Nonlinear BEC Dynamics Induced by Harmonic Modulation of Atomic *s*-wave Scattering Length

Ivana Vidanović[1], Antun Balaž[1], Hamid Al-Jibbouri[2], and Axel Pelster[3]

[1] Scientific Computing Laboratory, Institute of Physics Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

[2] Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany

[3] Fachbereich Physik, Universität Duisburg-Essen, Lotharstrasse 1, 47048 Duisburg, Germany

Motivation: In the recent experiment [1], a harmonic modulation of the atomic *s*-wave scattering length induced a nonlinear dynamics of a ^{7}Li BEC, and the resulting resonance curve for the excited quadrupole mode was measured. By combining a perturbative calculation with a numerical approach for solving the underlying Gross-Pitaevskii equation, we study in detail the frequency shift of collective BEC modes which arises due to nonlinear interaction effects [2].

BEC dynamics

⋆ At zero temperature, BEC can be described by the time-dependent GP equation

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\Delta+V(\vec{r})+g(t)|\psi(\vec{r},t))|^2\right]\psi(\vec{r},t),$$

where $V(\vec{r})=\frac{1}{2}m\omega_\rho^2(\rho^2+\lambda^2z^2)$ is a trap with anisotropy λ, $g(t)=\frac{4\pi\hbar^2Na(t)}{m}$ is a nonlinear interaction defined by the *s*-wave scattering length $a(t)$ and number of atoms N.

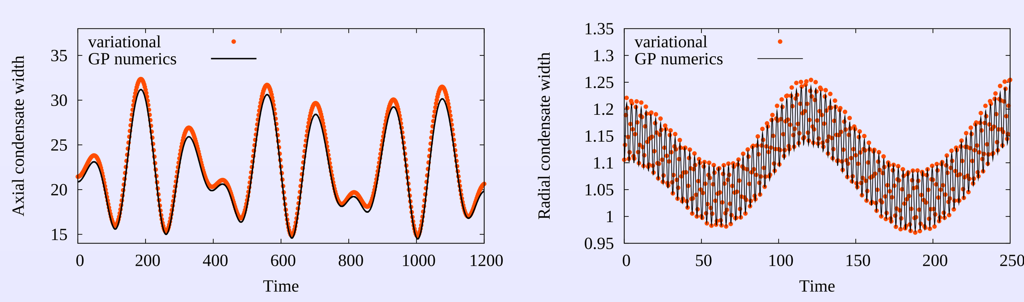
⋆ GP equation can be studied using a Gaussian variational ansatz [3], yielding

$$\frac{d^2u_\rho(t)}{dt^2}+u_\rho(t)-\frac{1}{u_\rho(t)^3}-\frac{p(t)}{u_\rho(t)^3u_z(t)}=0\,,$$
$$\frac{d^2u_z(t)}{dt^2}+\lambda^2u_z(t)-\frac{1}{u_z(t)^3}-\frac{p(t)}{u_\rho(t)^2u_z(t)^2}=0\,,$$

where $u_\rho(t)$ and $u_z(t)$ are condensate widths, $p(t)=\sqrt{\frac{2}{\pi}}\frac{Na(t)}{l}$ and $l=\sqrt{\frac{\hbar}{m\omega}}$.

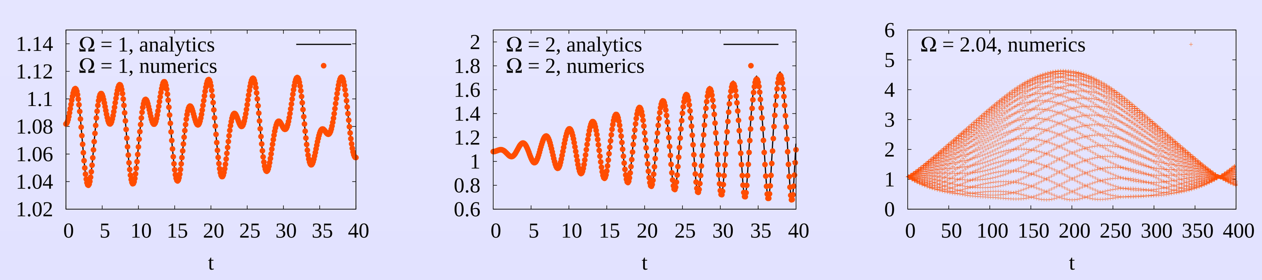
⋆ Using Feshbach resonances, scattering length was harmonically modulated [1], yielding the time-dependent interaction $p(t)=p+q\cos\Omega t$.

