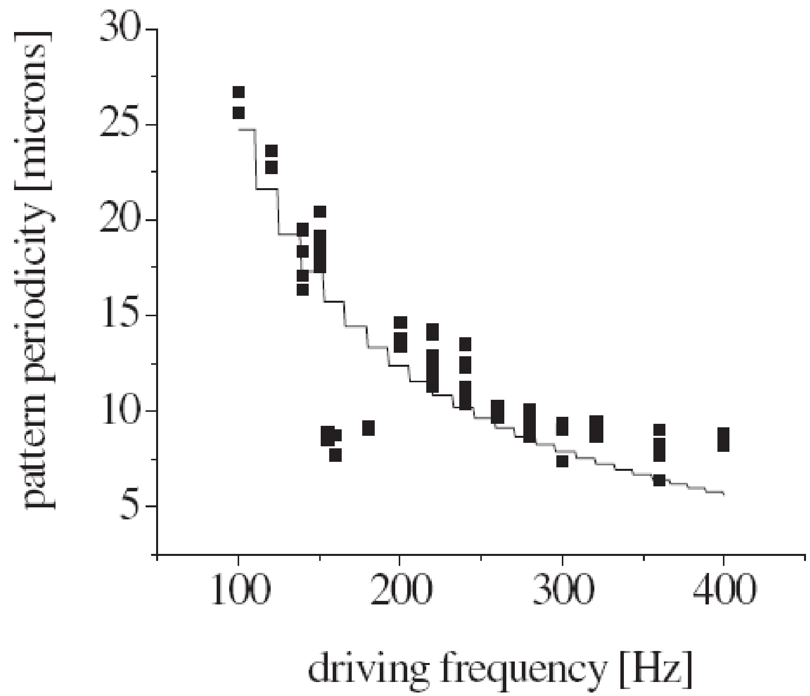


FIG. 2. Average spacing of adjacent maxima of the longitudinal patterns plotted versus the transverse driving frequency. Points are experimental data, while the line shows the theoretical values calculated for the longitudinal modes closest to half the driving frequency.

Faraday patterns in low-density condensates

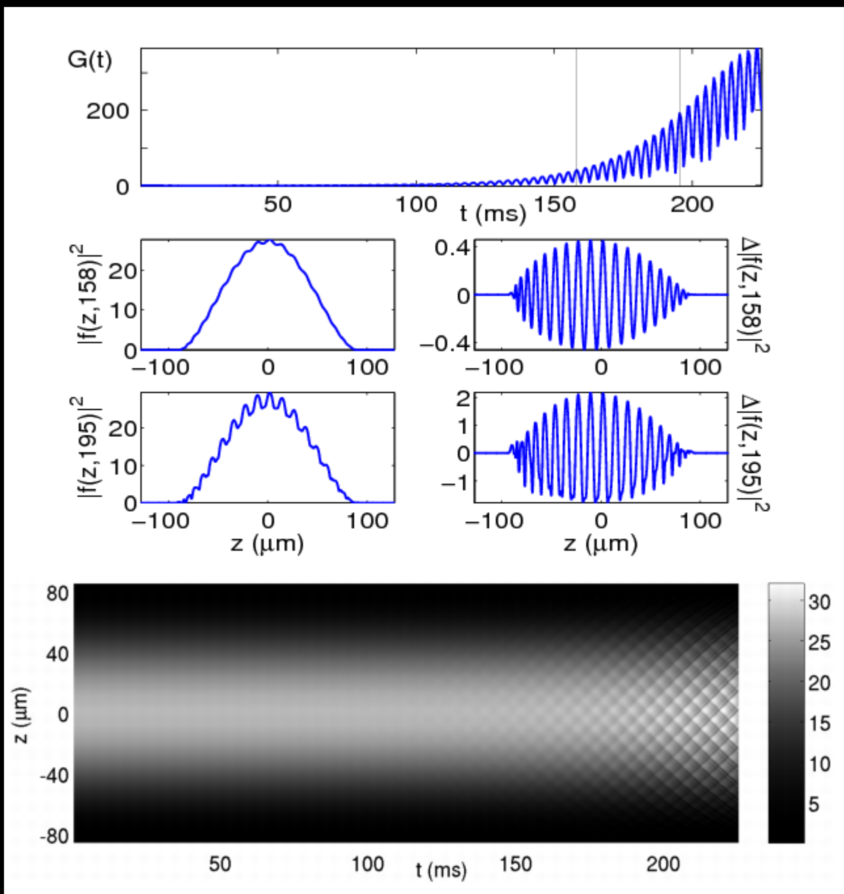
$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN\,|\psi|^2 \right]\psi$$

