Fractional quantum Hall effect in bilayers and wide quantum wells at $\nu = 1/2$

Z. Papić [1,2]

[1] Institute of Physics,
Belgrade, Serbia

[2] Laboratoire de Physique des Solides,
Université Paris-Sud, France

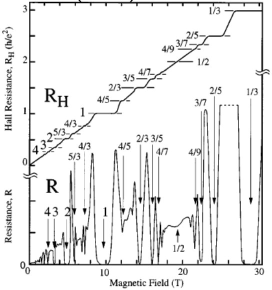
LPS Orsay, 25/06/2010

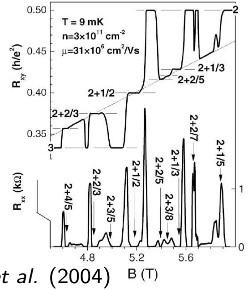
work done with M. Goerbig, M. Milovanović and N. Regnault

Outline

Trial wave functions for FQHE

H. Stormer (1999).





Xia *et al.* (2004)

- Laughlin $\nu = 1/m$, m-odd:
  $\Psi = \prod_{i<j}(z_i - z_j)^m$, $z = x - iy$,
  $l_B = \sqrt{\hbar c/eB}$
- Jain's CFs: $\nu = \nu^*/(2p\nu^* \pm 1)$, $\Psi_\nu = \mathcal{P}_{LLL} \prod_{i<j}(z_i - z_j)^{2p}\Phi_{\nu^*}$
- Composite Fermi liquid $\nu^* \to \infty$ i.e. $\nu = 1/2$:
  $\Psi_{CFL} = \mathcal{P}_{LLL} \det e^{i\mathbf{k}_i \mathbf{r}_j} \prod_{i<j}(z_i - z_j)^2$
- At $\nu = 2 + 1/2 = 5/2$ BCS instability of CFs: Moore-Read Pfaffian
  $\Psi_{\text{Pf}} = \text{Pf}(\frac{1}{z_i - z_j}) \prod_{i<j}(z_i - z_j)^2$

BCS effective description

Read and Green, PRB 61, 10267 (2000)

- near $\mathbf{k}=0$ and $\Delta_{\mathbf{k}} = \Delta(k_x - ik_y)$:
  $H_{\text{eff}} = \sum_{\mathbf{k}}\{(\epsilon_{\mathbf{k}} - \mu)c^\dagger_{\mathbf{k}}c_{\mathbf{k}} + (\Delta_{\mathbf{k}}c^\dagger_{\mathbf{k}}c^\dagger_{-\mathbf{k}} + h.c.)\}$
- Solutions:
  $E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2}, \Psi_0 = \prod_{\mathbf{k}}(u_{\mathbf{k}} + v_{\mathbf{k}}c^\dagger_{\mathbf{k}}c^\dagger_{-\mathbf{k}})|vacc\rangle$
- strong pairing phase: $\epsilon_{\mathbf{k}} - \mu > 0, |u_{\mathbf{k}}| \rightarrow 1, |v_{\mathbf{k}}| \rightarrow 0$
- weak pairing phase: $\epsilon_{\mathbf{k}} - \mu < 0, |u_{\mathbf{k}}| \rightarrow 0, |v_{\mathbf{k}}| \rightarrow 1$
- in the weak pairing phase, $g_{\mathbf{k}} = v_{\mathbf{k}}/u_{\mathbf{k}} \rightarrow$ asymptotically $g(\mathbf{r}) \propto 1/z$ (Moore-Read state)
- Supp. $\mu > 0$ and large, $\Delta \approx \text{const} \rightarrow \mathrm{E}_{\mathbf{k}} \approx \mu - \epsilon_{\mathbf{k}} + \frac{|\Delta_{\mathbf{k}}|^2}{2\mu}$
- Excitations are unstable around $\mathbf{k}=0 \rightarrow$ minimum moves to $\mathbf{k} = \mathbf{k}_F$ (CF liquid)
- but Ψ_0 still describes an unstable point (excited state of CFL)

Can we engineer non-Abelian states?

