Influence of interface roughness on relaxation rates and optical gain in a quantum cascade laser

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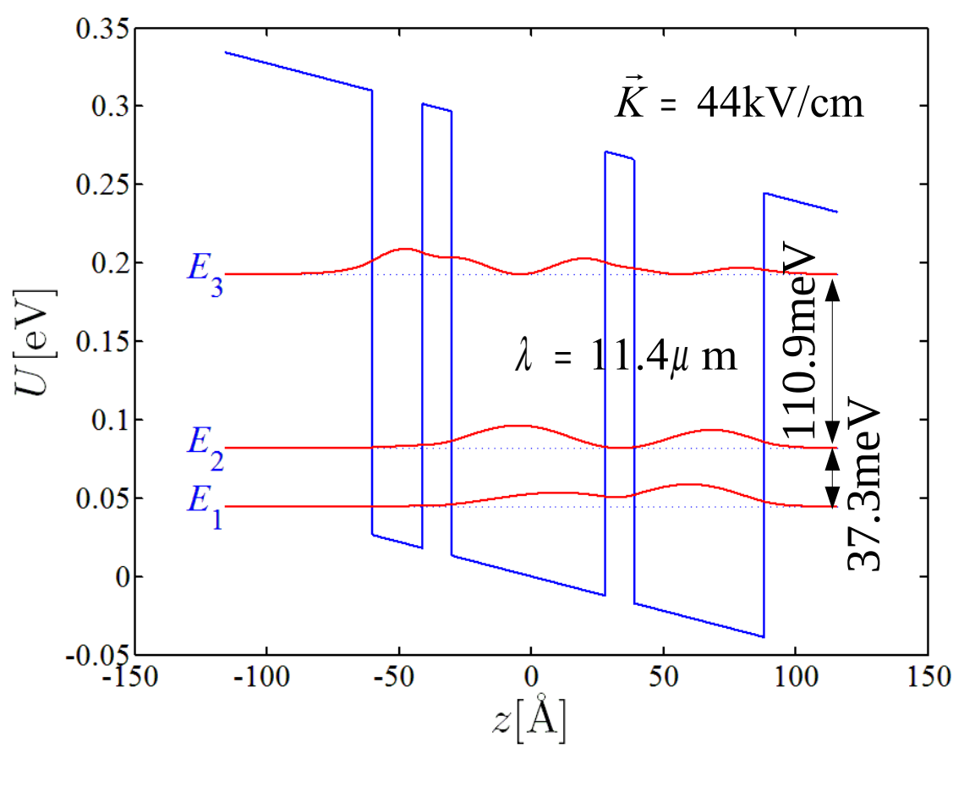
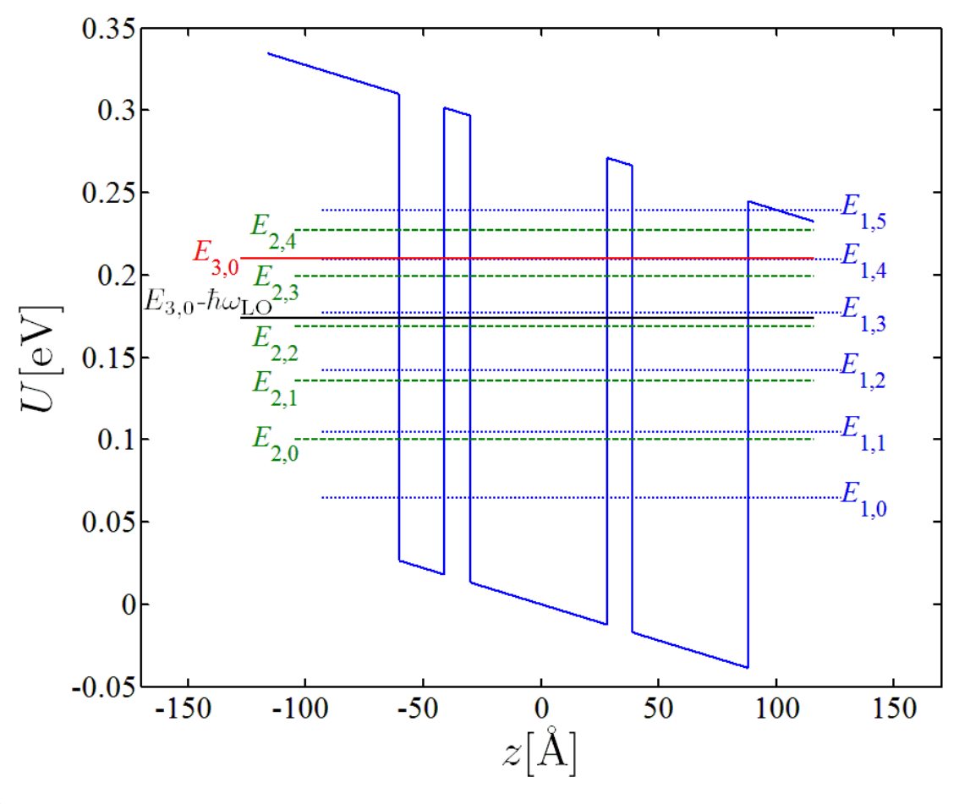
Introduction: We present a detailed theoretical analysis of LO-phonon and interface roughness scattering influence on the operation of GaAs/AlGaAs quantum cascade laser (QCL) in the presence of an intense external magnetic field. We observe strong variations in the life time of the upper state, population inversion and optical gain when magnetic field is increased. The positions and magnitude of peaks are found to be a result of the combined action of both studied scattering mechanisms and strongly influenced by temperature. Sill, incorporation of the interface roughness scattering mechanism into the model did not create new resonant peaks of the optical gain. We show that the decrease in the optical gain at elevated temperatures, is moderated by the occurrence of interface roughness scattering, which remains unchanged with increasing temperature.

