Merons in a finite quantum Hall system

M. V. Milovanović
Institute of Physics, P.O. Box 56, 11001 Belgrade, Yugoslavia
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We study quasiparticle states in a finite $\nu=1$ spin (as an additional degree of freedom) quantum Hall system. The skyrmion (topological) quasiparticle states are well established in the infinite system, but in finite systems, in the effective mean-field pictures that we have, as a rule, they are not eigenstates of hamiltonian and some other generators of the system symmetries. We present a class of states, which are eigenstates of certain combinations of the generators and are similar in structure to the orbital angular momentum eigenstates of the Laughlin quasihole in the same system. We refer to the states as states of a meron. They have an upper limit on the mean orbital angular momentum, and the limit defines an effective edge, at a distance less than the system radius, in an effective quasiparticle description of the meron. Remarkably, the meron edge (as the usual quasihole edge) is characterized by a power-law decay of the single-particle, static correlator defined between positional coherent states of the meron.

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I. INTRODUCTION

Merons appear in the description of $\nu=1$ (Ref. 1) spin (as an additional degree of freedom) and $\nu=1$ (Refs. 2,3) bilayer (pseudospin) quantum Hall effect (QHE). In the former case, they are the constituents in the form of a meron pair of the elementary skyrmion excitation. As soon as we have a skyrmion excitation, there are two merons that can be recognized by a SU(2) transformation (see below) on the skyrmion wave function. In the bilayer case the merons also come in the form of bound pairs, distorted and less entangled than in the SU(2) case[2–5] which can be released at some finite temperature of the Kosterlitz-Thouless transition.[6]

A skyrmion (variational) wave function in the spin QHE can be modeled[2] as

$$\Psi_s(w)=\prod_{i=1}^{N}\begin{bmatrix} z_i-w \\ \lambda \end{bmatrix}\prod_{i<j}(z_i-z_j), \tag{1}$$

where w and λ are complex parameters, the filling factor is $\nu=1$, and $\{z_i;i=1,\ldots,N\}$ denote electron two-dimensional (2D) coordinates. For simplicity, we suppressed the Gaussian factors. The center of the skyrmion is at w, and λ is a parameter that determines the size of the skyrmion, the distance when S_z component of the electron spin starts to point up rather than down (as it does if we are at smaller distance, near the center). A SU(2) transformation of the form

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{2}$$

can be done on each spinor to obtain the following wave function:

$$\Psi_{2m}(w)=\prod_{i=1}^{N}\begin{bmatrix} z_i-w_1 \\ z_i-w_2 \end{bmatrix}\prod_{i<j}(z_i-z_j) \tag{3}$$

that describes two merons at positions w_1 and w_2. Namely, as $w_1\to w_2$ we get a Laughlin quasihole, so that each constituent in Eq. (3) carry half of the unit of charge and can be ascribed to half of the unit flux.

At this stage, we may ask whether the meron representation [Eq. (3)] is in any sense superfluous for the effective quasiparticle description of the system. Indeed any n skyrmion (variational) wave function,[7] studied in the infinite system, can be, similar to the single skyrmion case above, transformed by the application of the same SU(2) transformation [Eq. (2)] into a $2n$ meron wave function

$$\Psi_m(w_1,\ldots,w_n;v_1,\ldots,v_n)=\prod_{i=1}^{N}\begin{bmatrix} (z_i-w_1)\cdots(z_i-w_n) \\ (z_i-v_1)\cdots(z_i-v_n) \end{bmatrix}\prod_{i<j}(z_i-z_j), \tag{4}$$

with appropriate w and v variables, and vice versa. If the constructions are relevant and appropriate in the finite system, we can claim the same equivalence there. Nevertheless, we will argue that in the finite system, with no Zeeman coupling, some of the states of the form given in Eq. (3) can be interpreted, approximately, as characteristic quasiparticle states of the system, and that their representation is most transparent and convenient. We will call the associated quasiparticles as merons, although due to the strong entanglement inside the pair in the state, Eq. (3), it is hard to apply a particle picture with definite charge and finite extent of the excitation.

Thus, we will be considering coherent states, states that describe quasiparticles of the system localized or centered around a single point, which we usually denote by the letter w. These may be hamiltonian eigenstates in the infinite system like the Laughlin construction of the Laughlin quasihole in the $\nu=1/m$, m odd case, but usually, almost as a rule, in the finite system these are not. In the Laughlin case, in the finite rotationally symmetric system, there are the quasiparticle orbital angular momentum eigenstates instead, which are simultaneously (or very nearly) hamiltonian eigenstates. In the case of the spin QHE and the underlying skyrmion quasiparticle, this description in the finite system has not been achieved. Although the skyrmion excitations are firmly, experimentally,[8] and theoretically[1,9,10] established in the bulk of the large thermodynamic systems; in the theoretical studies of large but finite systems, we only approximately model quasiparticle states (coherent or other) because they are not the eigenstates of hamiltonian and some other generators of the system symmetries.[11,12]

