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DISORDERING OF THE CORRELATED STATE OF THE QUANTUM HALL BILAYER AT FILLING FACTOR $\nu = 1$

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The phase diagram of a quantum Hall bilayer at total filling $\nu = 1$ contains an incompressible superfluid for small distances d between the layers, as well as the compressible phase corresponding to two uncoupled Fermi liquids for large d. Using exact diagonalization on the sphere and torus geometry, we investigate a long-standing question of the nature of the transition between the two regimes, and the possibility for the existence of a paired phase in the transition region. We find considerable evidence for a direct transition between the superfluid and the Fermi liquid phase, based in particular on the behavior of the ground state energy on the sphere (including appropriate finite-size corrections) as a function of d. At the critical distance $d_C \approx 1.6\ell_B$ the topological number ("shift") of the ground state changes, suggesting that tuning the layer separation d in experiment likely leads to a direct transition between the superfluid and the Fermi liquid phase.

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Quantum Hall bilayer (QHB) is a semiconductor structure that consists of two quantum wells spatially separated by an insulating barrier that is of the same order of magnitude as the width of each of the wells. When QHB is placed in the perpendicular magnetic field, adjusted in such a way that that the ratio of the number of electrons in the system (N) and the magnetic flux quanta (N_ϕ) is exactly $\nu = N/N_\phi = 1$, remarkable manifestations of quantum-mechanical coherence take place on the macroscopic scale.[1] These interesting effects occur upon varying a single parameter, d/ℓ_B, the ratio of the center-to-center distance between the wells

d to the magnetic length $\ell_B = \sqrt{\hbar c/eB}$. Much of the QHB physics has been well established in the extreme cases when d is (1) much smaller, or (2) much larger than ℓ_B.

When $d \ll \ell_B$, i.e. the Coulomb interaction between electrons in the same layer and in the opposite layers is of about the same magnitude, a good starting point for the physical description is the Halperin state Ψ_{111},[2] explicitly defined below in Eq. (1). The physics contained in Ψ_{111} is that of exciton binding:[1,3] an electron in one layer and a correlation hole directly opposite to it in the other layer, are in a coherent quantum-mechanical superposition dictated by the form of the Ψ_{111} wave function. This exciton description can be a viewpoint of the phenomenon of superfluidity found in these systems,[4] and is closely connected to the concept of composite bosons (CBs)[5–7] that can be used as natural quantum Hall quasiparticles in the small d/ℓ_B regime.

On the other hand, when $d \gg \ell_B$ we have the case of the decoupled layers and the ground state (GS) is a product of two Fermi seas, each described by the Rezayi–Read wave function $\Psi_{\rm RR}$,[8] defined below in Eq. (3). The underlying quasiparticles in this case are composite fermions (CFs), the usual quasiparticles of the single-layer quantum Hall physics.[9]

To address the range of intermediate d, when the system is a disordered superfluid, one may try to capture the basic physics by interpolating between the two limits described above. In other words, one may describe the physics by using mixed states of CBs and CFs.[10] This is a phenomenological approach in which we start from the identical underlying electrons, split them into a group of those that correlate as CBs, and a group of those that correlate as CFs. The wave function for the superfluid state at small d will involve mainly CBs; the disordering of the CB superfluid can be viewed as caused by "nucleation" of CF quasiparticles as d is increased.

In the remainder of this paper, we first introduce and systematically review the construction of mixed states of CBs and CFs,[11,12] paying special emphasis on the kind of pairing between CFs that might be relevant for the bilayer system to produce a paired state. After presenting the analytic arguments that motivate the existence of a paired state in the transition region between superfluid and Fermi liquid phases, we present the results of exact diagonalization calculations on the torus and sphere, and conclude by discussing some of their implications.

The basic ingredients for the construction of mixed CB/CF states are the Halperin 111 state, describing the bilayer ground state for very small distances d, and the Rezayi–Read wave function that describes the Fermi liquid state in a single-layer quantum Hall system at $\nu = 1/2$. The 111 state is given by

$$\Psi_{111}(\{z^{\uparrow}, z^{\downarrow}\}) = \prod_{i<j}^{N_{\uparrow}} (z_i^{\uparrow} - z_j^{\uparrow}) \prod_{k<l}^{N_{\downarrow}} (z_k^{\downarrow} - z_l^{\downarrow}) \prod_{m=1}^{N_{\uparrow}} \prod_{n=1}^{N_{\downarrow}} (z_m^{\uparrow} - z_n^{\downarrow}) \,. \tag{1}$$