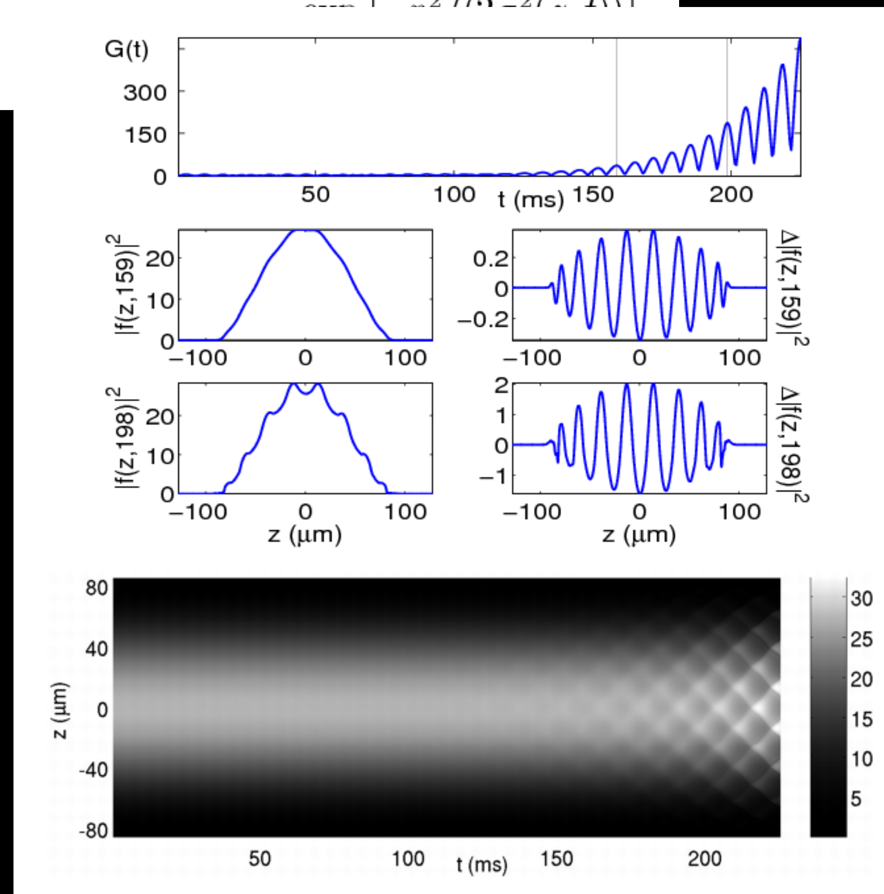
$$V(r,z) \;=\; \frac{1}{2}m\omega_r^2 r^2$$