A classification of the elementary particle-hole eigenstates in the case of the spin quantum Hall droplet was done in Ref. 11, where a connection (see Appendix A) between these and some of the coherent states was established. The coherent states reside in the center and have the skyrmion interpretation. In Appendix B, we classify all rotationally symmetric states (found in Ref. 11) on the droplet which are made by the same procedure as the skyrmion coherent states. Some of them are encircling the edge with yet no clear quasiparticle interpretation. The question we raise is what are the coherent states away from the central region? In other words we want to address the question how topological excitations, which are firmly established in the infinite system or in the bulk of large systems, are described (at least in mean field, effectively) as they near the boundary. In Sec. II we propose the case for the meron states to be appropriate, candidate quasiparticle states for the edge region. In the same section, we establish a characteristic distance from the center of the droplet at which, in fact, further onto the edge, even the merons, described effectively as quasiparticles, do not exist. We may consider this distance as a definition for a meron edge (just as the radius of the system defines the edge of the Laughlin quasihole). In Sec. III using the ansatz wave function, Eq. (3), we calculate, analytically, static correlations of a single meron along the newly defined edge (keeping the other at the center of the system). Section IV is devoted to discussion and conclusions.

II. MERON AND THE EDGE EXCITATIONS

Let us consider the construction, Eq. (3), of two merons, where we place one meron with a coordinate w_2 in the center of the disk $(w_2=0)$ and denote w_1 as w,

$$\Psi(w)=\prod_{i=1}^{N}\begin{bmatrix} z_i-w \\ z_i \end{bmatrix}\prod_{i<j}(z_i-z_j). \tag{5}$$

We want to decompose the excitation into a series similar to the well-known decomposition of the Laughlin quasihole,

$$\begin{aligned}\Psi_L(w)&=\prod_{i=1}^{N}\begin{bmatrix} z_i-w \\ 0 \end{bmatrix}\prod_{i<j}(z_i-z_j)\\ &=\left\{\begin{bmatrix} -w \\ 0 \end{bmatrix}^N+\sum_{i=1}^{N}\begin{bmatrix} z_i \\ 0 \end{bmatrix}\begin{bmatrix} -w \\ 0 \end{bmatrix}^{N-1}\right.\\ &\quad\left.+\cdots+\begin{bmatrix} z_1 \\ 0 \end{bmatrix}\cdots\begin{bmatrix} z_N \\ 0 \end{bmatrix}\right\}\prod_{i<j}(z_i-z_j),\end{aligned} \tag{6}$$

in the completely polarized case.[13] The coefficients of z's in the series are the elementary symmetric polynomials, and the decomposition, Eq. (6), can be considered as the one over eigenstates, w^n, of the hole orbital angular momentum. The larger the n, the closer to the edge the hole is. The electronic wave function, in which the Laughlin part is multiplied with the symmetric polynomial of degree n, describes the eigenstate of this hole.

In the case of the wave function given in Eq. (5), we have

$$\begin{aligned}\Psi(w)=&\left\{\prod_{i=1}^{N}\begin{bmatrix} -w \\ z_i \end{bmatrix}+\sum_{i=1}^{N}\begin{bmatrix} z_i \\ 0 \end{bmatrix}\prod_{k\neq i,=1}^{N}\begin{bmatrix} -w \\ z_k \end{bmatrix}+\cdots\right.\\ &\left.+\begin{bmatrix} z_1 \\ 0 \end{bmatrix}\cdots\begin{bmatrix} z_N \\ 0 \end{bmatrix}\right\}\prod_{i<j}(z_i-z_j).\end{aligned} \tag{7}$$

The first term is of the form of the so-called edge spin texture,

$$\Psi_{tex}(\delta)=\prod_{i=1}^{N}\begin{bmatrix} 1 \\ \dfrac{\delta}{\sqrt{2}}z_i \end{bmatrix}\prod_{i<j}(z_i-z_j), \tag{8}$$

found in Ref. 11, see also Ref. 14, where δ, a complex parameter, is equal to $\delta=-\sqrt{2}(1/w)$ in our case. In Appendix A we show[11] that Eq. (8) is a mean-field (BCS) construction of the condensation of the system eigenstates, particle-hole (exciton) pairs of opposite spin. In Appendix B we classify all rotationally symmetric (excited) states of the Laughlin disk, made by the Hartree-Fock ansatz, and find the state in Eq. (8) on the edge, and the skyrmion coherent states at the center of the disk. Only the latter have clear topological quasiparticle interpretation.

Thus an edge state [Eq. (8)] (centered not in a point but around the edge) enters the description of a coherent state of the meron with coordinate w in Eq. (5). As $|w|\to R^*$—radius characteristic for the excitation (8) in the expansion (7), to be calculated below—the meron description reduces, in the first approximation, to the edge state with a complex boundary spin structure and bulk polarization that approaches the one of the ground state (all spins up).