Here $z_\sigma = x_\sigma + iy_\sigma$ is the complex 2D coordinate of an electron in the layer $\sigma \in \{\uparrow, \downarrow\}$ (containing N_σ particles), and we have set the magnetic length ℓ_B equal to 1, supressing the spinor part of the wave function and the ubiquitous lowest Landau level (LLL) Gaussian factors. Because it describes identical electrons, this wave function is subject to the constraint that there is the same number of magnetic flux quanta per particle. This translates into the following flux-counting relation

$$N_\phi = N_\uparrow - 1 + N_\downarrow = N_\downarrow - 1 + N_\uparrow \,, \tag{2}$$

which necessitates $N_\uparrow = N_\downarrow = N/2$.

On the other hand, the Rezayi–Read CF-sea state[8] at $\nu = 1/2$ is given by

$$\Psi_{\rm RR}(z) = \mathcal{P}_{\rm LLL}\, \mathcal{F}(z, \overline{z}) \prod_{i<j} (z_i - z_j)^2 \,, \tag{3}$$

where $\mathcal{F}$ stands for the Slater determinant of free waves. Because $\mathcal{F}$ contains the terms involving $\overline{z}$, we need to project those by $\mathcal{P}_{\rm LLL}$ to obtain a holomorphic LLL wave function. The wave function for two decoupled layers is then simply given by the product $\Psi_{\rm RR}(z_\uparrow) \times \Psi_{\rm RR}(z_\downarrow)$.

Possible corrections to the 111 state, resulting from increasing the distance d that leads to superfluid disordering, have been the subject of numerous previous works in the literature. For example, an approach based on the traditional Chern–Simons theory[7] of CBs in the RPA approximation finds the following correction to Ψ_{111}:[13]

$$\Psi_{\rm ph} = \exp\left\{ -\frac{1}{2} \sum_{\mathbf{k}} \frac{\sqrt{\frac{V_S(\mathbf{k})}{\overline{\rho}/m}}}{|\mathbf{k}|} \rho^S_{\mathbf{k}} \rho^S_{-\mathbf{k}} \right\} \Psi_{111} \,, \tag{4}$$

where $\rho^S_k \equiv \rho^\uparrow_{\mathbf{k}} - \rho^\downarrow_{\mathbf{k}}$ is the difference of the densities of two layers, $V_S(\mathbf{k}) = \frac{V_{\uparrow\uparrow}(\mathbf{k}) - V_{\uparrow\downarrow}(\mathbf{k})}{2}$ is the interaction in the neutral channel, m is the electron mass and $\bar{\rho}$ is the uniform total density. As usual, the bilayer problem at $\nu = 1$ has been decomposed into the charge and neutral channel, and the latter reduces to the problem of an ordinary superfluid with the phonon contribution, hence our notation for the correction $\Psi_{\rm ph}$. In the small d limit $V_S(\mathbf{k}) = \pi d$, and we can expand the expression $\Psi_{\rm ph}$ as

$$\Psi_{\rm ph} = \Psi_{111} - \left(\sum_{\mathbf{k}} \frac{c\sqrt{d}}{|\mathbf{k}|} \rho^S_{-\mathbf{k}} \rho^S_{\mathbf{k}} \right) \Psi_{111} + \cdots \,, \tag{5}$$

where c is a positive constant. The terms after the first one represent corrections, in the order of importance, to the Ψ_{111} ansatz as d increases. The form of the correction is fixed by the basic phenomenology and sum rules for a superfluid in two dimensions.[14]

The previous correction can be recovered as a special case of the mixed CB-CF ansatz, as we now show. For small distances d, it was argued in Refs. 10–12 that the low-energy physics of the bilayer should be captured by the following mixed state

of CBs and CFs:

$$\Psi_1 = \mathcal{A}_\uparrow \mathcal{A}_\downarrow \Bigg\{ \Psi_{111}(z_\uparrow, z_\downarrow) \Psi_{\rm RR}(w_\uparrow) \Psi_{\rm RR}(w_\downarrow) \prod_{i,j} (z_{i\uparrow} - w_{j\uparrow}) \prod_{k,l} (z_{k\uparrow} - w_{l\downarrow}) \times \prod_{p,q} (z_{i\downarrow} - w_{q\uparrow}) \prod_{m,n} (z_{m\downarrow} - w_{n\downarrow}) \Bigg\}, \tag{6}$$