$$\psi(\mathbf{r},t) = \phi(r,t;\sigma(z,t))$$

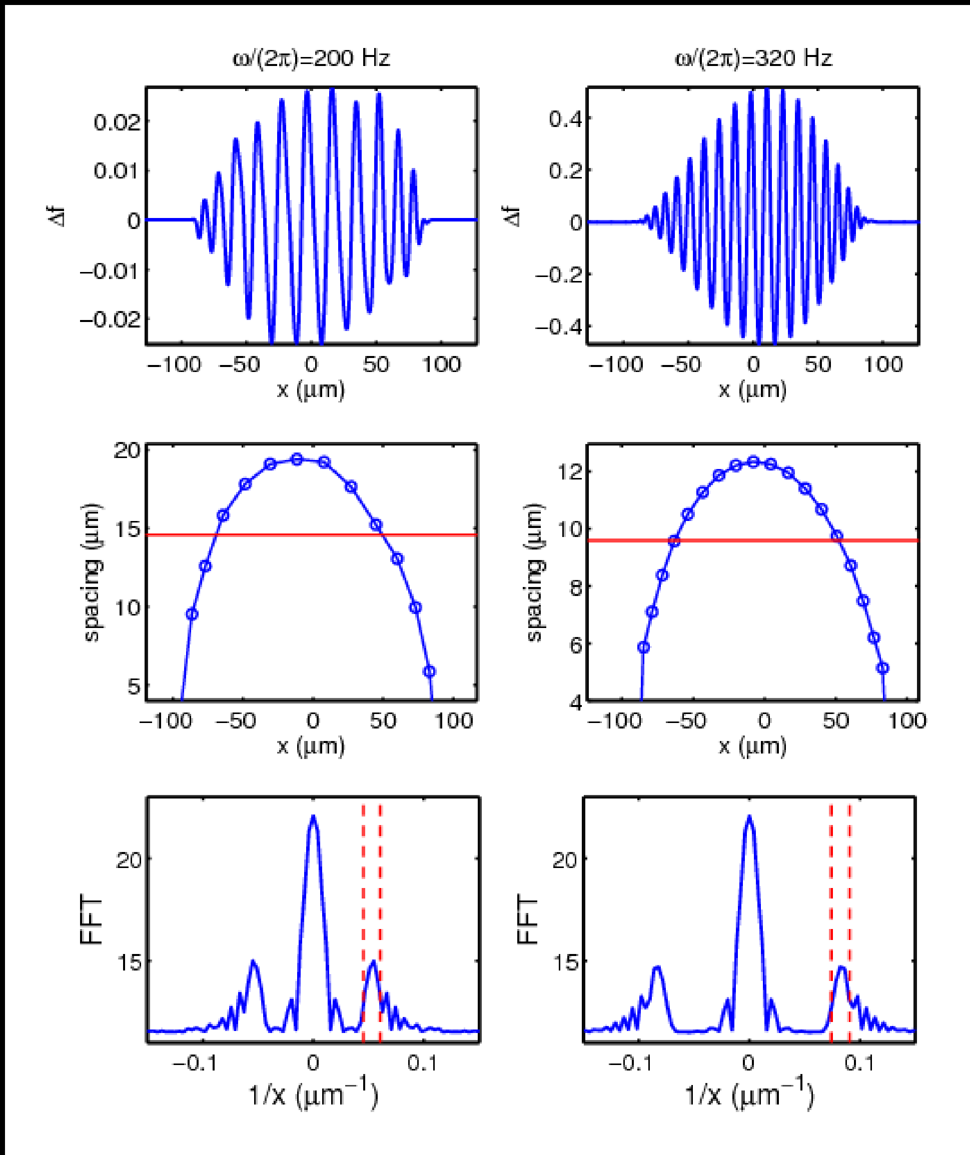
$$i\hbar\frac{\partial f}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hbar\omega_r \frac{1+3\,a_sN\,|f|^2}{\sqrt{1+2\,a_sN\,|f|^2}} \right] f$$

L. Salasnich, A. Parola and L. Reatto, Phys. Rev. A 65, 043614 (2002)





Faraday patterns in low-density condensates



- Since this is really a one-dimensional equation the FFT is one-dimensional as well.
- Due to the inhomogeneity of the condensate imposed by the magnetic trapping the peaks of the FFT are rather broad indicating the period of the Faraday patterns is not that "well defined."
- While there is good quantitative agreement between the observed and theoretically computed periods there is a rather large discrepancy when it comes to the time after which the Faraday waves become visible. This is due to the fact we "freeze" the radial dynamics; the full 3D numerics do not show this discrepancy.

Analytical calculations

$$i\hbar\frac{\partial f}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hbar\omega_r\frac{1+3\,a_sN\,|f|^2}{\sqrt{1+2\,a_sN\,|f|^2}}\right]f$$

$$\omega_r(t) = \omega_{r,0}\cdot(1+\epsilon\sin(\omega t))$$

$$f_0(t) = A\exp\left[-ic\left(t-\epsilon\frac{\cos(\omega t)}{\omega}\right)\right]$$

$$c = \frac{\omega_{r,0}}{\sqrt{1+2\,a_sNA^2}}(1+3\,a_sNA^2)$$

Let us now look at the perturbed solution and determine the leading order equation of the deviation.

