

Composite bosons in bilayer $\nu=1$ system: An application of the Murthy-Shankar formalism

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We calculate the dispersion of the out-of-phase mode characteristic for the bilayer $\nu=1$ quantum Hall system applying the version of Chern-Simons theory of Murthy and Shankar that cures the unwanted bare electron mass dependence in the low-energy description of quantum Hall systems. The obtained value for the mode when d , distance between the layers, is zero is in a good agreement with the existing pseudospin picture of the system. For d nonzero but small we find that the mode is linearly dispersing and its velocity to a good approximation depends linearly on d . This is in agreement with the Hartree-Fock calculations of the pseudospin picture that predicts a linear dependence on d , and contrary to the naive Hartree predictions with dependence on the square root of d . We set up a formalism that enables one to consider fluctuations around the found stationary point values. In addition we address the case of imbalanced layers in the Murthy-Shankar formalism.

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A major problem surfaced in the early Chern-Simons (CS) composite boson description¹ of $\nu=1$ quantum bilayer. Namely, a bare electron mass appeared in the Bogoliubov out-of-phase gapless mode dispersion, which is unwanted due to the expectation that the leading description of any intra-Landau-level collective mode depends solely on interactions. The same requirement applies for the description of any quasiparticles that may exist in the lowest Landau level, namely their mass should stem from interactions.² Murthy and Shankar^{3,4} put forward an extended CS theory that was able to prescribe such a mass for composite particles. The theory underwent major additions and improvements (including an extension to higher momenta),⁵ but in this paper we will use the early version⁴ best suited for our needs, i.e., the calculation of the dispersion relation of the out-of-phase mode in a small-momentum, low-energy window.

On the other hand, what is believed to be the first calculation of the dispersion of the gapless mode in the scope of the pseudospin picture (in which an electron can be in states that are superpositions of localized layer states) was done by Fertig.⁶ He obtained an interaction-dependent velocity of a linearly dispersing mode when d is nonzero and a spin wave quadratic dispersion with interaction-dependent spin stiffness when d is zero. The problem at $d=0$ was again addressed in Ref. 7 where a quantum ferromagnet picture for this case was established. Then followed the pseudospin picture of Ref. 8, a standard reference for the bilayer $\nu=1$ problem, in which small-momentum dispersion of the gapless mode was in agreement with Fertig's.

Our Hamiltonian is

$$\mathcal{H} = \sum_{i,\sigma} \frac{|\vec{p}_{i,\sigma} + e\vec{A}(\vec{r}_{i,\sigma})|^2}{2m} + V_E + V_A, \quad (1)$$

where \vec{A} is the vector potential of the constant external magnetic field $-B_0\vec{e}_z$, so that the average total density is $n = \nu/2\pi l_B^2$ with $\nu=1/(2s+1)$, $s=0,1,\dots$ (for generality), and $l_B=1/\sqrt{eB_0}$ is the magnetic length. We take $l_B=1$ and $\hbar=1$. m is the "bare" electron mass, which is precisely the effective

mass of electron in GaAs. V_A and V_E denote intralayer and interlayer interactions, respectively. $\sigma=\uparrow, \downarrow$ is the layer index.

What follows is a brief account of a simple generalization of the Murthy and Shankar CS theory extended to the case of the bilayer. Missing details and explanations that are relevant also to the single-layer case can be found in Refs. 3–5. As usual in the CS theory we make the unitary transformation,⁹

$$U = \exp\left\{i(2s+1)\sum_{i<j}\Phi_{ij}\right\}, \quad (2)$$

where Φ_{ij} is the phase of the difference, $z_i - z_j$, of any two coordinates z_i and z_j . Therefore, we emphasize the transformation is the same, irrespective of the layer indices. The corresponding Hamiltonian is

$$\mathcal{H}_{CS} = U^{-1}\mathcal{H}U = \sum_{i,\sigma} \frac{|\vec{p}_{i,\sigma} + e\vec{A}(\vec{r}_{i,\sigma}) + \vec{a}_{CS}|^2}{2m} + V_E + V_A, \quad (3)$$

where the new "gauge" field satisfies the following connection with the total density, $\rho(r)$:

$$\vec{\nabla} \times \vec{a}_{CS} = 2\pi(2s+1)\rho(r)\vec{e}_z. \quad (4)$$

Then we consider averaged and fluctuating values of \vec{a}_{CS} and ρ ,⁹ rewriting Eq. (4) as

$$\vec{\nabla} \times \langle \vec{a}_{CS} \rangle + \vec{\nabla} \times : \vec{a}_{CS} := 2\pi(2s+1)n + 2\pi(2s+1):\rho: \quad (5)$$

so that equivalently, due to the cancellation of the external and averaged CS field, we can rewrite Eq. (3) as

$$\mathcal{H}_{CS} = \sum_{i,\sigma} \frac{|\vec{p}_{i,\sigma} + : \vec{a}_{CS} :|^2}{2m} + V_E + V_A. \quad (6)$$