where $\mathcal{A}_\sigma$ stands for the anti-symmetrization in the layer σ, and we omitted the projection to the LLL. By using the expressions for the densities of electrons in each layer, $\rho^\sigma(\eta) = \sum_i \delta^2(\eta - z_i^\sigma) + \sum_i \delta^2(\eta - w_i^\sigma)$, we can further rewrite the wave function in the following way:[12]

$$\Psi_1 = \int \prod_{n \in \rm CF} d^2\eta_{n\sigma} \Bigg\{ \frac{\prod_{k<l}(\eta_{k\uparrow} - \eta_{l\uparrow}) \prod_{p<q}(\eta_{p\downarrow} - \eta_{q\downarrow})}{\prod_{i,j}(\eta_{i\uparrow} - \eta_{j\downarrow})} \mathcal{F}(\eta_\uparrow) \times \mathcal{F}(\eta_\downarrow) \times \rho^\uparrow(\eta_{1\uparrow}) \cdots \rho^\downarrow(\eta_{n\downarrow}) \Psi_{111}(z_\uparrow, z_\downarrow) \Bigg\}, \tag{7}$$

where n is the total number of electrons that correlate as CFs. This expression is exactly equivalent to Eq. (6) (up to an unimportant numerical factor).

Let us compare the first phonon corrections in both approaches to find out which possibilities for the pairing are allowed amongst the most simple choices for the (weak) pairing function. Based on the usual Chern–Simons approach, the first phonon correction is $\sim \sum_{\mathbf{k}} \frac{1}{|\mathbf{k}|} \rho^\uparrow_{\mathbf{k}} \rho^\downarrow_{-\mathbf{k}}$. On the other hand, the mixed wave function including pairing suggests the following simplest correction when there are two CFs:

$$\int d^2\eta_{1\uparrow} \int d^2\eta_{2\downarrow} \frac{1}{(\eta_{1\uparrow} - \eta_{2\downarrow})} g(\eta_{1\uparrow} - \eta_{2\downarrow}) \rho^\uparrow(\eta_{1\uparrow}) \rho^\downarrow(\eta_{2\downarrow}), \tag{8}$$

where g is the pairing function. If we choose $g(z) = 1/z$, we obtain *no correction whatsoever* to the 111 state. Among other simple choices, the next candidate for the pairing function could be $g(z) = \sqrt{z/\bar{z}}$ ($\bar{z}$ is the complex conjugate of z). When substituted in Eq. (8), this reduces to the form of the first phonon contribution in the long-distance limit with the $\frac{1}{|\mathbf{k}|}$ singularity, Eq. (5). Thus $g(z) = \sqrt{z/\bar{z}}$ accommodates the usual (on the level of RPA) superfluid description given in Eq. (5). It can be shown that $g(z) = $ const. i.e. no pairing, also produces a trivial correction; see Table 1 caption. We can continue exploring the simple choices for pairing, e.g. the next possibility in the order of weakness of the pairing that retains the same angular momentum for the pairing as $g(z) = \sqrt{z/\bar{z}}$ is $g(z) = 1/\bar{z}$. The phonon contribution in this case turns out to be $\sim \sum_{\mathbf{k}} \ln(|\mathbf{k}|\ell_B) \rho^\uparrow_{\mathbf{k}} \rho^\downarrow_{-\mathbf{k}}$.[12] Our results can be summarized as in Table 1.

Having identified some simple pairing functions allowed in the bilayer system starting from Ψ_1 and at small d/ℓ_B, we can ask whether any of those may lead to a paired phase in the intermediate range of d/ℓ_B and can we find a simple wave function to describe this phase. If the translation symmetry remains unbroken as we

Table 1. Phonon corrections for different choices of the pairing function. The functions f_1, f_2 and f_3 define the weight of each correction in terms of the bilayer distance d. In the first case (no pairing) the correction is proportional to $\sum_{\mathbf{k}} \frac{1}{(k_x+ik_y)} \rho^{\uparrow}_{\mathbf{k}} \rho^{\downarrow}_{-\mathbf{k}}$, but we expect that with no constraint on the number of CFs [as in Eqs. (4) and (5)], this will correspond to $\sum_{\mathbf{k}} \frac{1}{(k_x+ik_y)} \rho^{S}_{\mathbf{k}} \rho^{S}_{-\mathbf{k}}$ i.e. zero (no correction) due to the anti-symmetry under $\mathbf{k} \to -\mathbf{k}$ exchange.