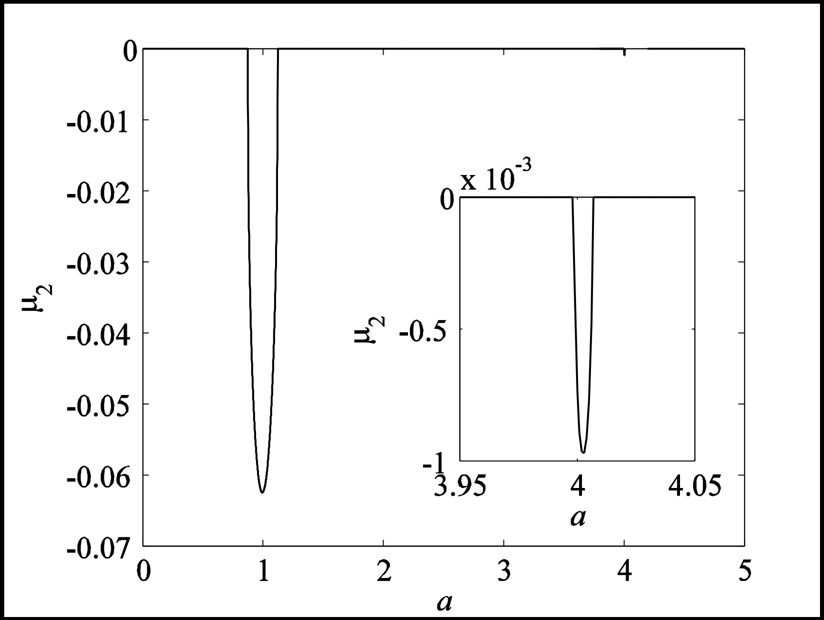
$$f(t) = f_0(t)\,[1+(u(t)+iv(t))\cos(kz)]\,.$$

$$\frac{d^2u}{d\tau^2} + (a(k,\omega)+b(k,\omega)\sin(2\tau))\,u = 0,$$

$$a(k,\omega) = \frac{k^2}{2\hbar\pi m^2\omega^2}\,\frac{6\pi a_s^2\hbar^2mN^2\omega_{r,0} + a_sN\,2\hbar^3k^2\pi\sqrt{L^2+La_sN} + 2\hbar^3k^2\pi\sqrt{L^4+L^3a_sN} + 2gLm^2N\omega_{r,0}}{a_sN\sqrt{L^2+La_sN}+\sqrt{L^4+L^3a_sN}},$$

$$b(k,\omega) = \frac{k^2\omega_{r,0}\epsilon N}{2\hbar\pi m\omega^2}\,\frac{2gmL+6a_s^2\hbar^2N\pi}{a_sN\sqrt{L^2+La_sN}+\sqrt{L^4+L^3a_sN}}.$$

Analytical calculations



To determine the most unstable mode we have to solve the equation $a(k,\omega)=1$.

$$k = \omega \frac{m^{1/2}}{\omega_{r,0}^{1/2}\hbar^{1/2}}(2a_s\rho)^{-1/2}(1+2a_s\rho)^{3/4} \times (4+6a_s\rho)^{-1/2},$$