In the case of a single Laughlin quasihole [Eq. (6)], the largest radius that the quasihole takes in the expansion is given by the radius of the state with the largest angular momentum $\left[\begin{smallmatrix} -w \\ 0 \end{smallmatrix}\right]^N$. The radius is $r_o=\sqrt{2(N+1)}\approx\sqrt{2N}\approx R$—radius of the system. In the Laughlin case, we can generate[13] edge excitations (in the form of multiples of symmetric polynomials) by placing and expanding coherent states of Laughlin quasiholes near the edge. The states are

$$\Psi_L(w_1,\ldots,w_n)=\prod_{i=1}^{N}\begin{bmatrix} (z_i-w_1)\cdots(z_i-w_n) \\ 0 \end{bmatrix}\prod_{i<j}(z_i-z_j), \tag{9}$$

where n (number of quasiholes) = 1,2, . . . , and $|w_i|\to R$, $i=1,\ldots,n$. By an analogy, for the present system, besides these constructions (multiplication by symmetric polynomials), we would expect that nontrivial edge constructions can be expressed through coherent states of merons,

$$\Psi(w_1,\ldots,w_m)=\prod_{i=1}^{N}\left[\frac{(z_i-w_1)\cdots(z_i-w_m)}{z_i^m}\right]\prod_{i<j}(z_i-z_j), \tag{10}$$

which define various sectors of the (edge) theory for fixed positive integers m. (The reason we select these states is also because their polarization in the center (bulk) is of the ground state.) The question immediately raised is whether in this case also $|w_i|\sim R, i=1,\ldots,m$, because merons are extended objects and the notion that they are right at the system boundary is not well defined. Indeed, we should look more carefully into the single (pair) meron case [Eqs. (5) and (7)].

We can view Eq. (7) as an expansion over unnormalized states of a meron (which by themselves are superpositions of the orbital angular momentum eigenstates). A representation of the absolute square of a normalized meron state solely in the variable w can be obtained by tracing out z variables in the absolute square of the corresponding state in the expansion, and dividing by the trace in w and z variables of the same square. Similarly we can get the averaged (mean) orbital angular momentum for the state in the expansion by sandwiching wd/dw by the same state, tracing out both z and w variables, performing the sum, and dividing the sandwich by the same trace and the sum of the square (only) of the state. In this way we can get that for the first state in the expansion, the mean orbital angular momentum is $N/2$, so the effective radius of the meron is $R^*\approx\sqrt{N}\approx R/\sqrt{2}$. This is a considerable distance away from the usually defined edge region. Therefore, we have a different type of edge of different radius for the single excitation meron. Thus we should have cautiously called the first term in the expansion, Eq. (7), the edge state, although indeed it has the form (and underlying construction) of the state [Eq. (8)] so called in the reference.[11]

The states in the expansion, Eq. (7), orthogonal with respect to the scalar product defined above, are not eigenstates of the hamiltonian; they are the eigenstates of the combined symmetry generators, $L_m^z=M+mS^z-(m/2)N$, $(m=1)$ where M is the orbital angular momentum and S^z is the z projection of the spin of the system. The eigenvalues are M_o+k, $k=0,1,2,\ldots$, where M_o is the (orbital) angular momentum of the $\nu=1$ polarized ground state. It is easy to generalize these considerations to $m=\pm1,\pm2,\ldots$ cases, where we also consider antimeron (see Appendix B) constructions, and it is interesting to note that similar combinations of the generators as symmetries are characteristic for skyrmion costructions on a sphere,[10] and in the center of a droplet [see Eq. (1) with $w=0$]. The high symmetry of the states singles them and corresponding coherent states (generators [Eq. (10)]) out as, maybe, the most we can get in the search for an effective, mean-field quasiparticle picture of the system near the boundary.

At this point, we want to emphasize our line of reasoning. We begin with a reasonable meron coherent state [Eq. (5)], and then expand it mimicking the Laughlin quasihole expansion [Eq. (6)]. Explicitly, we see that it is not an expansion over the orbital momentum eigenstates of a single free particle in the lowest Landau level. Therefore, we do not deal with a well-defined particle. Nevertheless, we want to see what as a description comes out, if we approximate and model the expansion in the form [similar to the Laughlin quasihole expansion (6)]

$$\Psi(w)\sim\sum_k w^{m(k)}\Psi_k(z_1,\ldots,z_N), \tag{11}$$

where Ψ_k's are electronic wave functions (that include the description of spin). (Therefore, we enforce a particle interpretation.) Each term in the expansion approximates a state in the expansion (7) by a state of meron mean angular momentum $m(k)$ (given above in the $k=0$ case), and an electronic wave function that does not depend on w. A way to obtain the function in the $k=0$ case will be shown below and other functions can be obtained similarly.