Now Shankar and Murthy,^{3,4} analogously to what Bohm and Pines¹⁰ did in the case of a three-dimensional (3D) Coulomb gas, introduce magnetoplasmon degrees of freedom as separate and elementary but necessarily satisfying certain

constraints with particle degrees of freedom in order to avoid overcounting. They do this by introducing a pair of conjugate fields, $a(\vec{q})$ and $P(\vec{q})$, for each \vec{q} in a disk in the momentum space,

$$[a(\vec{q}), P(\vec{q}')] = i(2\pi)^2 \delta(\vec{q} + \vec{q}') \quad (7)$$

[reminding us (for fixed \vec{q}) of the harmonic oscillator commutation relation in (x, p) representation] and further, defining a longitudinal and a transverse field, $\vec{P}(\vec{q})$ and $\vec{a}(\vec{q})$, respectively, as

$$\vec{P}(\vec{q}) = i\hat{q}P(\vec{q}) \quad \text{and} \quad \vec{a}(\vec{q}) = -i\vec{e}_z \times \hat{q}a(\vec{q}). \quad (8)$$

They rewrite the Hamiltonian (density) as

$$\mathcal{H} = \frac{1}{2m} \sum_{\sigma} \Psi_{\text{CS},\sigma}^{\dagger} (-i\vec{\nabla} + \vec{a}_{\text{CS}} + \vec{a})^2 \Psi_{\text{CS},\sigma} + \tilde{V}_A + \tilde{V}_E, \quad (9)$$

in the second-quantized language, with the requirement (constraint) that

$$a(\vec{q})|\text{physical state}\rangle = 0, \quad (10)$$

for each \vec{q} such that $|\vec{q}| < Q$ where Q is the radius of the disk. Now it is convenient to eliminate \vec{a}_{CS} in favor of a and P , and Shankar and Murthy do that³ applying the following unitary transformation,

$$U = \exp \left\{ i \sum_{|\vec{q}| < Q} P(-\vec{q}) \frac{2\pi(2s+1)}{q} \rho(\vec{q}) \right\}, \quad (11)$$

where $\rho(\vec{q}) = \rho_{\uparrow}(\vec{q}) + \rho_{\downarrow}(\vec{q})$, i.e., the total charge density. Now for the Hamiltonian density we have

$$\mathcal{H} = \sum_{\sigma} \frac{1}{2m} \Psi_{kb,\sigma}^{\dagger} [-i\vec{\nabla} + \vec{a} + 2\pi(2s+1)\vec{P} + \delta\vec{a}]^2 \Psi_{kb,\sigma} + \tilde{V}_A + \tilde{V}_E, \quad (12)$$

where $\Psi_{kb,\sigma}$'s denote the transformed fields, which describe modified, transformed quasiparticles, and $\delta\vec{a}$ is the remnant of the CS field left uncanceled for $|\vec{q}| > Q$. The constraint gets the following form:

$$\left[a(\vec{q}) - \frac{2\pi(2s+1)}{q} \rho(\vec{q}) \right] |\text{physical state}\rangle = 0 \quad \text{for } |\vec{q}| < Q. \quad (13)$$

Neglecting³ $\delta\vec{a}$ from the start and introducing³

$$A(\vec{q}) = \frac{a(\vec{q}) + i2\pi(2s+1)P(\vec{q})}{\sqrt{4\pi(2s+1)}} \quad (14)$$

and

$$c(\vec{q}) = \hat{q} \cdot \sum_{j,\sigma} \vec{p}_{j,\sigma} \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\}, \quad (15)$$

where $V_{\pm} = V_x \pm iV_y$ for an arbitrary vector $\vec{V} = V_x \hat{e}_x + V_y \hat{e}_y$, we can rewrite our Hamiltonian as

$$\begin{aligned} \mathcal{H} = & \sum_{i,\sigma} \frac{\vec{p}_{i,\sigma}^2}{2m} + \sum_{|\vec{q}| < Q} \omega_c A^{\dagger}(\vec{q}) A(\vec{q}) \\ & + \sqrt{\frac{\pi(2s+1)}{m}} \sum_{|\vec{q}| < Q} [c(\vec{q}) A^{\dagger}(\vec{q}) + c^{\dagger}(\vec{q}) A(\vec{q})] + V_A + V_E. \end{aligned} \quad (16)$$

As expected $\omega_c = eB_0/m$, i.e., equal to the cyclotron frequency and in deriving the magnetoplasmon term we neglected also total density fluctuations.⁴