$$S_0 = \frac{2\pi}{k},$$

$$\rho(x) = 3\frac{L^2-x^2}{4L^3},$$

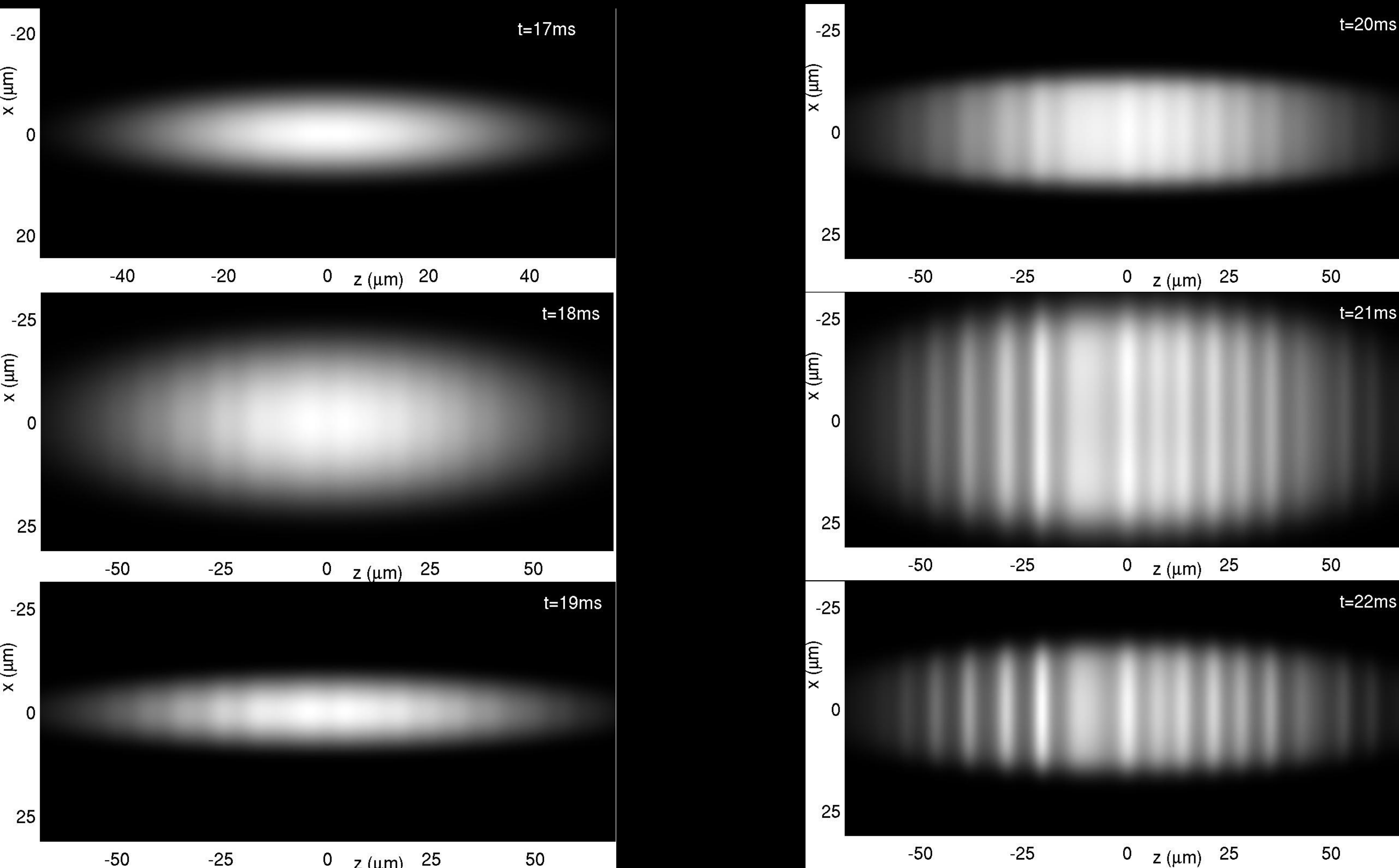
$$\mathcal{S}_1 = \frac{2\pi}{\bar{k}},$$