- using 3-body, 4-body etc. interaction (coupling with excited LLs)
- try with Zhang-Das Sarma finite thickness interaction $V_{ZDS} = 1/\sqrt{r^2 + w^2}$

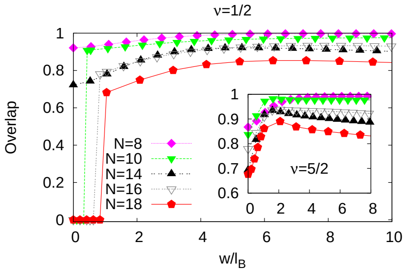


Figure: $\langle \Psi_{\mathrm{Pfaffian}} | \Psi_{\mathrm{exact}} \rangle$

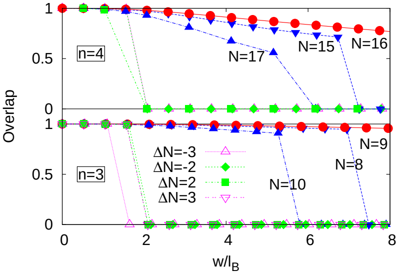
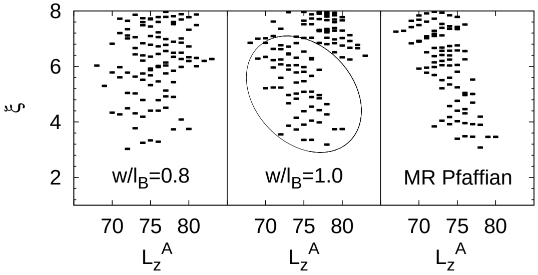
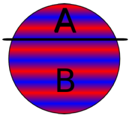


Figure: $\langle \Psi_{\mathrm{exact}}(w=0) | \Psi_{\mathrm{exact}}(w) \rangle$: Destruction of the CF sea

[Z.Papić *et al.*, Phys. Rev. B **80**, 201303 (2009)]

- look at one ground state $|\Psi\rangle$ on the sphere
- cut the system into two parts, A and B, in orbital space ($\sim$ real space, geometrical partition)
- reduced density matrix $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$
- *entanglement spectrum* [Li & Haldane] : plot $\xi = -\log \lambda_A$ vs L_A^z for fixed cut and N_A

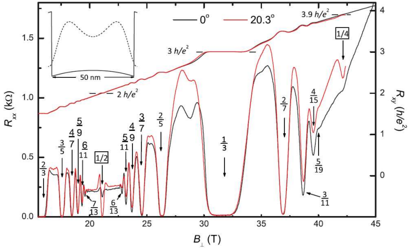


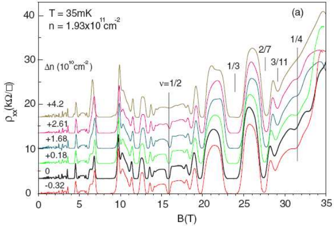


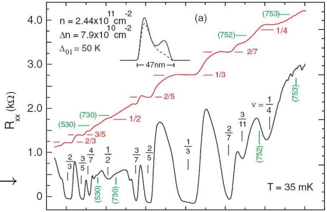
Multicomponent and non-Abelian states in LLL

Halperin states

$$\Psi_{mm'n} = \prod_{i<j}(z_{i\uparrow} - z_{j\uparrow})^m \prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})^{m'} \prod_{p,q}(z_{p\uparrow} - z_{q\downarrow})^n$$







Luhman *et al.*,PRL 101, 266804 (2008)

Shabani *et al.*,PRL 103, 256802 (2009) ⟶

BCS description for the bilayer $\nu = 1/2$

- Coulomb bilayer (d – distance between layers)
  $H = -\Delta_{SAS} S_x + \sum_{i<j,\sigma\in\uparrow,\downarrow} \frac{e^2}{\epsilon|\mathbf{r}_{i\sigma}-\mathbf{r}_{j\sigma}|} + \sum_{i,j} \frac{e^2}{\epsilon\sqrt{(\mathbf{r}_{i\uparrow}-\mathbf{r}_{j\downarrow})^2+d^2}}$
- effective BCS description for neutral fermions, $\tilde{\epsilon}_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$,

$$H = \sum_{\mathbf{k}} \tilde{\epsilon}_{\mathbf{k}}(c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k}\uparrow} + \uparrow\leftrightarrow\downarrow) + (\Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}) - \frac{\Delta_{SAS}}{2}(c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k}\downarrow} + \uparrow\leftrightarrow\downarrow)$$

