Intensive Week SFB/TR 185: Vortex Physics in Atomic and Photonic Matter

Exercises: Numerical simulation of vortices

In order to solve the problems, you can use Fortran/OpenMP or C/OpenMP programs published in Comput. Phys. Commun. **240**, 74 (2019), or C/OpenMP/MPI program available at: http://www.scl.rs/IWeek2022/.

1. Following guidelines from the lecture, derive forward recursion and backward substitution equations for a time propagation of the wave function with respect to $\hat{H}_2$ and $\hat{H}_3$, i.e., parts of the Hamiltonian that contain derivatives over x and y coordinate, respectively. Using those results, set up a complete semi-implicit Crank-Nicolson scheme for evolution of a rotating Bose-Einstein condensate in real and imaginary time.

2. Using imaginary-time propagation and parameters mentioned in the lecture, calculate ground state Ψ_n of a rotating Bose-Einstein condensate containing $n = 0, 1, 2$ vortices by varying value of the angular velocity.

3. Using real-time propagation, verify that the ground states obtained in Exercise 2 are stable, i.e., that the calculated physical quantities remain constant.

4. Modify the program such that it reads state of the system saved into a file and calculates the corresponding expectation value of the operator $\hat{L}_z$. Use this program to calculate the expectation values $\langle \Psi_n | \hat{L}_z | \Psi_n \rangle$ for the states obtained in Exercise 2.

5. Using imaginary-time propagation and parameters mentioned in the lecture, determine the critical angular velocity for the emergence of $n = 1, 2, 3$ vortices in the system.

6. Modify the program such that the trapping potential includes a moving obstacle along y axis and consider the case of a non-rotating condensate. Determine the critical velocity of the obstacle for the emergence of a vortex in the system. Is it possible to obtain a single vortex using this protocol?

7. Using the program written in Exercise 6, again consider the case of a non-rotating condensate and determine the critical velocity of the obstacle for the emergence of a vortex dipole in the system.