$$\bar{k} = \frac{1}{2L}\int_{-L}^{L} k(x)dx.$$

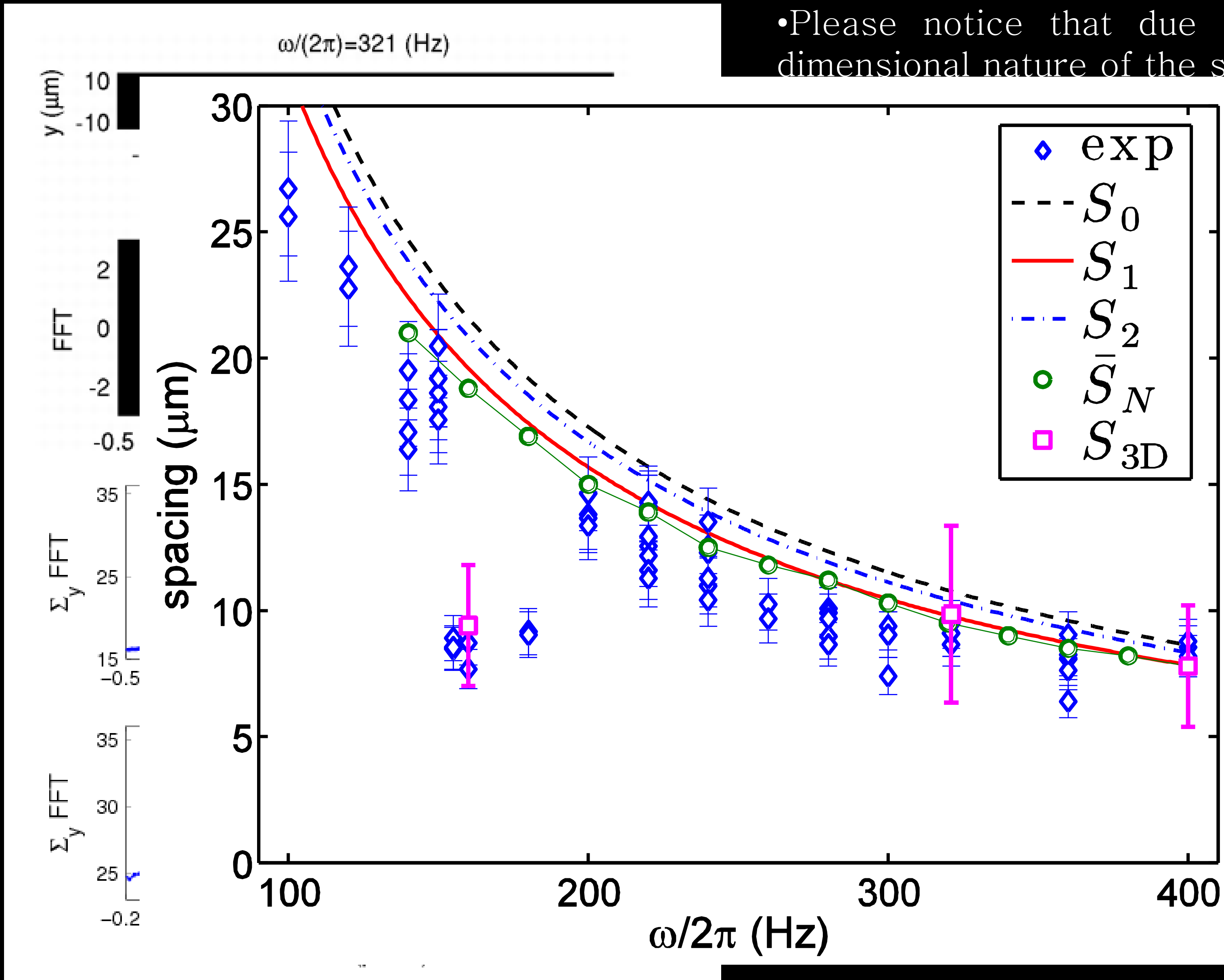
$$\mathcal{S}_2 = \frac{1}{2L}\int_{-L}^{L} \frac{2\pi}{k(x)}dx,$$

•Of course, the above results are obtained using a Gaussian radial ansatz, while the experiments of Engels et al. are really in the TF regime, but still they give good quantitative results.

Full 3D numerics



Full 3D numerics



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High-density condensates

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Faraday waves in high-density cigar-shaped Bose–Einstein condensates