$g(z) = \text{const.}$ (no pairing)	$\sum_{\mathbf{k}} f_1(d) \frac{1}{(k_x + ik_y)} \rho^{\uparrow}_{\mathbf{k}} \rho^{\downarrow}_{-\mathbf{k}}$
$g(z) = 1/z$	no correction when multiplies Ψ_{111}
$g(z) = \sqrt{z/\bar{z}}$	$\sum_{\mathbf{k}} f_2(d) \frac{1}{\lvert\mathbf{k}\rvert} \rho^{\uparrow}_{\mathbf{k}} \rho^{\downarrow}_{-\mathbf{k}}$
$g(z) = 1/\bar{z}$	$\sum_{\mathbf{k}} f_3(d) \ln(\lvert\mathbf{k}\rvert \ell_B) \rho^{\uparrow}_{\mathbf{k}} \rho^{\downarrow}_{-\mathbf{k}}$

increase d, one of the viable candidates is the mixed CB/CF wave function with the pairing $g(z) = 1/\bar{z}$. This pairing has the same angular momentum as $g(z) = \sqrt{z/\bar{z}}$, but it also has an additional amplitude factor to it. If we take the choice $g(z) = 1/\bar{z}$ and examine the final form of the mixed state when there are no CBs, we are lead to its following forms (see Ref. 12 for details),

$$\begin{aligned} \Psi_2 &= \det\left(\frac{1}{\bar{z}_{i\uparrow} - \bar{z}_{j\downarrow}}\right) \prod_{i<j} (z_{i\uparrow} - z_{j\uparrow})^2 \prod_{k<l} (z_{k\downarrow} - z_{l\downarrow})^2 \\ &= \det\left(\frac{1}{\bar{z}_{i\uparrow} - \bar{z}_{j\downarrow}}\right) \det\left(\frac{1}{z_{k\uparrow} - z_{l\downarrow}}\right) \Psi_{111}\,, \end{aligned} \tag{9}$$

where we used the Cauchy determinant identity in going from the first to the second line. The neutral part of Ψ_2 (i.e. the two determinants which do not carry a net flux through the system as Ψ_{111} does) can be viewed as a correlator of vertex operators of a single *nonchiral* bosonic field. According to Ref. 15, CFT correlators not only describe quantum Hall ground state wave functions, but can also be used to find out about the excitation spectrum and connect its edge and bulk theories. Using CFT analogy, one can construct the neutral excitations for Ψ_2 in terms of the vertex operators that multiply the ground state wave function (see Ref. 12 for the precise form of these operators). These vertex operators are parametrized by the exponents β_1 and β_2; if the low-lying spectrum were consisting only of $\beta_1 = \frac{1}{2}$ and $\beta_2 = \frac{1}{2}$ quasiparticle excitations, our system would be described by the so-called BF Chern–Simons theory or the theory of the 2D superconductor.[16] Combining the analysis with the charge part (Ψ_{111}) in which only charge-1 excitations are allowed (half-flux quantum excitations are strongly confined[17]), we arrive at the conclusion that the degeneracy of the system's ground state on the torus must be four.[16,18] However, the vertex operators yield a single-valued expression acting on the ground state also for any real β_1, including $\beta_1 = 0$ (no excitation), and therefore one can expect a gapless branch of excitations parametrized by a continuum of β_1,

and compressible (gapless) behavior of the system in the neutral sector (the charge channel, being described by Ψ_{111}, is incompressible).

In the following, we explore the prospects for the paired phase in finite-size systems using exact diagonalization for different choices of boundary conditions. There have been many numerical studies of the quantum Hall bilayer at $\nu = 1$.[10,19–27] In particular, elaborate studies in Refs. 10 and 20 demonstrated the relevance of CB/CF constructions for the clean systems (no impurities). Trial wave functions of this kind describe a continuous crossover between the CB superfluid and the two decoupled CF liquids via a possible intermediate p-wave paired phase that in our analysis corresponds to Ψ_2, Eq. (9). Here we would like to focus on addressing the question whether such a phase has a clear signature in small finite systems that can be studied numerically. This question is relevant in light of the new experimental results which indicate that the CF liquid phase in the usual samples is partially spin-polarized.[28] Since the 111 state is a QH ferromagnet, the experiments appear to preclude the possibility of a smooth crossover and instead suggest a first-order transition .[29] For larger Zeeman fields, the transition becomes smooth and the critical point drifts to larger values of d.[28,30]