- H can be split into an *even* and *odd* channel ($\uparrow \pm \downarrow$)
- $\mu^e = \mu + \Delta_{SAS}/2, \mu^o = \mu - \Delta_{SAS}/2$
- in the even channel: $|\psi_{\text{BCS}}\rangle = \prod_{\mathbf{k}}(1 + g_{\mathbf{k}} c^\dagger_{\mathbf{k},e}c^\dagger_{-\mathbf{k},e})|\text{vacuum}\rangle$
- but in the interacting system $\mu^{\text{eff}} = P\mu^e + (1-P)\mu^o \rightarrow$ what is P?
- if μ^e is very large, unstable excitations $\rightarrow$ CFL liquid

Exact diagonalization: Sphere ($d = l_B$)

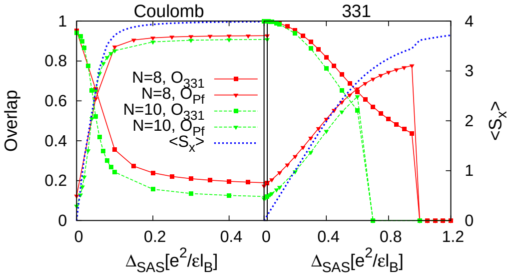


Figure: Overlaps, $d = l_B, \delta = -3$

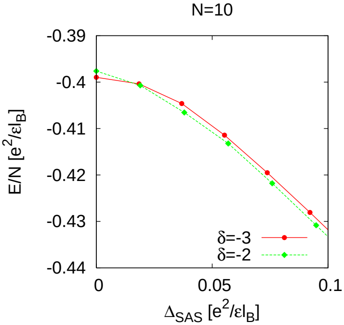


Figure: GS energies, $d = l_B$

[Z.Papić *et al.*, arXiv:0912.3103]

Exact diagonalization: Torus

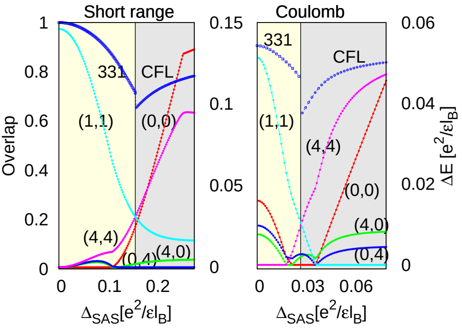


Figure: $N = 8,\ d = l_B$

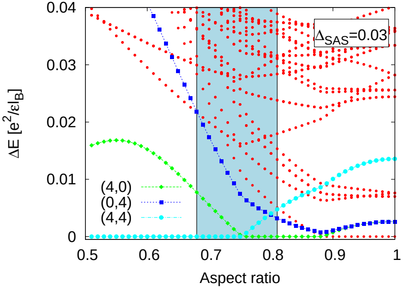
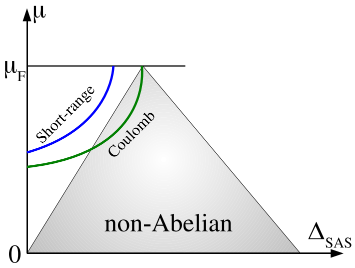


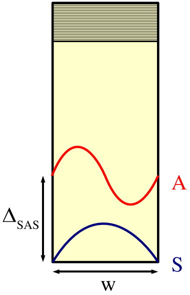
Figure: $\Delta_{SAS} = 0.03e^2/\epsilon l_B$

[Z.Papić *et al.*, arXiv:0912.3103]

Bilayer results: phase diagram



Model of the wide quantum well



- $H = -\Delta_{SAS} S_z - \Delta\rho S_x + V^{\text{Coulomb}}$
- Calculate overlaps with trial states
- Mean values
  - $\langle S_x \rangle$ – creates charge imbalance in the well
  - $\langle S_z \rangle$ – Zeeman field
- At $\nu = 1/2$ we expect transition between 331 and Pfaffian as the density is made more asymmetric

[Z.Papić *et al.*, PRB 79, 245325 (2009).]