Our quasiparticles are bosons and again for the sake of completeness and easy reference we give brief account of the so-called final representation in the Murthy-Shankar approach applied to the case of two species of composite bosons. First Murthy and Shankar always approximate as

$$\sum_{i,\sigma} \exp\{i(\vec{q} - \vec{k}) \cdot \vec{r}_i\} \approx n(2\pi)^2 \delta^2(\vec{q} - \vec{k}) \quad (17)$$

in the long-wavelength approximation so that also in this bosonic representation we consistently have

$$[c(\vec{q}), c^{\dagger}(\vec{q}')] \approx 0. \quad (18)$$

To decouple the oscillators and particles they apply the following canonical transformation,

$$\begin{aligned} U(\lambda_0) = & \exp\{iS_0\lambda_0\} \\ = & \exp \left\{ \lambda_0 \theta \sum_{|\vec{q}| < Q} [c^{\dagger}(\vec{q}) A(\vec{q}) - A^{\dagger}(\vec{q}) c(\vec{q})] \right\}, \end{aligned} \quad (19)$$

where

$$\theta = \frac{1}{\sqrt{4\pi(2s+1)n}}, \quad (20)$$

and the parameter λ_0 should be determined. As we have new variables Ω defined through

$$\Omega^{old} = \exp\{-iS_0\lambda_0\} \Omega \exp\{iS_0\lambda_0\}, \quad (21)$$

Murthy and Shankar also define

$$\Omega(\lambda) = \exp\{-iS_0\lambda\} \Omega \exp\{iS_0\lambda\}, \quad (22)$$

so that in this case we have

$$\begin{aligned} A(\vec{q}, \lambda) &= A(\vec{q}) - \theta \lambda c(\vec{q}), \\ c(\vec{q}, \lambda) &= c(\vec{q}), \quad |\vec{q}| < Q. \end{aligned} \quad (23)$$

It is easy to check that $\lambda_0 = 1$ does the job of decoupling and now we concentrate how variables $\rho_{\sigma}^{old}(\vec{q})$, $\sigma = \uparrow, \downarrow$ are connected with new ones. We use their definitions $\rho_{\sigma}^{old}(\vec{q}) = \sum_i \exp\{i\vec{q} \cdot \vec{r}_{i,\sigma}\}$, $\sigma = \uparrow, \downarrow$ apply Eqs. (21) and (22), find $d\rho_{\sigma}(\vec{q}, \lambda)/d\lambda$, and integrate over λ to get

$$\rho_{\sigma}^{old}(\vec{q}) = \rho_{\sigma}(\vec{q}) + \frac{q}{\sqrt{4\pi(2s+1)}} \frac{n_{\sigma}}{n} \left[A(\vec{q}) + A^{\dagger}(-\vec{q}) - \frac{\theta}{2} [c(\vec{q}) + c^{\dagger}(-\vec{q})] \right]. \quad (24)$$

Immediately we can conclude that the spin density $\rho_s^{old}(\vec{q}) = \rho_{\uparrow}^{old}(\vec{q}) - \rho_{\downarrow}^{old}(\vec{q})$ is invariant,

$$\rho_s^{old}(\vec{q}) = \rho_s(\vec{q}), \quad (25)$$

under the transformation of the final representation [Eq. (19)], when we assume that we have the same fixed number of particles, $n_{\uparrow} = n_{\downarrow} = n/2$, in each layer. This is our main claim in the Murthy-Shankar formalism for the bilayer system.

The analysis for the charge density and the form of the constraint in new variables proceeds as in Refs. 3 and 4 and finally for the form we get

$$\rho(\vec{q}) = -\frac{i}{2} \sum_{j,\sigma} (\vec{q} \times \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\}. \quad (26)$$

In a few lines but also using an assumption that we deal with an infinite system (without boundary) we can prove that, in the second-quantized language, the constraint is

$$\int d\vec{r} \exp\{-i\vec{q} \cdot \vec{r}\} \rho(\vec{r}) = \frac{i}{2} \int d\vec{r} \exp\{-i\vec{q} \cdot \vec{r}\} \sum_{\sigma} [\vec{\nabla} \times \Psi_{\sigma}^{\dagger}(\vec{r}) \vec{\nabla} \Psi_{\sigma}(\vec{r})] \quad (27)$$

for $|\vec{q}| < Q$ and as a shorthand notation we use $\Psi_{kb,\sigma} \equiv \Psi_{\sigma}$ also in the following. The proof starts by expressing the single-particle operator,

$$\sum_{j,\sigma} (\vec{q} \times \vec{\nabla}_j) \exp\{-i\vec{q} \cdot \vec{r}_j\}, \quad (28)$$

in the second-quantized language as

$$\sum_{\sigma} \int d\vec{r} \Psi_{\sigma}^{\dagger}(\vec{r}) (\vec{q} \times \vec{\nabla}) \exp\{-i\vec{q} \cdot \vec{r}\} \Psi_{\sigma}(\vec{r}), \quad (29)$$

then followed by simple regroupings and the neglect of a surface term. So we find that in the long-wavelength approximation we use, the charge density fluctuations exist only if there are vortex excitations in the system. [The charge density on the right-hand side (rhs) of Eq. (27) is proportional to the vortex densities of the two kinds of fields. Strictly speaking the vortex density is defined only by the phase part of a bosonic field, but in the small-momentum limit we will neglect the difference. Later a relaxation of the constraint and thereby generation of terms quadratic in momenta will be justified by this difference.] If our system is a stable 2D Bose system that would mean that we have the case for incompressibility, because to excite a vortex a finite energy is needed so all charge fluctuations are suppressed. But do we have a stable system? We have to get back to the Hamiltonian expressed in new variables. The Hamiltonian is

$$\mathcal{H} = \sum_{i=1,\sigma} \frac{\vec{p}_i^2}{2m} - \frac{1}{2mn} \sum_{i,\sigma,j,\sigma'} \sum_{|\vec{q}| < Q} \vec{p}_{i,\sigma} \exp\{-i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)\} \times \vec{p}_{j,\sigma'+} + \omega_c \sum_{|\vec{q}| < Q} A^{\dagger}(\vec{q}) A(\vec{q}) + V_E + V_A. \quad (30)$$