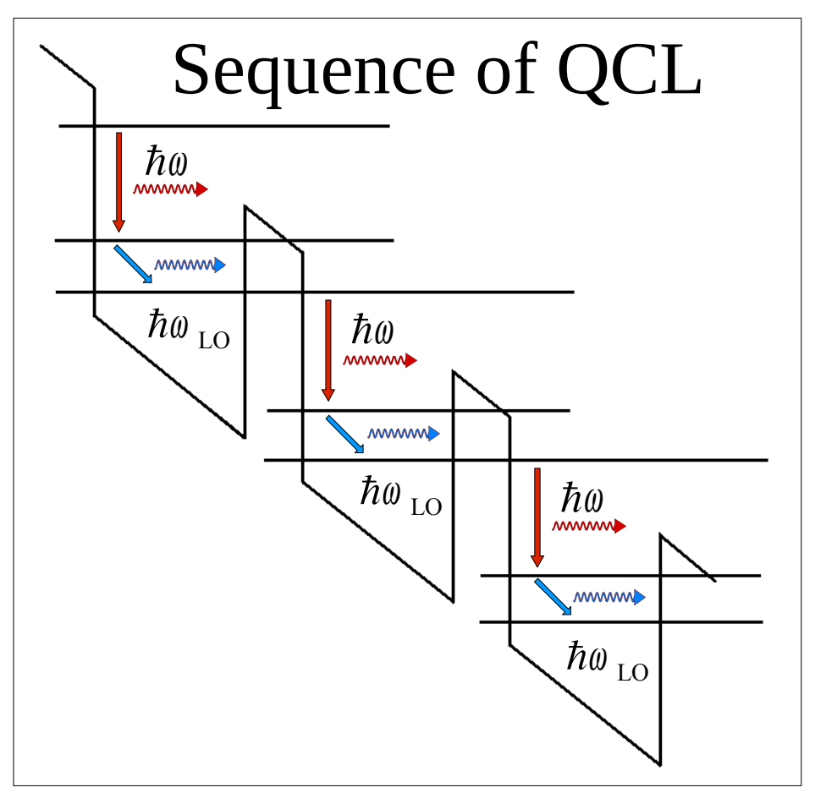
(1) Quantum cascade laser (QCL) in magnetic field