Phase diagram $\nu = 1/2, w = 4l_B$ (Sphere)

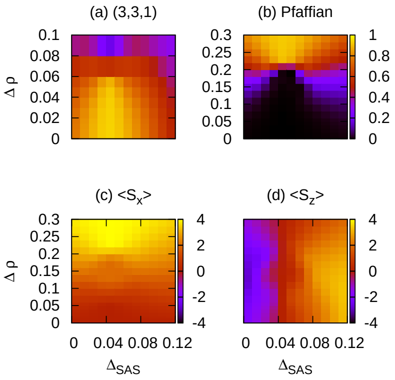


Figure: LLL

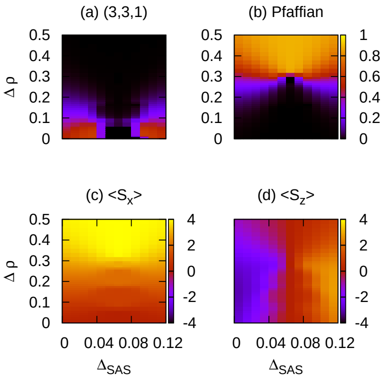


Figure: 2nd LL

[Z.Papić (unpublished)]

Neutral gap

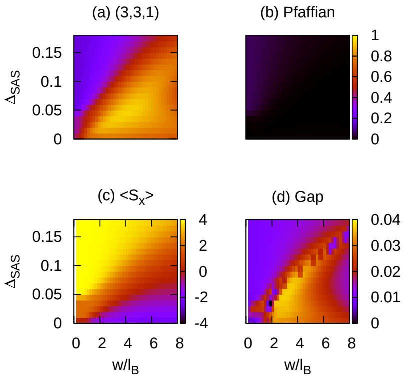


Figure: $\Delta\rho = 0$

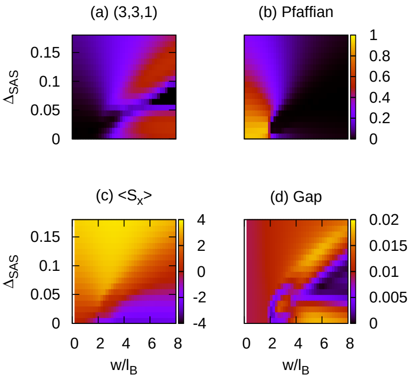


Figure: $\Delta\rho = 0.1e^2/\epsilon l_B$

[Z.Papić (unpublished)]

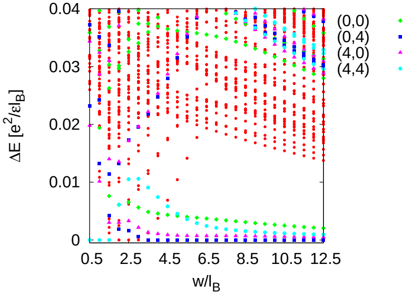


Figure: Spectrum as a function of w ($\Delta_{SAS} = \Delta\rho = 0$)

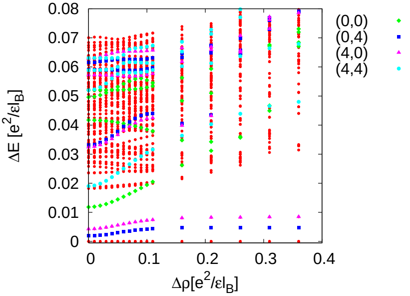


Figure: Effect of $\Delta\rho$ ($\Delta_{SAS} = 0.004e^2/\epsilon l_B, w = 4l_B$)

[Z.Papić (unpublished)]

How to create the Pfaffian in the LLL?