We model $\Psi_{k=0}$ as the edge spin texture in Eq. (8) with parameter $c=\sqrt{2}/\delta$ (which is not a variable such as w) demanding that the mean value of S_z (or the orbital angular momentum) in the state of Eq. (7) corresponding to $k=0$ is equal to the spin (or orbital angular momentum) in a state which we get from the same state by substituting variable w with the constant c. In this way we find that $\langle S_z\rangle|_{k=0}=0$ and $|c|^2=N=(R^*)^2$ (for a large N). We should not be surprised by the zero of spin; in our work we do not introduce Zeeman energy as a cutoff for the extent of possible excitations.

Similarly, we can get other functions, Ψ_k's, and corresponding $m(k)$'s in the expansion (11). For the second term in the expansion $m(k=1)=N/3$ (for a large N) and the corresponding mean radius is $R/\sqrt{3}$. Therefore we are getting a discrete series, $R(1/\sqrt{2},1/\sqrt{3},\ldots)$, for the mean radii of the wave functions in the expansion (7) [or Eq. (11)]. We should not take the discrete mean radius (and angular momentum) series too seriously because the mean values are far from being sharply defined, and the distribution in question (over the angular momentum states), for example, for the first state is uniform. Nevertheless, the beginning of the series marks an unusual behavior for a quasiparticle correlator in the bulk as we will see in the following section.

We may easily construct operators that act on the first state in the expansion (7) and give the states further on in the expansion. These are multiplets of

$$\rho_1=\sum_{i=1}^{N}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \tag{12}$$

and its inverse operator is

$$\rho_{-1}=\sum_{i=1}^{N}\begin{bmatrix}-\frac{w}{z_i} & -\frac{w^2}{z_i^2}\\ 1 & \frac{w}{z_i}\end{bmatrix}. \tag{13}$$

Their commutator is

$$[\rho_{-1},\rho_1]=\sum_{i=1}^{N}\begin{bmatrix}-1 & -2\frac{w}{z_i}\\ 0 & 1\end{bmatrix}. \tag{14}$$

For a large N, the right-hand side operator is the identity times N in the subspace of the first, low-lying states in the expansion and ρ_{-1} annihilates the first state. Therefore, we have a single simple harmonic oscillator algebra for these states of the meron (pair). Similarly, we can construct the same algebra for the states at the other end of the expansion.

III. STATIC CORRELATIONS OF A MERON

We consider the construction (5) of two merons, where one meron is at the center of the disk. The wave function we will use is

$$\Psi_{\gamma}(w)=\prod_{i=1}^{N}\begin{bmatrix} z_i-w \\ \gamma z_i \end{bmatrix}\prod_{i<j}(z_i-z_j), \tag{15}$$

where we introduced a (real) parameter γ which we will set to 1 at the end of calculation.

It is instructive to first do the calculation in the case of a single Laughlin quasihole, which is simply a hole at the filling $\nu=1$. In both the cases, we first calculate the norm of the excitation at point w and then by the way of analytical continuation to $\overline{w}\rightarrow\overline{w}_1=r$ and $w\rightarrow w_2=r\exp\{ix/r\}$, where $r=R$, the radius of the system in the Laughlin case, and $r=R^*$ in the meron case, find the correlator at distance x.[15] The Laughlin quasihole can be written as

$$\Psi_{L.qh.}(w)=\prod_{i=1}^{N}(z_i-w)\Psi_v, \tag{16}$$

where, in the case of filling factor $\nu=1$,

$$\Psi_v=\prod_{i<j}(z_i-z_j) \tag{17}$$

is the Vandermonde determinant. Its norm is easy to calculate,

$$\begin{aligned}\int\int \Psi_v^*\Psi_v dz_1\cdots dz_N &= \sum_{\sigma}\int\int |z_{\sigma(1)}|^{2(N-1)}\cdots|z_{\sigma(N)}|^0 \\ &= N!\int\int |z_1|^{2(N-1)}|z_2|^{2(N-2)}\cdots|z_N|^0\end{aligned} \tag{18}$$

(where we, for simplicity, dropped the signs of differentials). What we learned in this simple exercise is that only nonzero contribution to the integral comes from the equal powers of z^* and z for a given coordinate. We may expand Eq. (16) as

$$\begin{aligned}\Psi_{L.qh.}(w)=\{&(-w)^N+s_1(-w)^{N-1} \\ &+s_2(-w)^{N-2}+\cdots s_N\}\Psi_v,\end{aligned} \tag{19}$$

where $\{s_n; n=1,\ldots,N\}$ are elementary symmetric polynomials, i.e.,

$$s_n=\sum_{\langle i_1\cdots i_n\rangle} z_{i_1}\cdots z_{i_n}, \tag{20}$$

where sum goes over all possible multiplets of n z's. Then, to calculate the norm, we may consider

$$\begin{aligned}s_n\Psi_v=&\left(\sum_{\langle i_1,\ldots,i_n\rangle} z_{i_1}\cdots z_{i_n}\right) \\ &\times\left(\sum_{\sigma}\operatorname{sgn}\sigma z_{\sigma(1)}^{N-1}z_{\sigma(2)}^{N-2}\cdots z_{\sigma(N)}^{0}\right),\end{aligned} \tag{21}$$

which, after some insight, can be reduced to

$$s_n\Psi_v=\sum_{\sigma}\operatorname{sgn}\sigma z_{\sigma(1)}^{N}z_{\sigma(2)}^{N-1}\cdots z_{\sigma(n)}^{N-n+1}(1)z_{\sigma(n+1)}^{N-n-1}\cdots z_{\sigma(N)}^{0}. \tag{22}$$