The topological content of Ψ_2 is the four-fold ground state degeneracy on the torus. In Ref. 20 this degeneracy was analyzed as a function of d, and different shapes of the torus unit cell, but no definite conclusion was drawn due to the strong finite-size effects. We corroborate this finding by diagonalizing a larger system of $N = 16$ particles, Fig. 1. N electrons are placed on the surface of a torus i.e. we impose periodic boundary conditions in the presence of $N_\phi = N/\nu = N$ quanta of

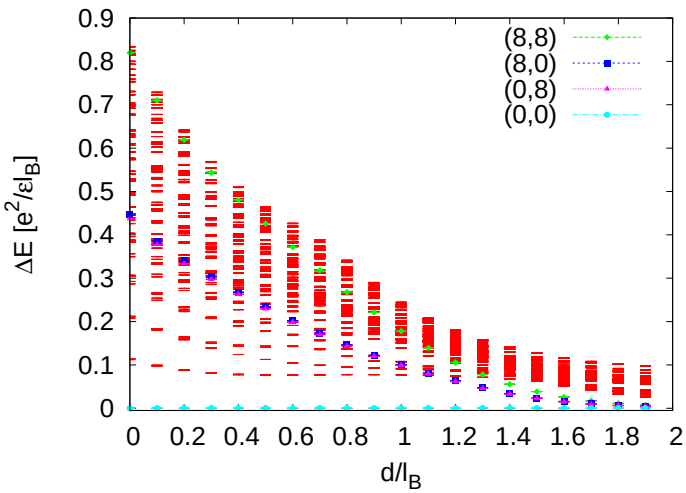

Fig. 1. (Color online) Energy spectrum of the quantum Hall bilayer at total filling $\nu = 1$ on the torus. The system contains $N = 16$ electrons in a rectangular domain $a \times b$ with the aspect ratio $a/b = 0.99$. Spectrum is plotted relative to the ground state at each d/ℓ_B, and special symbols denote the momentum sectors where the paired phase is expected to be degenerate.

the perpendicular magnetic field. The interaction between the electrons in the same layer is given by $V_c^{\uparrow\uparrow}(r) = e^2/\epsilon r$ and between those in opposite layers $V_c^{\uparrow\downarrow}(r) = e^2/\epsilon\sqrt{r^2+d^2}$. Note that, for simplicity, in numerical calculations we consider a fixed number of electrons in each layer (negligible interlayer tunneling). However, the CF/CB construction in Eq. (6) can easily accommodate the charge imbalance by a redistribution of CBs, which was also revealed in experiments. On the other hand, it can be shown[12] that compressible states cannot easily accommodate such a redistribution.

The bilayer Hamiltonian is numerically diagonalized for each d/ℓ_B, and eigenenergies are plotted in Fig. 1. Four seemingly degenerate states can be identified in the lowest-lying spectrum starting from $d = 1.4\ell_B$, but the gap decreases smoothly with the increase of d, which suggests that these states belong to the compressible CF liquid. Two decoupled CF liquids are allowed to display a four-fold degeneracy due to their center-of-mass motion.[31] This degeneracy, contrary to the one of Ψ_2, has no topological content, but in a finite system it may nonetheless persist for some variation of the aspect ratio or other parameters i.e. it may appear quasi-robust. However, since the gap of the system smoothly decreases as a function of d, it is unlikely that there is a third phase, distinct from the 111 state and the decoupled CF liquids.

We can also change the boundary condition and place N electrons on the surface of a sphere[32] with a magnetic monopole in the center. In order to probe a given many-body state ψ at the filling factor ν, the number of flux quanta generated by the monopole has to be adjusted in such a way that $N_\phi = N/\nu - \mathcal{S}$, where $\mathcal{S}$ is the *shift*, a topological number that characterizes each ψ. Since the Hilbert space is defined by (N, N_ϕ), two different states ψ_1 and ψ_2, which describe the same filling factor $\nu_1 = \nu_2 = \nu$, may be realized in different Hilbert spaces if $\mathcal{S}_1 \neq \mathcal{S}_2$. As an example, take $\tilde{\Psi}_2$ which is characterized by the shift $\mathcal{S} = 1$, like the 111 state, whereas CF liquid state occurs at $\mathcal{S} = 2$. Therefore, one cannot directly compare e.g. the overlaps of the exact ground state with the 111 state and CF liquids for a fixed N. Instead, one must perform an extrapolation to the thermodynamic limit to discriminate between phases. Overlaps are not useful from this point of view because they would extrapolate to zero in the thermodynamic limit, however ground state energy is an example of a quantity that is meaningful in this sense. It defines the transition point d_C between the 111 state and CF liquids as the value of d above which the ground state energy is lower at the shift $\mathcal{S} = 2$ than at $\mathcal{S} = 1$. We estimate d_C from the crossing point of the ground state energies for the two shifts, $\mathcal{S} = 1$ and 2, for various system sizes $N = 6 - 16$, Fig. 2. In doing so, it is essential to include the background charge correction and rescale the magnetic length in order to carefully compare the energies of the systems living on two slightly different FQH spheres.[33] It can be shown[33] that beyond $d \sim 1.5\ell_B$, which we identified as the critical value for the appearance of the four-fold degeneracy on the torus, one should no longer describe the system at the shift of $\mathcal{S} = 1$. Therefore, it is likely that the 111 phase goes directly into the CF liquids even at this finite value of d,