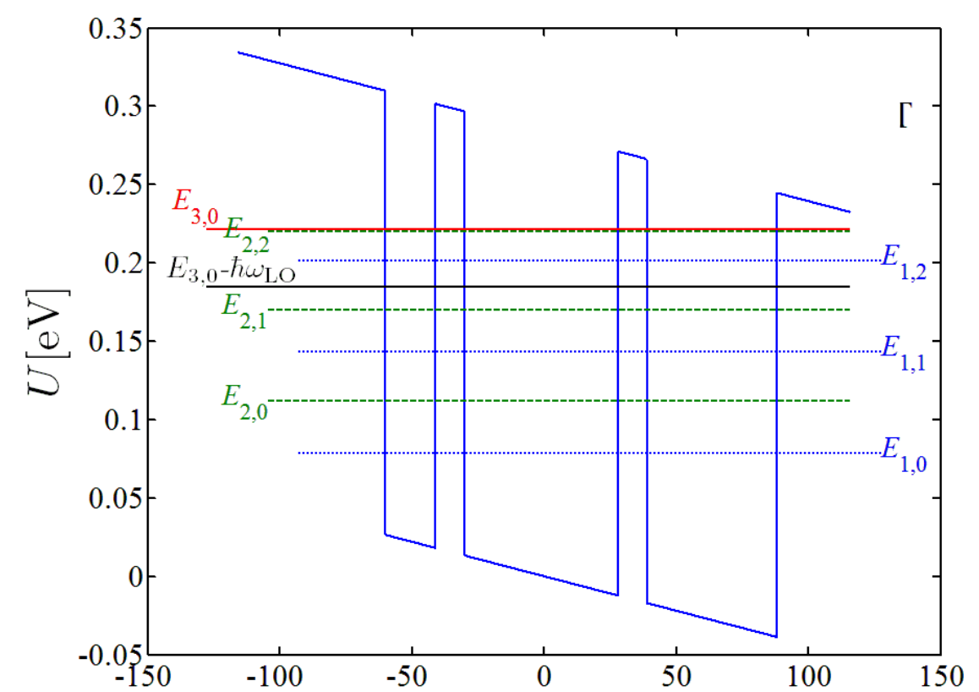


***B* = 0**



***B* = 24.2 T**





***B* = 41.4 T**

Schrödinger equation for B = 0:

$$\frac{d^2}{dz^2}\alpha_0\frac{d^2\eta_n}{dz^2}-\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m}\frac{d\eta_n}{dz}+U(z)\eta_n=E_n\eta_n$$

effective mass at the conduction band minimum

nonparabolicity parameter [1]

Magnetic field influence:

$$E_{n,l}\approx E_n+(l+1/2)\hbar\omega_c$$

Landau index: $l=0,1,2,\ldots$

cyclotron frequency: $\omega_c=eB/m$

(2) Electron scattering rates

1) electron - LO - phonon scattering emission, absorption [1]:

$$\frac{1}{\tau^{\mathrm{LO}}_{(n_i,l_i)\to(n_f,l_f)}}=\frac{2\pi}{\hbar}\sum_{\vec{q}}\left|\left\langle n_f,l_f,k_{x_f},n_q\pm1\middle|\hat{H}_{e-ph}(\vec{q})\middle|n_i,l_i,k_{x_i},n_q\right\rangle\right|^2 J^{LO}$$

$$\frac{1}{\delta\sqrt{2\pi}}e^{-\frac{\left(E_{n_i,l_i}-E_{n_f,l_f}-\hbar\omega_{\mathrm{LO}}\right)^2}{2\delta^2}}$$

$$\hat{H}_{e-ph}(\vec{q})=\sum_{\vec{q}}i\frac{g}{q}\left(e^{-i\vec{q}\cdot\vec{r}}a^{+}_{\vec{q}}-e^{i\vec{q}\cdot\vec{r}}a_{\vec{q}}\right),$$

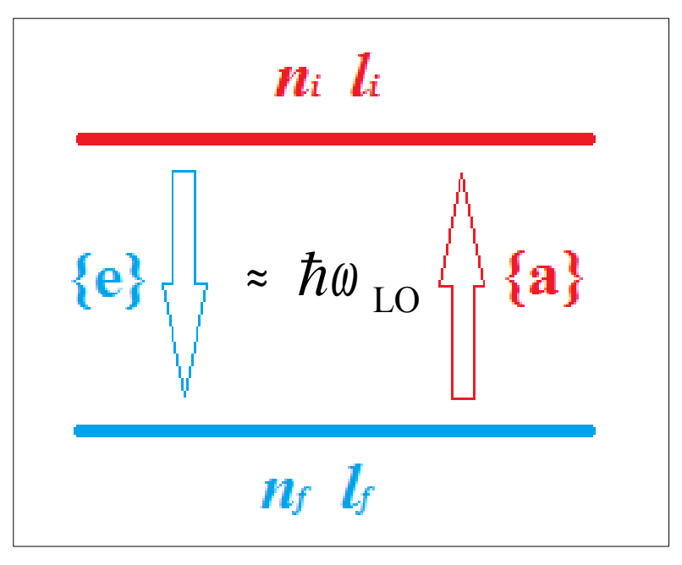
Frölich factor: $g^2\sim\hbar\omega_{\mathrm{LO}}$

phonon wave vector

creation and annihilation operators

emission and absorption (temperature dependence):

$$\frac{1}{\tau^{\mathrm{LO},\{a\}}_{(n_f,l_f)\to(n_i,l_i)}}=\frac{1}{\tau^{\mathrm{LO},\{e\}}_{(n_i,l_i)\to(n_f,l_f)}}\frac{1}{e^{\frac{\hbar\omega_{\mathrm{LO}}}{kT}}}$$



2) electron - interface roughness (IR) scattering [2]:

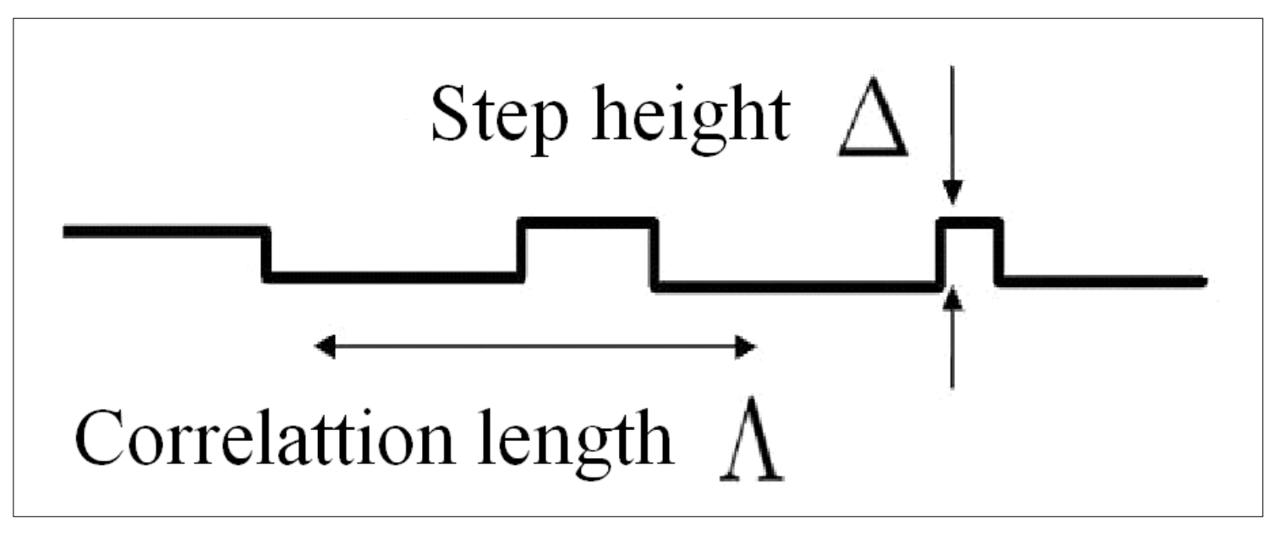
$$\left\langle\frac{1}{\tau^{\mathrm{IR}}_{(n_i,l_i)\to(n_f,l_f)}}(z_i)\right\rangle=\frac{2\pi}{\hbar}\left\langle\sum_{k_{x_i},k_{x_f}}\left\langle\left|\left\langle n_f,l_f,k_{x_f}\,|\,\hat{H}_{IR}\,|\,n_i,l_i,k_{x_i}\right\rangle\right|^2\right\rangle\right\rangle J^{\mathrm{IR}}$$

$$\frac{1}{\delta\sqrt{2\pi}}e^{-\frac{\left(E_{n_i,l_i}-E_{n_f,l_f}\right)^2}{2\delta^2}}$$

$$\hat{H}_{\mathrm{IR}}=U_0\delta(z-z_i)\Delta(\vec{r})$$

Gaussian correlation function:

$$\left\langle\Delta(\vec{r})\Delta(\vec{r}\,')\right\rangle=\Delta^2 e^{-\frac{|\vec{r}-\vec{r}\,'|}{\Lambda^2}}$$



Step height Δ

Correlattion length Λ

3) total scattering (LO - phonon + interface roughness):

$$\frac{1}{\tau_{(n_i,l_i)\to(n_f,l_f)}}=\frac{1}{\tau^{\mathrm{LO}}_{(n_i,l_i)\to(n_f,l_f)}}+\sum_{z_i}\left\langle\frac{1}{\tau^{\mathrm{IR}}_{(n_i,l_i)\to(n_f,l_f)}}(z_i)\right\rangle$$

(3) Optical gain and rate equations

$$g_{3\to2}=\frac{2e^2\pi^2}{\bar{n}\varepsilon_0}\frac{d^2_{3\to2}}{\lambda}\sum_i\delta\left(E_{3,i}-E_{2,i}-\hbar\omega\right)\underbrace{\left(N_{3,i}-N_{2,i}\right)}$$