(Here 1 stands for the empty orbit $N-n$.) Therefore, if we consider norm

$$\begin{aligned}\mathcal{N}_{L.qh.}&=\int\int \Psi_{L.qh.}^*\Psi_{L.qh.}dz_1\cdots dz_N \\ &=\sum_{n,m}\int\int s_n^*s_m\Psi_v^*\Psi_v dz_1\cdots dz_N(-w)^{N-n} \\ &\quad(-w)^{N-m} \\ &=\sum_{n}\int\int |s_n|^2|\Psi_v|^2dz_1\cdots dz_N|-w|^{2(N-n)}\end{aligned} \tag{23}$$

and take $\mathcal{N}_m$ to denote

$$\mathcal{N}_m=\int\Psi_m^*(z)\Psi_m(z)dz, \tag{24}$$

where

$$\Psi_m(z)=z^m e^{-1/4|z|^2}, \tag{25}$$

we have that

$$\mathcal{N}_{L.qh.}\sim\sum_{n=0}^{N}\frac{|w|^{2(N-n)}}{\mathcal{N}_{N-n}}=\sum_{n=0}^{N}\frac{1}{2^{N-n+1}\pi(N-n)!}|w|^{2(N-n)}. \tag{26}$$

(As a conclusion, in the thermodynamic limit,

$$\mathcal{N}_{L.qh.}\sim e^{+|w|^2/2}, \tag{27}$$

which is a well-known result.) In the finite-size system, we may rewrite

$$\mathcal{N}_{L.qh.}\sim\sum_{k=0}^{N}\frac{1}{k!2^k}|w|^{2k}, \tag{28}$$

and consider two points on the edge of the disk: $w_1^*=R$, and $w_2=Re^{ix/R}$, to which we can analytically continue $|w|^2$ as $|w|^2\rightarrow w_1^*w_2$. Then, if we approximate that the most important contribution comes from $k\sim N$, we have

$$k!\sim\sqrt{2\pi}k^{k+1/2}e^{-k}\sim\sqrt{2\pi}N^{k+1/2}e^{-N}. \tag{29}$$

On the other hand, the averaged square of distance is equal to $2(m+1)$. Therefore, square of the radius of the disk is proportional to $2N$. All these things together give for $f(x)$—static one-particle correlator of a Laughlin quasihole along the edge,

$$f(x)\sim\sum_{k=0}^{N}\frac{1}{k!2^k}R^{2k}e^{ik\,x/R}\approx\frac{1}{\sqrt{2\pi N}e^{-N}}\sum_{k=N_\Lambda}^{N}e^{ikx/R}, \tag{30}$$

which is proportional to nothing, but the correlator of one electron in the one-dimensional Fermi liquid theory with the Fermi momentum is propotional to $N-N_\Lambda$. When $R/(N-N_\Lambda)\ll x\ll R$,

$$f(x)\sim\frac{1}{1-e^{ix/R}}\sim\frac{1}{\sin x/2R}\sim\frac{1}{x}. \tag{31}$$

On the other hand, to accomplish an analysis of the correlations in the case of the meron excitation (15), we might consider, in the beginning, that

$$\prod_{i=1}^{N}|z_i-w|^2=\prod_{i=1}^{N}(|z_i|^2-w^*z_i-wz_i^*+|w|^2) \tag{32}$$

can be symbollically expressed as a polynomial in the following way:

$$\sum_{a+b+c+d=N}\frac{N!}{a!b!c!d!}\{|z_i|^2\}^a\{-w^*z_i\}^b\{-wz_i^*\}^c\{|w|^2\}^d. \tag{33}$$

Here each multiple comes from a different term (particle) i in the product (32). We learned from Eq. (23) that in the integration of

$$\sum_{n,m}s_n^*s_m\Psi_v^*\Psi_v(-w^*)^n(-w)^m, \tag{34}$$

effectively, we were calculating overlaps between the same permutations in Ψ_v^* and Ψ_v. To introduce nonzero overlaps, terms under b and c in Eq. (33) have to mix the permutations, which are different, and, therefore, in the final calculation their contribution does not appear.