- two spin species, $\Delta_{\mathbf{k}}^{\uparrow\downarrow}$ pairing and a constraint $\lambda(\mathbf{r})\Psi_{\rm odd}^{\dagger}(\mathbf{r})\Psi_{\rm odd}(\mathbf{r}) = \lambda(\mathbf{r})\left[\Psi_{\uparrow}^{\dagger}(\mathbf{r}) - \Psi_{\downarrow}^{\dagger}(\mathbf{r})\right]\left[\Psi_{\uparrow}(\mathbf{r}) - \Psi_{\downarrow}(\mathbf{r})\right]$
- λ bosonic multiplier, similar to Δ_{SAS}
- Only difference from tunneling Hamiltonian: $\mu \to \mu^t = \mu - \lambda$
- diagonalize H using Bogoliubov, again obtain two channels:

$$\tilde{\mu}^e = \mu \text{ and } \tilde{\mu}^o = \mu - 2\lambda$$

- stationary point $\frac{\partial H}{\partial \lambda} = 0 \to \lambda \to \infty$
- constant $\tilde{\mu}^e = \mu$ (avoids CFL), strong coupling in odd channel $=$ Pfaffian for $\lambda \to \infty$

Physical condition

- We seek a Hamiltonian which has the property $\delta\mu^t = -\lambda$
- $\rightarrow \frac{\partial\Omega}{\partial\lambda} = N$
- Partial differential equation for the variation of density (k_F) with tunneling λ (Δ_{SAS})
- large tunneling limit, $\Delta_{SAS} \gg \hbar^2 k_F^2/2m^*$: $\frac{\partial N(k_F)}{\partial \Delta_{SAS}} \propto -N(k_F)$

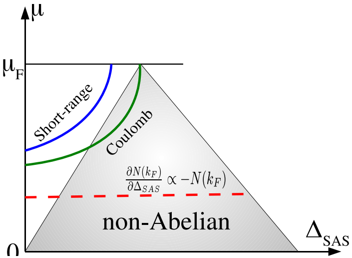


Figure: 331-Pf transition, $\Delta^{\uparrow\downarrow} \neq 0, \Delta^{\uparrow\uparrow} = 0$

Summary

- Pfaffian can arise as an excited state of the CFL which becomes the ground state in the systems with sufficiently softened Coulomb interaction or it can be a phase with a small gap
- in bilayers and wide quantum wells, there is possibility for a critical Moore-Read phase
- we can have a transition from a compressible state (with some p-wave pairing) to the Moore-Read Pfaffian
- if we want to generate the Pfaffian from the 331 state, the total density should be reduced as Δ_{SAS} is increased

Possibility for the "critical" Pfaffian

- if we add $\Delta_{\mathbf{k}}^{\uparrow\uparrow}$ pairing $\rightarrow E_e = \sqrt{(\tilde{\epsilon}_{\mathbf{k}} - \Delta_{SAS}/2)^2 + |\Delta_{\mathbf{k}}^{\uparrow\downarrow} + 2\Delta_{\mathbf{k}}^{\uparrow\uparrow}|^2}$, $E_o = \sqrt{(\tilde{\epsilon}_{\mathbf{k}} + \Delta_{SAS}/2)^2 + |\Delta_{\mathbf{k}}^{\uparrow\downarrow} - 2\Delta_{\mathbf{k}}^{\uparrow\uparrow}|^2}$
- odd channel can be gapless and we can have a CFL - Pfaffian transition with the Pfaffian having a bigger gap
- more likely to have $\Delta^{\uparrow\uparrow} \neq 0$ in the 2nd LL

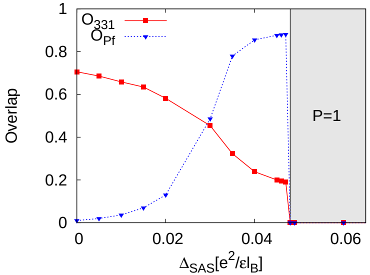


Figure: Possibility for the critical Pfaffian before complete polarization ($d = 0.4 l_B$)