To eliminate the bare electron mass in the kinetic energy and low-energy description we choose, as in Refs. 3 and 4, that the number of the oscillators is the same as the number of particles, so that the diagonal part ($i=j, \sigma=\sigma'$) of the second term in Eq. (30) exactly cancels the first kinetic energy term. The bare mass is still present in the off-diagonal part of the second term, and if we decompose the q sum as^{3,4}

$$\sum_{|\vec{q}| < Q} \exp\{-i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)\} = \delta^2(\vec{r}_i - \vec{r}_j) - \sum_{|\vec{q}| > Q} \exp\{-i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)\}, \quad (31)$$

we are left with a δ -function interaction among particles and another short-range interaction that may be grouped⁴ with previously neglected short-range pieces. We are assuming all along that the same kind (layer) of bosons (transformed electrons) behave as hard core bosons, so for $\sigma=\sigma'$ we see that the δ function is ineffective. To eliminate the bare mass in the δ -function interaction between the opposite kind bosons we require that they also behave mutually as hard core bosons. As we will see, this additional requirement (not due to the fermionic statistics) will be very important in the derivation of the low-lying spectrum.

Therefore, as a result of the transformations made, our Hamiltonian has a free oscillator and Coulomb interaction part only. The interaction part in the old variables with the introduced cutoff is

$$V \equiv V_A + V_E = \frac{1}{2} \sum_{\sigma, |\vec{q}| < Q} \rho_{\sigma}^{old}(-\vec{q}) V_A(\vec{q}) \rho_{\sigma}^{old}(\vec{q}) + \frac{1}{2} \sum_{\sigma, |\vec{q}| < Q} \rho_{\sigma}^{old}(-\vec{q}) V_E(\vec{q}) \rho_{-\sigma}^{old}(\vec{q}), \quad (32)$$

with $V_A(\vec{q}) = 2\pi e^2/|\vec{q}|$ and $V_E(\vec{q}) = (2\pi e^2/|\vec{q}|) \exp\{-d|\vec{q}|\}$, where d is the distance between layers. If we introduce

$$V_c(\vec{q}) = V_A(\vec{q}) + V_E(\vec{q}) \quad \text{and} \quad V_s(\vec{q}) = V_A(\vec{q}) - V_E(\vec{q}), \quad (33)$$

and

$$\rho_c^{old}(\vec{q}) = \rho_{\uparrow}^{old}(\vec{q}) + \rho_{\downarrow}^{old}(\vec{q}) \quad \text{and} \quad \rho_s^{old}(\vec{q}) = \rho_{\uparrow}^{old}(\vec{q}) - \rho_{\downarrow}^{old}(\vec{q}), \quad (34)$$

we can rewrite Eq. (32) as

$$V = \sum_{|\vec{q}| < Q} \frac{1}{4} \rho_c^{old}(-\vec{q}) V_c(\vec{q}) \rho_c^{old}(\vec{q}) + \sum_{|\vec{q}| < Q} \frac{1}{4} \rho_s^{old}(-\vec{q}) V_s(\vec{q}) \rho_s^{old}(\vec{q}). \quad (35)$$

To get the expression in new variables for $\rho_c^{old}(\vec{q})$ we should

compare Eq. (24), Eq. (26), and Eq. (15) and find

$$\rho_c^{old}(\vec{q}) = \frac{q}{\sqrt{4\pi(2s+1)}} [A(\vec{q}) + A^\dagger(\vec{q})] - i \sum_{j,\sigma} (\vec{q} \times \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\} \quad (36)$$

and also, as we already found out, we have

$$\rho_s^{old}(\vec{q}) = \rho_s(\vec{q}). \quad (37)$$

Further decoupling of the oscillators and particles in V would amount to higher-order corrections to the expressions found^{3,4} and we can safely neglect the presence of oscillators (terms with A 's) in Eq. (36) when discussing the low-energy excitations. Then the charge part [the first term in Eq. (35)] can be decomposed into a diagonal and off-diagonal part. The diagonal part can be rewritten as

$$\sum_{j,\sigma} \frac{|\vec{p}_{j,\sigma}|^2}{2m_c}, \quad (38)$$

i.e., as a kinetic term of particles with mass m_c , where

$$\frac{1}{m_c} = \sum_{|\vec{q}| < Q} \frac{V_c(\vec{q})}{2} q^2 \sin^2 \theta_{\vec{q}, \vec{p}_{j,\sigma}} \quad (39)$$