In the case of the excitation (15), parallel to Eq. (32), we have

$$\prod_{i=1}^{N}(|z_i-w|^2+\gamma^2|z|^2)=\prod_{i=1}^{N}(1+\gamma^2)\left\{|z_i|^2-\frac{w^*}{(1+\gamma^2)}z_i-\frac{w}{(1+\gamma^2)}z_i^*+\frac{|w|^2}{(1+\gamma^2)}\right\}. \tag{35}$$

Further on we will neglect $(1+\gamma^2)$ in front of the curly brackets. Let us consider the expansion of the norm of Eq. (15) in the powers of $|w|^2$. We consider the most important terms for the physics of the edge that are of the order $\sim|w|^{2N}$. The highest power contribution to the norm is

$$\frac{|w|^{2N}}{(1+\gamma^2)^N}T_0\frac{\mathcal{N}_0}{\mathcal{N}_N}, \tag{36}$$

where

$$T_0=N!\prod_{n=1}^{N}\mathcal{N}_n \tag{37}$$

(the norm of the Laughlin quasihole that resides in the center of the disk). The contribution, Eq. (36), when $\gamma=0$ is the same as in the Laughlin case, and is the result of the multiplication of $|w|^2$ in Eq. (32), or (the term under d) in Eq. (33).

Now, let us consider the contribution of the order of $|w|^{2N-2}$. The contributing terms from Eq. (33) are of the form $|w|^{2N-2}|z_i|^2$ where $|z_i|^2$, can be from the crossed terms, under b and c, or solely from under a. The contributions that we get from the latter case are

$$N!\frac{|w|^{2N-2}}{(1+\gamma^2)^{N-1}}\mathcal{N}_N\mathcal{N}_{N-2}\cdots\mathcal{N}_0=\frac{|w|^{2N}}{(1+\gamma^2)^{N-1}}T_0\frac{\mathcal{N}_0}{\mathcal{N}_{N-1}}, \tag{38}$$

which is equal to the contribution in the Laughlin case when $\gamma=0$, and

$$\begin{aligned}N!\frac{|w|^{2N-2}}{(1+\gamma^2)^{N-1}}\sum_{k=0}^{N-2}\mathcal{N}_{k+1}\prod_{i=0,\neq k}^{N-1}\mathcal{N}_i\\=\frac{|w|^{2N-2}}{(1+\gamma^2)^{N-1}}T_0\frac{\mathcal{N}_0}{\mathcal{N}_N}\sum_{k=0}^{N-2}\frac{\mathcal{N}_{k+1}}{\mathcal{N}_k},\end{aligned} \tag{39}$$

which, in the Laughlin $(\gamma=0)$ case, is canceled by the contribution that we get from the terms under b and c, crossed terms in the Eq. (33). (This follows from the above analysis of the Laughlin case.) Therefore, to order $|w|^{2N-2}$, we have

$$T_0\frac{|w|^{2N-2}}{(1+\gamma^2)^N}\frac{\mathcal{N}_0}{\mathcal{N}_N}\left\{1+\frac{1}{|w|^2}\times[(1+\gamma^2)(2N+2\Sigma)-2\Sigma]+\cdots\right\}, \tag{40}$$

where

$$\Sigma=\sum_{k=1}^{N-1}k=\frac{(N-1)(N-2)}{2}. \tag{41}$$

This contribution in the large-N limit has the following expression in the curly brackets of Eq. (40):

$$1+\frac{\gamma^2N^2}{|w|^2}+\cdots. \tag{42}$$

It is important to note in Eq. (40) that the contribution from the crossed terms is of the order of $(1+\gamma^2)$ smaller (for large γ) than from the terms under a in Eq. (33).

Due to the complexity of the algebra, next we will only consider contributions of the order of $1/(1+\gamma^2)^{N-2}$, γ being large, for the fixed $(2N-4)$th power of $|w|$. (Here underlying assumption is that the neglected terms are of the same order in N as retained ones. We will address this question of the neglected contributions below.) Similar to Eqs. (36) and (38), we have the contribution

$$\frac{|w|^{2N-4}}{(1+\gamma^2)^{N-2}} T_0 \frac{\mathcal{N}_0}{\mathcal{N}_{N-2}}, \tag{43}$$

and, to that order also,

$$\frac{|w|^{2N-4}}{(1+\gamma^2)^{N-2}} \frac{1}{2!} T_0 \frac{\mathcal{N}_0}{\mathcal{N}_N} \sum_{l,k=0(l\neq k)}^{N-2} \frac{\mathcal{N}_{l+1}}{\mathcal{N}_l} \frac{\mathcal{N}_{k+1}}{\mathcal{N}_k} + \frac{|w|^{2N-4}}{(1+\gamma^2)^{N-2}} \frac{1}{2!} T_0 \frac{\mathcal{N}_0}{\mathcal{N}_{N-1}} \sum_{m=0}^{N-3} \frac{\mathcal{N}_{m+1}}{\mathcal{N}_m} 2!, \tag{44}$$

the terms that come solely from the multiplication of $|z_i|^2$ under a in Eq. (33). In the large-N limit, the first term in Eq. (44) is dominant with respect to the second term and the one in Eq. (43). After the limit, the expansion in Eq. (42), with this new contribution, becomes

$$1+\frac{\gamma^2 N^2}{|w|^2}+\frac{1}{2!}\frac{\gamma^4 N^4}{|w|^4}+\cdots, \tag{45}$$