⋆ Real-time dynamics for $p=15$, $q=10$, $\lambda=0.021$ and $\Omega=0.05$:



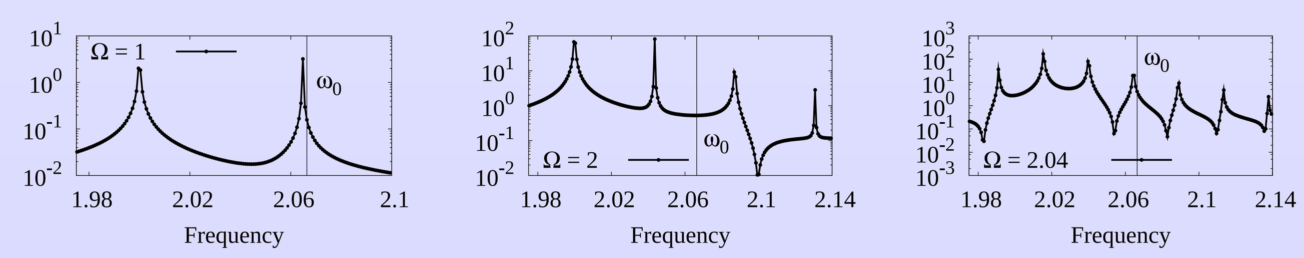
Excitation spectra

⋆ Condensate dynamics depends strongly on the value of Ω, (p=0.4, $q=0.2$, $\lambda=1$):



⋆ From the linear stability analysis we find equilibrium size u_0 via $u_0-\frac{p}{u_0^4}-\frac{1}{u_0^3}=0$ and collective oscillation mode $\omega_0=\sqrt{1+\frac{3}{u_0^4}+\frac{4p}{u_0^5}}$. For $p=0.4$: $u_0=1.08183$, $\omega_0=2.06638$.

⋆ In the corresponding Fourier spectra we observe nonlinear features - higher harmonics generation, nonlinear mode coupling and frequency shifts:



Poincaré-Lindstedt analysis

⋆ Linearization of the variational equation for vanishing driving $q=0$ yields zeroth order collective mode $\omega=\omega_0$ of oscillations around the time-independent solution u_0. To calculate the collective mode to higher orders, we rescale time as $s=\omega t$ [4]:

$$\omega^2\ddot{u}(s)+u(s)-\frac{1}{u(s)^3}-\frac{p}{u(s)^4}-\frac{q}{u(s)^4}\cos\frac{\Omega s}{\omega}=0\,.$$

⋆ Far from resonances, we assume perturbative expansions in q:

$$u(s)=u_0+q\,u_1(s)+q^2\,u_2(s)+q^3\,u_3(s)+\dots\,,$$
$$\omega=\omega_0+q\,\omega_1+q^2\,\omega_2+q^3\,\omega_3+\dots$$

⋆ This leads to a hierarchical system of equations in orders of q [4]. To the 3$^{\rm rd}$ order:

$$\omega_0^2\,\ddot{u}_1(s)+\omega_0^2\,u_1(s)=\frac{1}{u_0^4}\cos\frac{\Omega s}{\omega}\,,$$
$$\omega_0^2\,\ddot{u}_2(s)+\omega_0^2\,u_2(s)=-2\omega_0\,\omega_1\,\ddot{u}_1(s)-\frac{4}{u_0^5}\,u_1(s)\,\cos\frac{\Omega s}{\omega}+\alpha\,u_1(s)^2\,,$$
$$\omega_0^2\,\ddot{u}_3(s)+\omega_0^2\,u_3(s)=-2\omega_0\,\omega_2\,\ddot{u}_1(s)-2\beta\,u_1(s)^3+2\alpha\,u_1(s)u_2(s)-\omega_1^2\,\ddot{u}_1(s)+\frac{10}{u_0^6}\,u_1(s)^2\,\cos\frac{\Omega s}{\omega}-\frac{4}{u_0^5}\,u_2(s)\,\cos\frac{\Omega s}{\omega}-2\omega_0\,\omega_1\,\ddot{u}_2(s),$$