Therefore we came to a description of the system in terms of quasiparticles with a mass that is due to interactions. These are the expected bosonic dipole objects^{2,4} with interaction among them described by the off-diagonal part. As overall neutral objects they should make Bose condensate(s) in the ground state and we proceed by taking the Bogoliubov expansion of the quasiparticle operators $\Psi_\sigma, \sigma = \uparrow, \downarrow$ in the second-quantized language as

$$\Psi_\uparrow = \sqrt{\frac{n_0}{2}} + \eta_\uparrow \quad \text{and} \quad \Psi_\downarrow = \sqrt{\frac{n_0}{2}} + \eta_\downarrow, \quad (40)$$

where operators $\eta_\sigma, \sigma = \uparrow, \downarrow$ describe the small fluctuations around the mean field value, $\sqrt{n_0}/2$, where n_0 is the density of particles in each condensate. We also introduce

$$\eta_c = \frac{\eta_\uparrow + \eta_\downarrow}{\sqrt{2}} \quad \text{and} \quad \eta_s = \frac{\eta_\uparrow - \eta_\downarrow}{\sqrt{2}}, \quad (41)$$

new fields that, as we will find out soon, are appropriate for the low-energy description of the system.

In terms of the new variables, the constraint is, effectively,

$$\Psi_\uparrow^\dagger \Psi_\uparrow + \Psi_\downarrow^\dagger \Psi_\downarrow - n = n_0 - n + \sqrt{n_0}(\eta_c^\dagger + \eta_c) + \eta_c^\dagger \eta_c + \eta_s^\dagger \eta_s = 0. \quad (42)$$

Please note that the equality here should be understood as the equality of the Fourier transforms of lhs and rhs for \vec{q} small. It is also important to notice that although the constraint effectively is $\rho_c^{old}(\vec{q})=0$ for $\vec{q} \neq \vec{0}$ in the low-energy sector and constrains the first term in Eq. (35) to vanish, the underlying canonical variables η_c and η_s may assume nonzero values. To find them, especially η_s in which we are mostly interested, we do the following decoupling. Due to the small-

ness of η_c and η_s the constraint may be rewritten as

$$\eta_c^\dagger(\vec{q}) + \eta_c(-\vec{q}) \approx 0 \quad (43)$$

so that fields may effectively decouple, satisfying the constraint only approximately. As a result, from the first part in Eq. (35), by relaxing the constraint, we get a kinetic term for η_s . There are no other contributions to the second order in η_s . From the first part of Eq. (42) and in the spirit of the Bogoliubov expansion we may conclude that η_c is the field that couples to the external electromagnetic potential. In our decoupling ansatz η_c is only very weakly coupled. This coincides with the physical picture that we have for bosonic dipoles that (as dipoles) they weakly interact with external field and therefore as a system are incompressible.⁴

Applying the Bogoliubov expansion again, and neglecting the difference between n_0 and n ,

$$\rho_s \equiv \Psi_\uparrow^\dagger \Psi_\uparrow - \Psi_\downarrow^\dagger \Psi_\downarrow \approx \sqrt{n}(\eta_s^\dagger + \eta_s), \quad (44)$$

we are led to the following Hamiltonian for η_s fields,

$$\mathcal{H}_s = \sum_{\vec{q}} \frac{|\vec{q}|^2}{2m_c} \eta_s^\dagger(\vec{q}) \eta_s(\vec{q}) + \frac{n}{4} \sum_{\vec{q}} [\eta_s(-\vec{q}) + \eta_s^\dagger(\vec{q})] \times V_s(q) [\eta_s(\vec{q}) + \eta_s^\dagger(-\vec{q})]. \quad (45)$$

As before the hard core boson constraint makes the $V_s(0) = 2\pi e^2 d$ part of the interaction ineffective but leaves us with \mathcal{H}_s that describes an unstable system. Therefore we must impose separately the hard core constraint of composite bosons on fields η_s . That amounts locally to the following requirement,

$$\rho_s^2(r) = \Psi_\uparrow^\dagger \Psi_\uparrow + \Psi_\downarrow^\dagger \Psi_\downarrow, \quad (46)$$

where we used the hard boson properties $\Psi_\sigma^\dagger \Psi_\sigma \Psi_\sigma^\dagger \Psi_\sigma = \Psi_\sigma^\dagger \Psi_\sigma, \sigma = \uparrow$ and \downarrow , and $\Psi_\sigma^\dagger \Psi_\sigma \Psi_{-\sigma}^\dagger \Psi_{-\sigma} = 0$. Using the Bogoliubov expansion, Eq. (44), again this becomes

$$n[\eta_s^\dagger(r) + \eta_s(r)]^2 = n \quad (47)$$

that has to be imposed on η_s fields. Note that here we also used the incompressibility property of the system in the low-energy region for the charge degrees of freedom, on the rhs irrespective of the length scale [Eq. (42) Fourier transformed for any \vec{q}]. We had to make this assumption because we are incorporating a piece of short-range physics into the long-wavelength description. Please also note that this is an operator identity, where the automatic neglect of the quadratic terms on the lhs of the equation, in the Bogoliubov expansion, is not allowed.