where we approximated $1+\gamma^2$ with γ^2. Taking, further, only dominant contributions in N, we can obtain a close expression for Eq. (45),

$$\sum_{n=0}^{N} \frac{1}{n!}\left(\frac{\gamma N}{|w|}\right)^{2n}, \tag{46}$$

which after the same analytical continuation as in the Laughlin case and taking $\bar{w} \to \bar{w}_1 = R^*$ and $w \to w_2 = R^* \exp\{ix/R^*\}$, where $R^{*2} \approx N$, gives

$$\sum_{n=0}^{N} \frac{1}{n!}[\gamma^2 N e^{-ix/R^*}]^n. \tag{47}$$

The question that arises is whether we can continue this result to other, smaller values of γ including $\gamma=1$. We believe that we can claim and substantiate the same with the following.

We consider again the contribution to the norm of the excitation (15) of the order of $|w|^{2N-4}/(1+\gamma^2)^{N-2}$. The contribution without the one in Eq. (43) (the Laughlin case only contribution when $\gamma=0$) can be cast into the following form:

$$\frac{|w|^{2N-4}}{(1+\gamma^2)^{N-2}}[(1+\gamma^2)^2 r-(1+\gamma^2)p+q], \tag{48}$$

where $r,p,$ and q are positive numbers. The term with r is equal to the contribution given by Eq. (44). The terms with p and q come from the crossed terms in Eq. (33), which connect permutations with one and two exchanges, respectively. (The minus sign in front of p term stems from the overlap of permutations that differ for one exchange.) As we already emphasized, the contribution (48) should vanish as γ approaches zero. We were able to check this by simply counting the number of terms (single overlaps) that go into $r,p,$ and q, proving that the numbers satisfy

$$\mathcal{K}_r-\mathcal{K}_p+\mathcal{K}_q=0 \tag{49}$$

in the large-N limit. The numbers we found are

$$\mathcal{K}_r=\binom{N}{2}N!, \tag{50}$$

$$\mathcal{K}_p=\binom{N}{3}\binom{3}{2}2(N-1)!, \tag{51}$$

and

$$\mathcal{K}_q=\binom{N}{4}\binom{4}{2}2(N-4)!(N-1)(N-3), \tag{52}$$

which in the large N-limit give

$$\lim_{N\to\infty}\frac{\mathcal{K}_r}{\mathcal{K}_p}=\lim_{N\to\infty}\frac{\mathcal{K}_q}{\mathcal{K}_p}=\frac{1}{2}. \tag{53}$$

Therefore we may conclude that in the same limit, $p=2r$, $q=r$, and the Eq. (48) becomes

$$\frac{|w|^{2N-4}}{(1+\gamma^2)^{N-2}}(\gamma^2)^2 r, \tag{54}$$

which supports our conjecture, to the given order, that the expansion for an arbitrary γ can be brought to the form in Eq. (45) or (46).

Similar to the Eq. (30) (in the Laughlin case) the sum in Eq. (47) can be approximated, and, as a result, we get that the static correlator along the radius R^*, of the excitation in Eq. (15) is propotional to

$$\sim\frac{1}{1-\gamma^2 e^{ix/R^*}}. \tag{55}$$

This implies, approximately, that in the meron case, $\gamma=1$, the correlator behaves as

$$f_m(x)\sim\frac{1}{x} \tag{56}$$

(in the long-wavelength limit). It is interesting to note that the main contribution to the correlator was coming from the angular momentum states near the center where the other meron is placed. This is quite opposite to the case of the Laughlin quasihole [Eqs. (28) and (30)], and points out to the strong entanglement of the two merons.

IV. DISCUSSION AND CONCLUSIONS

To conclude, in order to understand and construct positional coherent states of the topological excitations away from the central region of the $\nu=1$ large quantum Hall droplet (at least in the case of zero Zeeman coupling), we studied the meron coherent state, given by Eq. (3), when one meron in the pair (w_1,w_2) is kept at the center. It turned out that the state may serve as a generator of states of high rotational symmetry, which can be classified by positive integers (eigenvalues of the symmetry generators) just like the states in the expansion of a Laughlin quasihole with orbital angular momentum eigenvalues. Then we considered an effective (approximative) expansion of the meron pair, where each term was modeled as an orbital momentum eigenstate with its mean orbital angular momentum as eigenvalue. Therefore, we effectively introduced mean radii around which the meron states are spread. In this way we found the maximum radius of the farthest meron state, R^*, at a distance considerably less than the radius of the system. Remarkably, this distance marks the usual edge behavior characterized by a power-law decay of the single-quasiparticle, meron static correlator.