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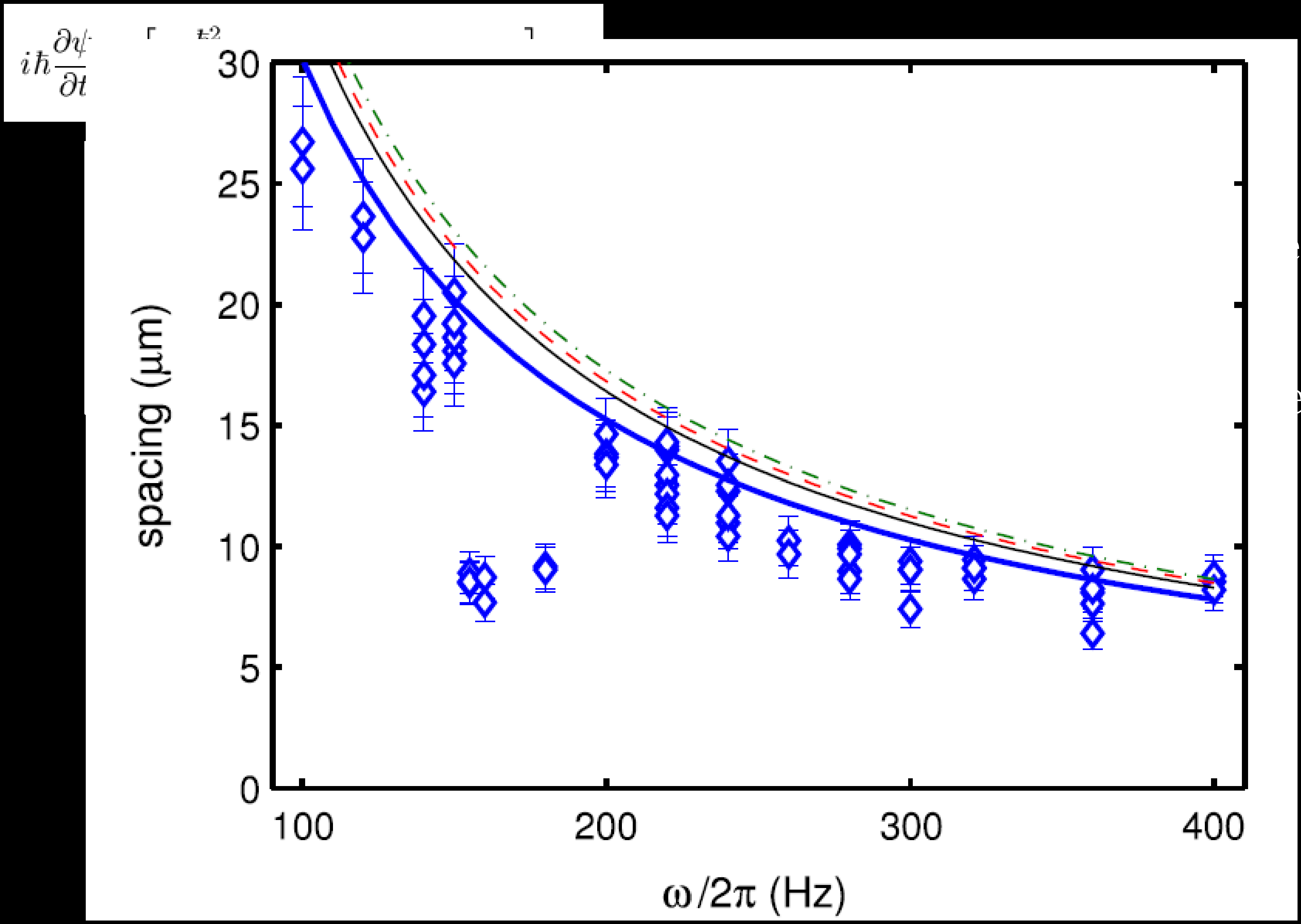
ABSTRACT

Motivated by the experimental achievement of Faraday waves in Bose–Einstein condensates and concurrent theoretical works, we investigate the dynamics of a high-density cigar-shaped condensate subject to periodic modulation of the strength of the transverse confinement. To this end, we propose a one-dimensional non-polynomial Schrödinger equation specifically designed for high-density cigar-shaped condensates and offer an analytical description of the observed patterns. The key ingredient of our equation is a *q*-Gaussian ansatz which accurately describes the high-density regime of a Bose–Einstein condensate.

One-dimensional condensates



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Conclusions

- We have obtained fully analytical results for low- and high-density condensates using the non-polynomial Schrodinger equations and the theory of the Mathieu equations

- We have performed extensive quasi one-dimensional and fully three-dimensional numerical computations

- Overall, we obtain good quantitative results, the main difference between the quasi 1D and the full 3D simulation is that in the former case the Faraday patterns sets in rather slowly because of the ansatz in the radial direction (which is too restrictive)

Thank you for your attention!