The constraint we handle in the usual way, switching to the Lagrangian formulation with fields η_s, η_s^\dagger and a field λ that enforces the constraint.¹¹ The generating functional is

$$\mathcal{Z} = \int \mathcal{D}\eta_s \int \mathcal{D}\eta_s^\dagger \int \mathcal{D}\lambda \exp \left\{ - \int_0^\beta d\tau \int d^2x (\eta_s^\dagger \partial_\tau \eta_s + \mathcal{H}_s(x) + \{[\eta_s(x) + \eta_s^\dagger(x)]^2 - 1\} i\lambda(x, \tau)) \right\}, \quad (48)$$

where

$$\mathcal{H}_s(\vec{x}, \tau) = \frac{1}{2m_c} \vec{\nabla} \eta_s^\dagger \cdot \vec{\nabla} \eta_s + \frac{n}{4} \int d^2y [\eta_s(x) + \eta_s^\dagger(x)] V_s(\vec{x} - \vec{y}) \times [\eta_s(y) + \eta_s^\dagger(y)]. \quad (49)$$

The constraint approximately commutes with the Hamiltonian in the long-wavelength limit (using this property we combined contributions into a single exponential), and so we will take λ , τ (imaginary time) to be independent. Also, at the mean-field level, we are allowed to assume that λ is space independent.

Introducing Bogoliubov transformations on $\eta_s(\vec{q}, \tau)$ fields,

$$\eta_s(\vec{q}, \tau) = \alpha(\vec{q}) \exp\{i\omega_q \tau\} \cosh \theta_{\vec{q}} + \alpha^\dagger(-\vec{q}) \exp\{-i\omega_q \tau\} \sinh \theta_{\vec{q}} \quad (50)$$

[where $\alpha(\vec{q})$ and $\alpha^\dagger(\vec{q})$ are new canonical fields], we get after standard transformations that diagonalize the problem (see also Ref. 12), the following mean-field expression for \mathcal{Z} :

$$\mathcal{Z}_{mf} = \int d\lambda \prod_{\vec{q}} \frac{1}{1 - \exp\{-\beta \epsilon(\vec{q}, \lambda)\}} \exp\{-\beta E_0(\lambda)\}, \quad (51)$$

where

$$\epsilon(\vec{q}, \lambda) = \sqrt{(\epsilon_q^c)^2 + [nV_s(q) + 4i\lambda] \epsilon_q^c}, \quad (52)$$

with $\epsilon_q^c = q^2/2m_c$, and the domain of \vec{q} 's is again the disk with radius Q . Also

$$E_0(\lambda) = \frac{1}{2} \sum_q [\epsilon(\vec{q}, \lambda) - \epsilon_q^c] - i\lambda \sum_q \quad (53)$$

with the q summations where the cutoff Q is understood. The $\epsilon(\vec{q}, \lambda)$'s are the usual Bogoliubov energies, the results of the Bogoliubov transformation, now requiring also a suitable λ to get the final expression for the mode dispersion we are looking for. We approximate λ in the saddle point approach (see, for example, Ref. 13) searching for a stationary point of $F(T, \lambda)$, from the following expression for \mathcal{Z}_{mf} ,

$$\mathcal{Z}_{mf} = \int d\lambda \exp\{-\beta F(T, \lambda)\}, \quad (54)$$

i.e., look for the solution of

$$\frac{\partial F(T, \lambda)}{\partial \lambda} = 0, \quad (55)$$

which effectively becomes

$$\frac{\partial E_0(\lambda)}{\partial \lambda} = 0, \quad (56)$$

in the $T \rightarrow 0$ limit.

We solved the equation numerically finding only solutions λ_0 with $i\lambda_0$ real and positive (therefore, as usual,¹³ we found a path and a saddle point in the complex λ plane), and results are depicted in Fig. 1. In Fig. 1 we also plotted the Hartree-Fock result of Ref. 8, and the same reference exact diagonalization results at $d=0.5l_B$ and $d=1l_B$. To a good approximation we can claim a linear dependence for small d of the

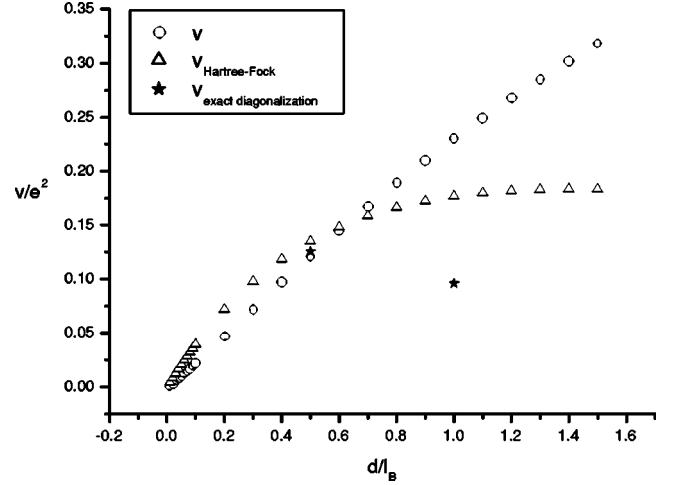


FIG. 1. The stationary point values for the velocity of the out-of-phase mode (○), the values from the pseudospin theory in the Hartree-Fock approximation from Ref. 8 (△), and exact diagonalization results (★) from Ref. 8.