population inversion

wavelength and the frequency of the emitted light

material refractive index

transition matrix element: $d_{3\to2}=\int\eta_3(z)\;z\,\eta_2(z)\,dz$

(4) Rate equations

extraction current

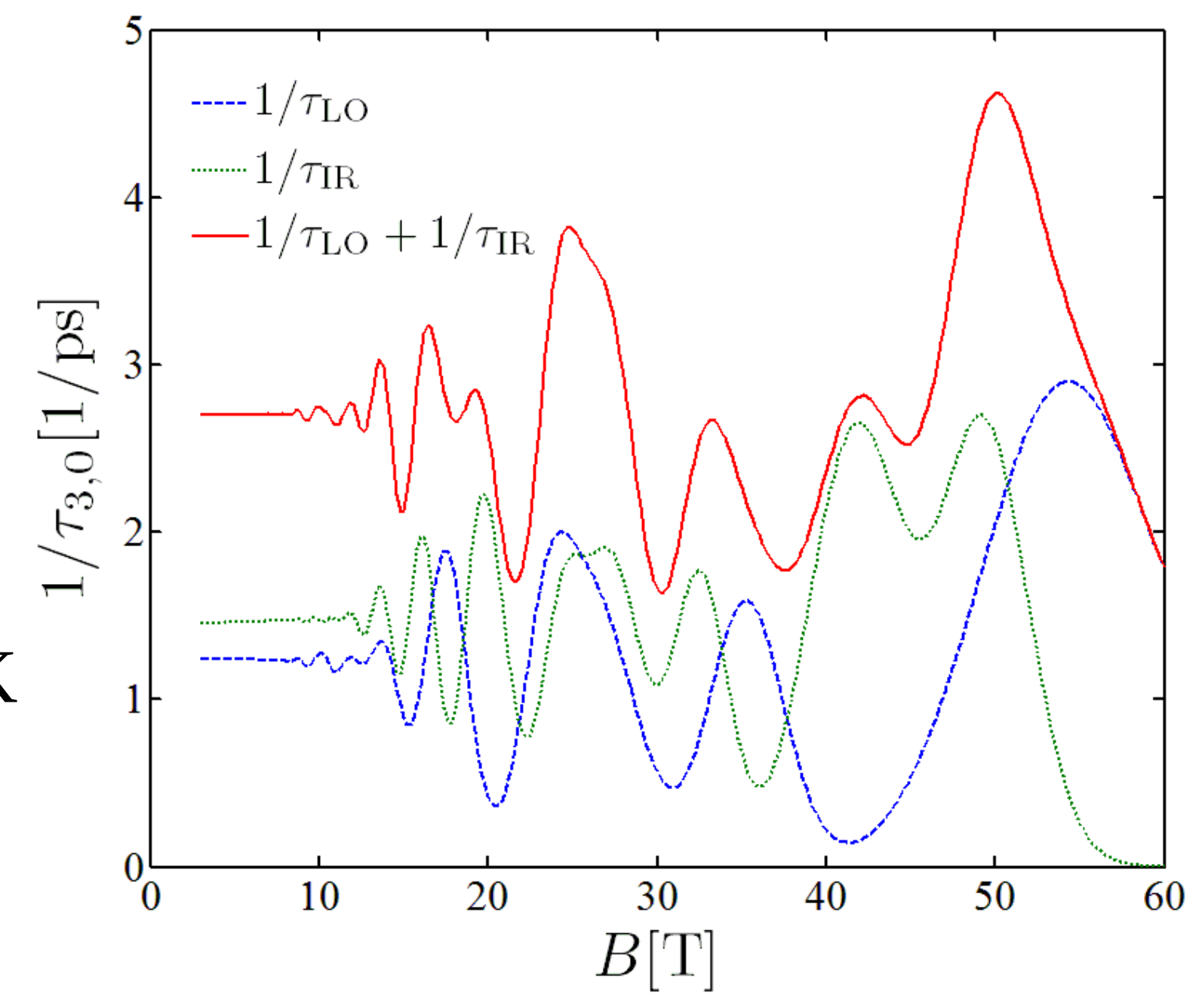
$$N_i\sum_{j\neq i}\frac{\bar{f}_j}{\tau_{i\to j}}-\bar{f}_i\sum_{j\neq i}\frac{N_j}{\tau_{j\to i}}+\frac{J_i}{e}=0,\qquad \bar{f}_i=1-\frac{\pi\hbar}{eB}N_i$$