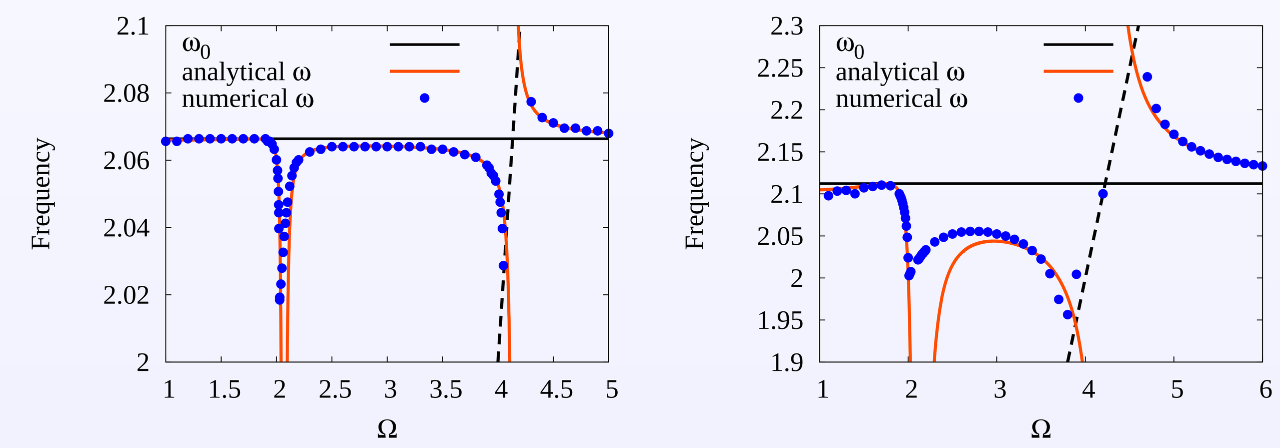
where $\alpha=10p/u_0^6+6/u_0^5$ and $\beta=10p/u_0^7+5/u_0^6$.

Frequency shift of collective modes - spherically sym. BEC

⋆ Frequency shift of the breathing mode is obtained by imposing the cancellation of secular terms according to the Poincaré-Lindstedt method. Up to third order in q it turns out that the 1$^{\rm st}$ order correction ω_1 vanishes, leading to a shift quadratic in q:

$$\omega=\omega_0+q^2\,\frac{\text{Polynomial}(\Omega)}{(\Omega^2-\omega_0^2)^2\,(\Omega^2-4\omega_0^2)}+\dots$$

⋆ Good agreement of numerical and analytical results is obtained for the frequency shift far from resonances. On the left plot $p=0.4$, $q=0.1$, on the right plot $p=1$, $q=0.8$:

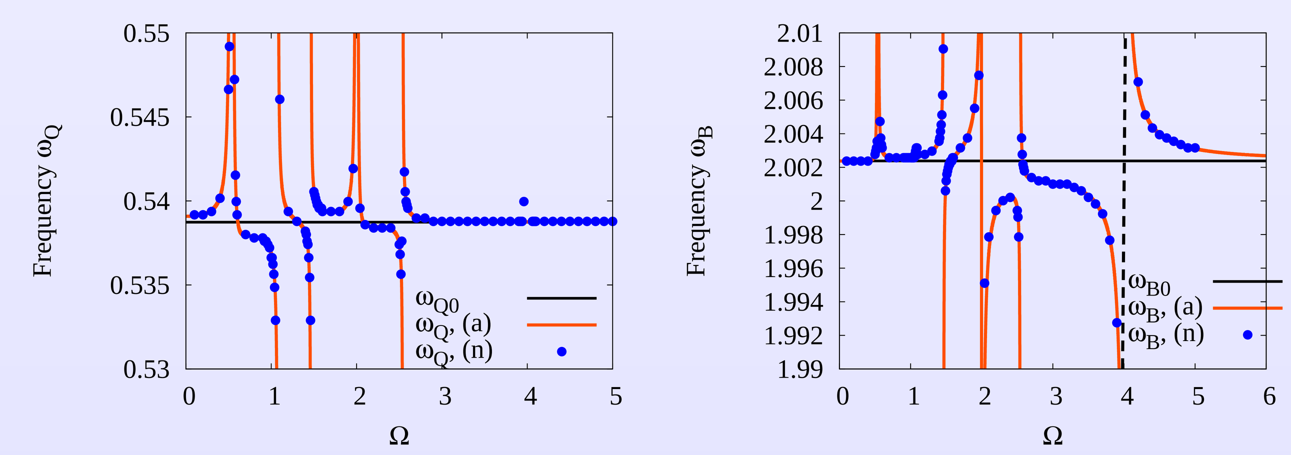


⋆ Frequency shifts of up to 10% are found for large modulation amplitudes.

Frequency shift of collective modes - axially sym. BEC

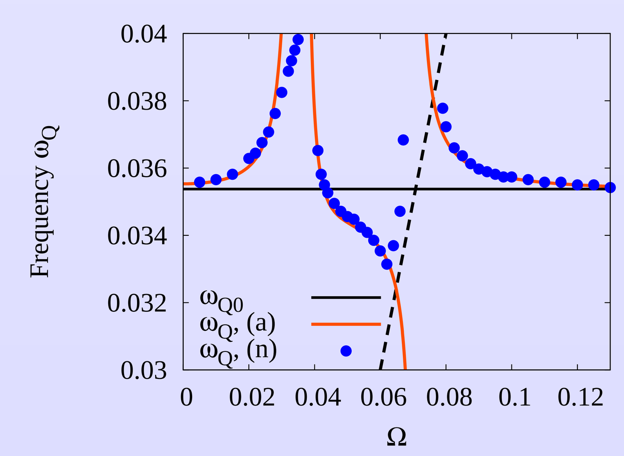
⋆ Harmonic modulation of interaction leads to the simultaneous excitation of quadrupole (ω_Q) and breathing (ω_B) mode, and their coupling.

⋆ Good agreement of numerical and analytical results for the frequency shift of quadrupole and breathing mode is obtained far from resonances, $p=1$, $q=0.2$ and $\lambda=0.3$:



⋆ Analytical solution in the 2$^{\rm nd}$ order in q exhibits poles; for the quadrupole mode poles are at ω_{Q0}, $2\omega_{Q0}$, $\omega_{B0}-\omega_{Q0}$, $\omega_{Q0}+\omega_{B0}$ and ω_{B0} , while for the breathing mode positions of poles are at ω_{Q0}, ω_{B0}, $2\omega_{B0}$, $\omega_{B0}-\omega_{Q0}$ and $\omega_{Q0}+\omega_{B0}$.

⋆ For the experimental setup [1]:
$p=15$, $q=10$, $\lambda=0.021$,
$\omega_{Q0}=0.035375$, $\omega_{B0}=2.00002$,
$\omega_{Q0}<<\omega_{B0}$, $\Omega\in(0,3\omega_{Q0})$;
strong excitation of quadrupole mode and significant excitation of breathing mode; frequency shift of about 10%.



Summary and outlook

⋆ Using numerical Fourier analysis and analytical Poincaré-Lindstedt method, we calculated the frequency shift of collective modes for a spherically and axially symmetric BEC excited by harmonic modulation of the scattering length.

⋆ To extend applicability of our analytical approach, perturbative expansion to higher order has to be performed, or some kind of resummation of perturbative series could be applied.

⋆ Clear experimental verification of such nonlinearity-induced frequency shifts may be possible using trap geometry with higher λ then the one used in Ref. [1].

• **References**

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• **Support:** Serbian Ministry of Science (ON171017, NAD-BEC), DAAD - German Academic and Exchange Service (NAD-BEC), and European Commission (EGI-InSPIRE, PRACE-1IP and HP-SEE).

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