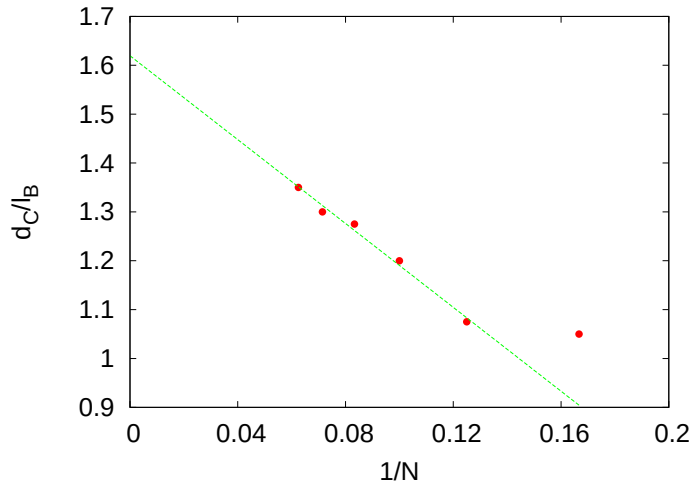

Fig. 2. Critical bilayer distance d_C defined as the crossing point of the ground state energies at shifts $\mathcal{S} = 1$ and $\mathcal{S} = 2$ on the sphere. Linear extrapolation for $N \to \infty$ yields $d_C \sim 1.6\ell_B$ and does not involve the smallest system $N = 6$ which shows strong finite-size effects.

and not via the p-wave paired state. Nevertheless, the two energies remain very close to each other and the paired wave function such as $\tilde{\Psi}_2$ is not conclusively ruled out as a candidate for the description of the system. It may either describe an excited state of the CF liquid or a phase with a tiny gap that would be hard to discern from an ordinary compressible state in the experiment.

Our estimate of critical $d_C \sim 1.6\ell_B$ roughly agrees with that obtained by complementary methods in the literature, e.g. in Ref. 21 where d_C was estimated by measuring the change in the pseudospin expectation value at a fixed shift $\mathcal{S} = 1$ and zero tunneling. Although the obtained d_C is in reasonable agreement with the experiments, it does not imply that we have proved a direct transition between the two shifts for the ground state ($\mathcal{S} = 1$ versus $\mathcal{S} = 2$). In order to do that, one would want, for each fixed d, to diagonalize the Hamiltonian for all the available system sizes and make the thermodynamic extrapolation of the energies (with the appropriate corrections) as a function of $1/N$. While this works nicely for the shift of $\mathcal{S} = 1$, in the case of CF shift $\mathcal{S} = 2$ the ground state energy has a nontrivial dependence on $1/N$ which reflects the shell-filling effect[33] The dependence of energy on $1/N$ is somewhat similar to that reported in Ref. 34 for the single layer at $\mathcal{S} = 2$, except that the energy minima occur for $N/2 = n^2, n = 2, 3, \ldots$. In between the minima, the energy has a local maximum. Therefore, in order to perform a reliable extrapolation, a few minima/maxima would be required, but since we are only able to diagonalize up to $N = 16$, that gives us a single minimum $n = 2$. However, the fact that for all the available systems we consistently obtain lower ground state energy at $\mathcal{S} = 2$ for sufficiently large d strongly suggests that the transition involves a change in shift at finite d.

In conclusion, we discussed how the ground state of the quantum Hall bilayer at $\nu = 1$ evolves with the changing distance between the layers in the light of trial wave functions describing the mixed states of CBs and CFs, as well as using numerical diagonalization on the sphere and torus. The study of the ground state energy on the sphere gives considerable support for the direct transition between superfluid and Fermi liquid phases. Paired state may only exist in the regions of the phase diagram where the interaction is significantly different from the pure Coulomb repulsion studied in this work.

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