electron areal density

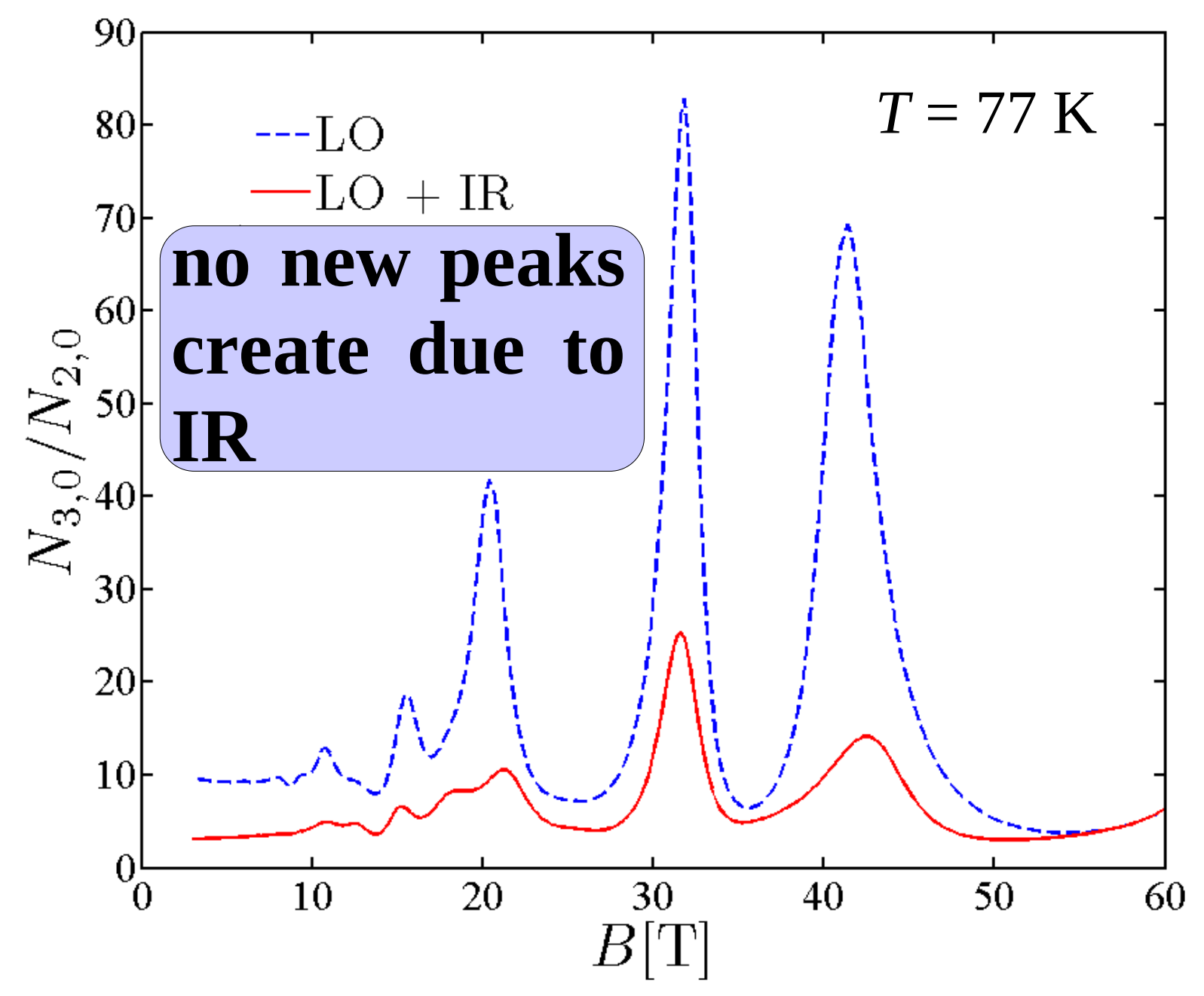
probability that the state is not occupied according to the Fermi-Dirac distribution

(5) Results

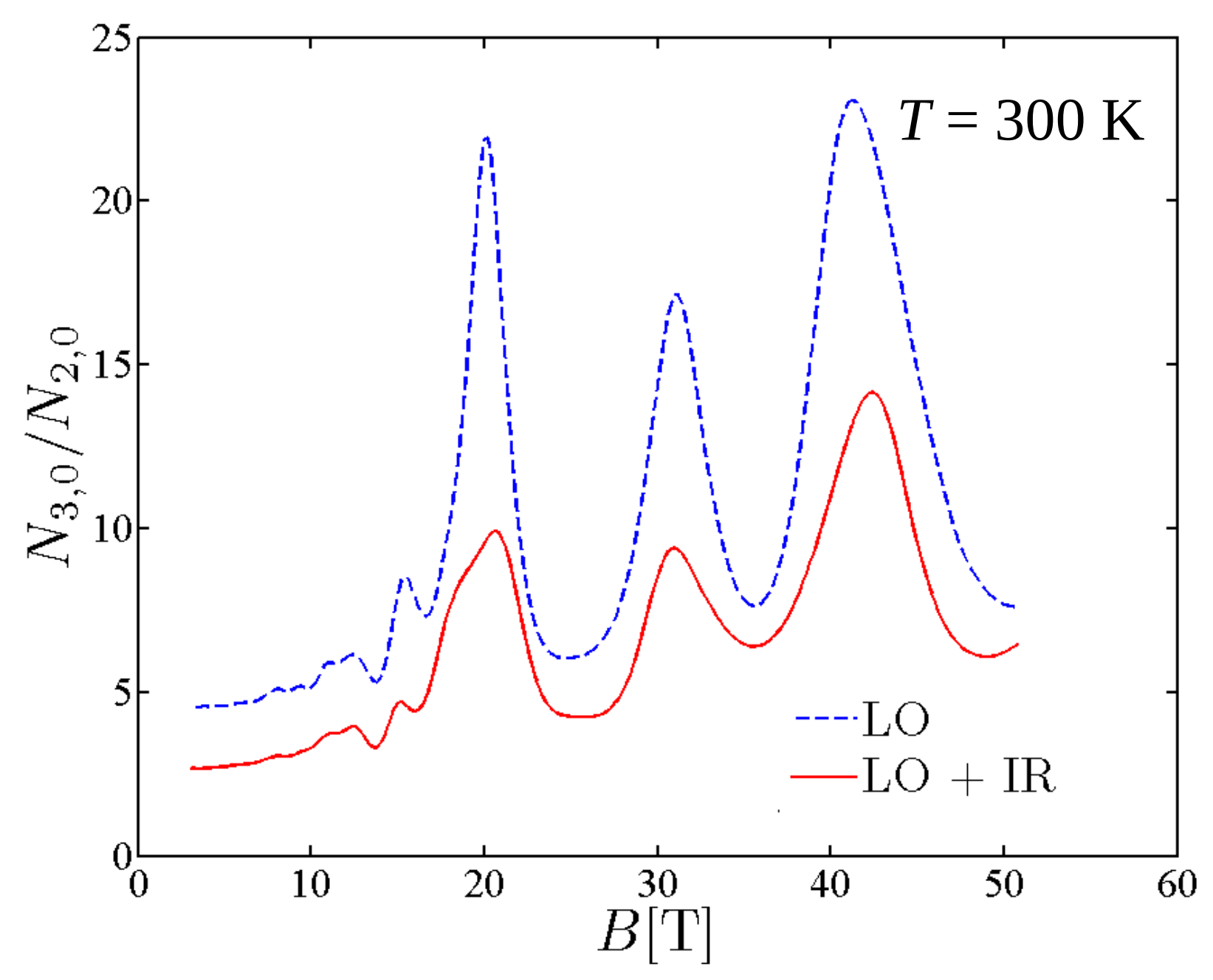
electron relaxation rates from level (3, 0)