Bogoliubov velocity though with the values significantly reduced from the Hartree-Fock results. But at $d=0.5l_B$ all three data points are very close to each other. For larger values of d , $d \sim 1l_B$ and larger, both approximation schemes fail to capture the quantum fluctuations that increase with d .⁸

The result for $\epsilon(q)$, the dispersion of the out-of-phase mode in the $d=0$ case,

$$\epsilon(q) = \epsilon_q^c = \frac{q^2}{2m_c}, \quad (57)$$

is fairly close to the estimate of Ref. 8 and 7. Namely $\epsilon(q) = e^2 l_B \frac{1}{6} \sqrt{2} q^2$, while the Hartree-Fock result is $e^2 l_B (\sqrt{\pi}/8) \sqrt{2} q^2$. It is interesting to note that if we use the expression for m_c conjectured in the generalized theory^{3,5} that includes higher-momentum physics, we exactly get the Hartree-Fock result.

It is also interesting to speculate about the discrepancy between our and the Hartree-Fock result for small d . Part of it might be due to our low-energy, low-momentum limited approach, but it might also well be due to the incompleteness of the underlying analogy^{6,14} of the $d \neq 0$ system description compared to the one of a repulsively interacting Bose gas. (The analogy of the $d=0$ case to an ideal Bose gas is complete, as we found out.) The incompleteness might follow from the modifications of the composite-boson picture due to the presence of composite fermions as proposed in Ref. 15. The composite fermions come into relevance very soon as d acquires a nonzero value, and their number rapidly increases with d .¹⁵ If we are allowed to view bosons and fermions to a first approximation as weakly interacting through a short-range interaction (more precisely here interacting are differences between up and down bosons and fermions, respectively) we can borrow considerations applied to the Bose-Fermi mixtures in optical traps; see, for example, Refs. 16 and 17. When we have a small number of fermions the Bogoliubov mode can, in fact, enhance its value (on the

other hand, decrease) and damping follows when fermions proliferate. In that sense we can expect that by considering fluctuations around our mean field solution (which maps the problem to an interacting Bose gas with a Bogoliubov mode), the value of the Bogoliubov velocity may increase for small d , stay almost the same for intermediate values, and decrease and even acquire damping for large d . In this sense here we have set up a formalism necessary to check these considerations in a future work.

It is important to address the case when we do not have the same density of particles in the up and down layer.^{18,19} Then in general we have, instead of Eq. (25),

$$\rho_s^{old} = \rho_s - (\nu_\uparrow - \nu_\downarrow) \frac{i}{2} \sum_{j,\sigma} (\vec{q} \times \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\}, \quad (58)$$

when, as before, we neglected the magnetoplasmon part. [ν_\uparrow and ν_\downarrow ($\nu_\uparrow + \nu_\downarrow = 1$) are the filling factors of each layer separately.] For $\rho_\sigma, \sigma = \uparrow, \downarrow$ in $\rho_s = \rho_\uparrow - \rho_\downarrow$ we assume the following form:

$$\begin{aligned} \rho_\sigma = & -\frac{i}{2} \sum_j (\vec{q} \times \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\} \\ & + C \sum_j (\vec{q} \cdot \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\}. \end{aligned} \quad (59)$$

The second part is the longitudinal component of the paramagnetic current, and the term should appear, in general, when compressible low-lying degrees of freedom are present. If C in Eq. (59) is the same for both layers then $\rho^c = \rho_\uparrow + \rho_\downarrow$, because the total component of the current is zero in the charge channel due to its incompressible nature. Substituting Eq. (58) with Eq. (59) in the projected Hamiltonian [Eq. (35)] and collecting all diagonal terms of the dipole expansion, we get

$$\frac{1}{m_{\delta\nu}} = \sum_{|\vec{q}| < Q} q^2 \sin^2(\theta_{\vec{q}, \vec{p}_{j,\sigma}}) \left\{ \frac{V_c}{2} - \frac{V_s}{2} \left(\frac{\nu_\uparrow - \nu_\downarrow}{2} \right)^2 \right\} \frac{1}{4} \left(\frac{1}{\nu_\uparrow} + \frac{1}{\nu_\downarrow} \right), \quad (60)$$

as a generalization of Eq. (39) to the case of imbalanced layers. This is the mass of the η_s field defined as $\eta_s = \sqrt{\nu_\uparrow} \eta_\uparrow - \sqrt{\nu_\downarrow} \eta_\downarrow$. So we assumed that we can apply the Bogoliubov theory and with neglect of some residual dipole-dipole interaction in the pseudospin channel, our problem reduces to the one expressed in Eq. (48) and (49) in which instead of the mass m_c , we have $m_{\delta\nu}$. The assumption is based on the expectation that the pseudospin channel is compressible. For not large $|\nu_\uparrow - \nu_\downarrow| \equiv \delta\nu$, the velocity of the Bogoliubov mode decreases quadratically with $\delta\nu$ as a consequence of Eq. (60), in agreement with Ref. 20. A more detailed investigation of the influence of the dipole-dipole interaction is needed for general $\delta\nu$.