The exponent of the decay does not correspond to the one, equal to $\frac{1}{4}$, of the correlator in the case of one Abelian quasiparticle, associated with the half of the unit of charge and the half of the unit of flux. We do not think that we have a case for the non-Abelian statistics,[16] rather we may note that there is a strong entanglement between merons in the pair, and also, the status of the quasiparticles in the finite system is far from being clear. Nevertheless, as with the correlator, the radius R^* might mark an unusual behavior for the tunneling into the bulk of the system at zero Zeeman coupling. The existence of the radius away from the real edge, in the effective quasiparticle picture, might also suggest that topological excitations do not contribute to the transport along the edge, and the transfer of charge and quantization of the Hall current in the edge region are ensured by the Laughlin (quasi) holes only.

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APPENDIX A

The exciton superposition,

$$\Sigma_1^\dagger = \sum_m \sqrt{m+1}\, c^\dagger_{m+1,\downarrow} c_{m,\uparrow}\,, \tag{A1}$$

with all well defined (commuting) quantum numbers,[11] and obviously edge excitation because of the weight $\sim\sqrt{m+1}$ (which grows as we are approaching the edge), can be used to describe a BCS condensate:

$$|\Psi_{tex}(\delta)\rangle = \sum_{n=0}^{N-2} \frac{\delta^n}{n!} (\Sigma_1^\dagger)^n |C_N\rangle, \tag{A2}$$

where $|C_N\rangle$ is the incompressible condensate (the filled lowest Landau level) at $\nu=1$.

The fact that Eq. (A2) leads to Eq. (8) can be proved by taking the thermodynamic limit $(N\to\infty)$ when

$$\begin{aligned}|\Psi_{tex}(\delta)\rangle &= \exp\{\delta\Sigma_1^\dagger\}|C_N\rangle \\ &= \prod_{j=0}^{\infty} \exp\{\delta\sqrt{j+1}\,c^\dagger_{j+1,\downarrow}c_{j,\uparrow}\}|C_N\rangle \\ &= \prod_{j=0}^{\infty} \{1+\delta\sqrt{j+1}\,c^\dagger_{j+1,\downarrow}c_{j,\uparrow}\}|C_N\rangle \\ &= \prod_{j=0}^{\infty} \{c^\dagger_{j,\uparrow}+\delta\sqrt{j+1}\,c^\dagger_{j+1,\downarrow}\}|0\rangle,\end{aligned} \tag{A3}$$

where $|0\rangle$ is vacuum. In the spinor language, we have the following Slater determinant of single-particle states:

$$\begin{bmatrix}\Phi_j \\ \delta\sqrt{j+1}\,\Phi_{j+1}\end{bmatrix} = \begin{bmatrix}\Phi_j \\ \dfrac{\delta}{\sqrt{2}} z\Phi_j\end{bmatrix} = \begin{bmatrix}1 \\ \dfrac{\delta}{\sqrt{2}} z\end{bmatrix}\Phi_j\,, \tag{A4}$$

where

$$\Phi_j = \frac{z^j}{\sqrt{2^{j+1}\pi j!}} \tag{A5}$$

is a single-particle, normalized wave function in the lowest Landau level. Therefore, we can prove that the BCS superposition of particle-hole pairs leads to the edge spin-texture (8).

APPENDIX B

In this Appendix we review known constructions of the excited states, made in the form of Slater determinants of single-particle states, from a unifying point of view.

As a way of relieving of, for example, the double occupancy of electrons at the center of the disk (in the case of the Laughlin quasiparticle excitation) we might consider the following superpositions of the single-electron states:

$$(c^\dagger_{m\uparrow}+\alpha c^\dagger_{m+1\downarrow}) \tag{B1}$$

for each m. In the single-particle spinor language, we have

$$\begin{pmatrix}\Phi_m \\ \alpha\Phi_{m+1}\end{pmatrix}. \tag{B2}$$

Here Φ_m's are defined in Eq. (A5) and α is an arbitrary complex number. Now we take the Hartree-Fock ansatz, by ensuring that the final state can be cast into the Slater determinant of single-particle states. We have as a possibility of Eq. (B2) in the form

$$\begin{pmatrix}\Phi_m \\ \alpha\dfrac{z}{\sqrt{(m+1)2}}\Phi_m\end{pmatrix}, \tag{B3}$$

where we chose $\alpha\sim\sqrt(m+1$ to get an edge (hole) excitation, or

$$\begin{pmatrix}\dfrac{\sqrt{2}}{\sqrt{(m+1)}}\partial_z\Phi_{m+1} \\ \alpha\Phi_{m+1}\end{pmatrix}, \tag{B4}$$

where we chose $\alpha \sim 1/\sqrt{m+1}$ to get a bulk (particle- skyrmion) excitation (in the center of the disk). Taking another superposition,

$$(c^{\dagger}_{m\uparrow} + \beta c^{\dagger}_{m-1\downarrow}), \tag{B5}$$

for $m > 0$, similarly we can get an edge (particle) excitation and a bulk (hole, antiskyrmion) excitation (in the center of the disk) [Eq. (1)].

The excitations that we found are rotationally symmetric and reside either in the center of the disk or on the edge uniformly extended. The question arises what (coherent) quasiparticle state, beside Laughlin quasihole, we have in between, or away from the central region.

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