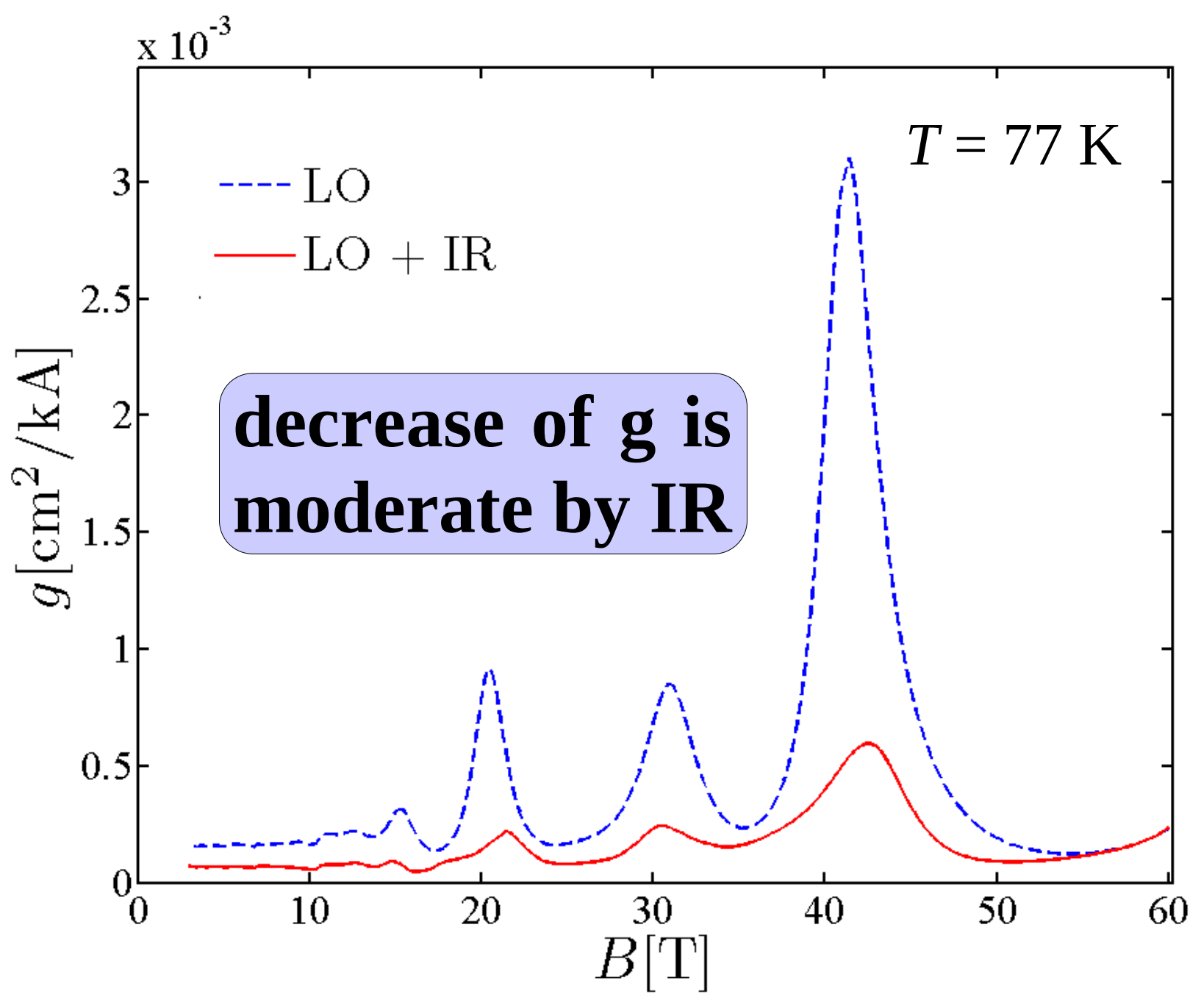


population inversion $N_{3,0}/N_{2,0}$ and optical gain g for T = 77 K and T = 300 K

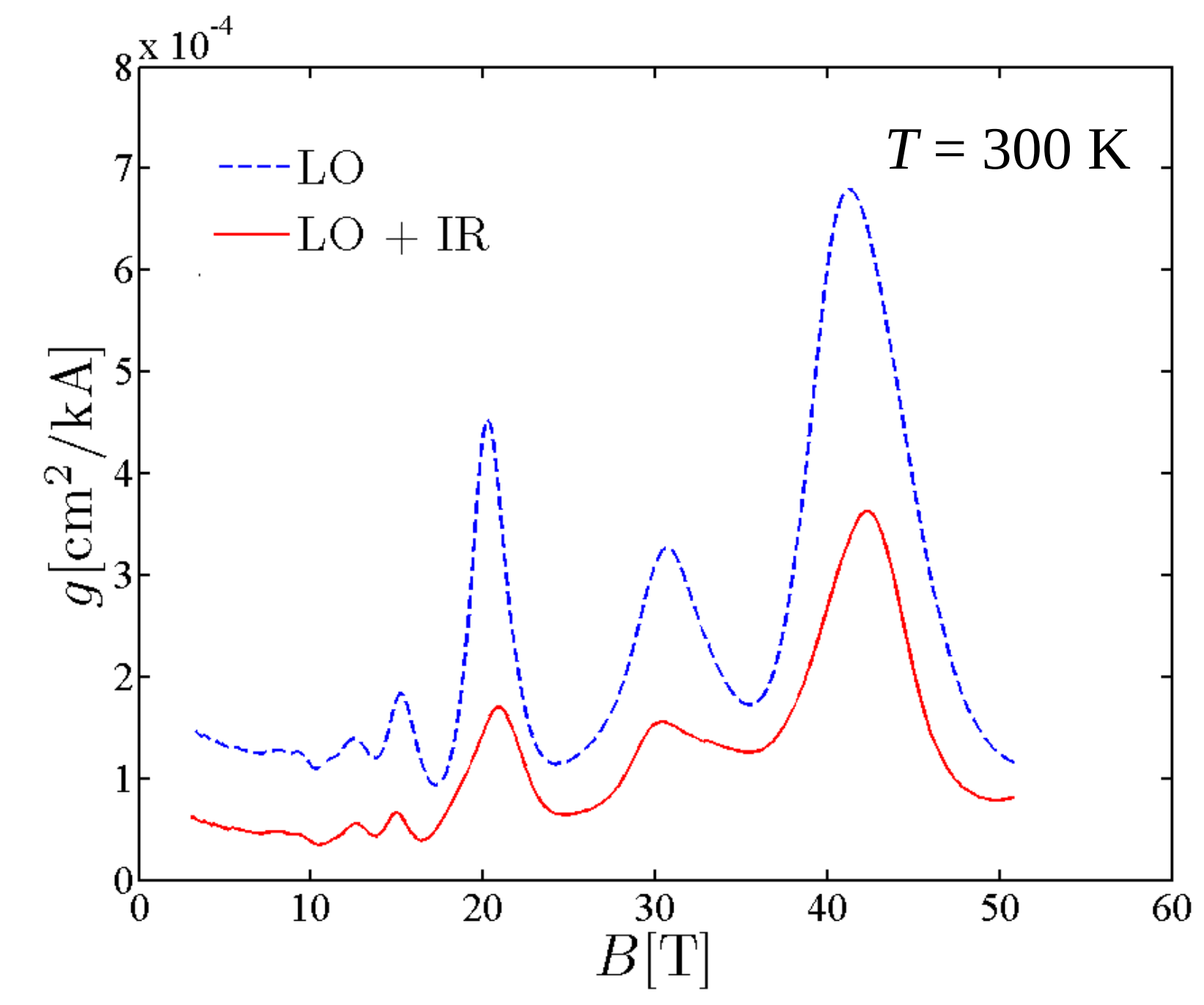


no new peaks create due to IR





decrease of g is moderate by IR



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[2] C. Becker, A. Vasanelli, C. Sirtori and G. Bastard, *Phys. Rev. B* 69, 115328 (2004).
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