We would like to address also the case of huge imbalance, when we take, for example, $\nu_\uparrow \gg \nu_\downarrow$.²¹ As $\nu_\downarrow \rightarrow 0$, $1/m_{\delta\nu} \rightarrow \infty$, and $\eta_s = \sqrt{\nu_\uparrow} \eta_\uparrow - \sqrt{\nu_\downarrow} \eta_\downarrow \approx \eta_\uparrow$, which probably signals the incompressible physics of the \uparrow layer. Therefore, to find out more about the physics of \downarrow quasiparticles, we must go back

to the beginning formulation, and apply a different decomposition. Namely we will take (in the limit $\nu_\uparrow \gg \nu_\downarrow$)

$$\rho_s^{old} \approx \rho_c^{old} - 2\rho_\downarrow = -i \sum_{j,\sigma} (\vec{q} \times \vec{p}_{j,\sigma}) \exp\{-i\vec{q} \cdot \vec{r}_{j,\sigma}\} - 2\rho_\downarrow. \quad (61)$$

It is easy to see that the first term in Eq. (61) would lead to the effective mass for all quasiparticles, in the first approximation, equivalent to what we would have if there was only one single layer with $\nu=1$. Next considering the cross term,

$$\sum_{q < Q} 2[-2\rho_\downarrow(-q)] \frac{V_s}{4} \rho_c^{old}(q), \quad (62)$$

and taking the expression in Eq. (59) for ρ_\downarrow , we get from Eq. (62) for the \downarrow quasiparticle mass,

$$\frac{1}{m_e} = \sum_{q < Q} V_E(\vec{q}) q^2 \sin^2 \theta_{\vec{q}, \vec{p}_{j,\sigma}}. \quad (63)$$

Because of the incompressible \uparrow background we can neglect \uparrow and \downarrow cross terms. If we again also assume irrelevance of the remaining \downarrow dipole-dipole interactions (for the low-momentum physics), our effective Hamiltonian for \downarrow quasiparticles is

$$\mathcal{H}_e = \sum_{q < Q} \frac{1}{2m_e} \Psi_\downarrow^\dagger \vec{p}^2 \Psi_\downarrow + \sum_{q < Q} \rho_\downarrow \frac{2V_s(q)}{2} \rho_\downarrow. \quad (64)$$

In Ref. 21 a physical picture of a Bose gas of excitons and dipoles with density n_\downarrow was developed for the case $\nu_\uparrow \gg \nu_\downarrow$. The mass m_e we derived is the result of the low-momentum theory. As in the $d=0$, $n_\uparrow = n_\downarrow$ case we expect that in the generalized theory^{3,5} the cutoff in Eq. (63) would be replaced by a Gaussian in the momentum space and the expression would coincide with the one in Ref. 21. In this sense with the assumption made, also in this case due to the comparison to Ref. 21, we can claim a complete analogy to a weakly interacting Bose gas in $d \rightarrow 0$ limit.

References 6, 8, and 22 and Ref. 23 (when not considering spiral states) agree on the dependence of the Bogoliubov velocity (in the $n_\uparrow = n_\downarrow$ case). Possible additions of quantum fluctuations to this value can be extracted from Ref. 24. There, due to the justified assumption of the suppression of charge fluctuations; a Schwinger boson mean field theory was used with the requirement on the single occupancy of the Schwinger boson in a lowest-Landau-level basis. In this work we were primarily concerned with the establishment of the concept of a composite boson, and we only set up the stage for considering fluctuations beyond generalized CS mean-field theory that is based on this concept. (A composite-boson approach may prove useful for the study of quantum phase transitions in the bilayer,^{15,25} and building of the physical picture of the bilayer in analogy with the picture based on composite fermions in the single layer.) Our mean-field theory and the usual theory do not agree somewhat, although they agree to a much better degree than the usual CS theory^{1,26} (linear dependence on small d and the absence of the bare mass). Inclusion of the fluctuations in our hard-core (belonging to different layer) CS boson model that is

probably related to the model with the single occupancy of Schwinger bosons and comparison to Ref. 24 are planned for future work. It would be important to probe the significance of the fluctuations around $d \sim l_B$. Any strong instability of the Bogoliubov mode velocity would signal, in the composite-boson–composite-fermion model (see above and Ref. 15), the phase separation of the two fluids²⁷ and the proposed first-order transition.^{28,29} Then, from the composite-boson

point of view, we would be able to address in more detail the extraordinary experiments done on the bilayer.^{30,31}

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